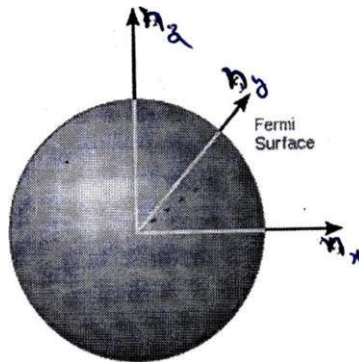




DENSITY OF STATES



The number of states with energy less than E_f is equal to the number of states that lie within a sphere of radius $|n_f|$ in a region of K-space where n_x, n_y and n_z are positive.

$$\therefore N = 2 \times \frac{1}{8} \times \frac{4}{3} \pi n_f^3$$

$$\therefore N = 2 \times \frac{1}{8} \times \frac{4}{3} \pi n_f^3 = \frac{3N}{\pi} \Rightarrow \boxed{n_f = \left(\frac{3N}{\pi}\right)^{\frac{1}{3}}}$$

So the Fermi energy

$$E_f = \frac{\hbar^2 \pi^2 n_f^2}{2m} = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3N}{\pi}\right)^{\frac{2}{3}}$$

$$E_f = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3N}{\pi}\right)^{\frac{2}{3}} = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3N}{\pi}\right)^{\frac{2}{3}} = \frac{\hbar^2}{2m} \left(\frac{3N\pi^2}{V}\right)^{\frac{2}{3}}$$

$$\therefore N^{\frac{2}{3}} = \frac{2m}{\hbar^2} \left(\frac{V}{3\pi^2}\right)^{\frac{2}{3}} E_f^{\frac{3}{2}}$$

$$\Rightarrow N = \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \left(\frac{V}{3\pi^2}\right) E_f^{\frac{3}{2}}$$

Therefore density of states: $D(E) = \frac{dN}{dE} = \frac{3}{2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \left(\frac{V}{3\pi^2}\right) E_f^{\frac{1}{2}}$

$$D(E) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E_f^{\frac{1}{2}}$$

Therefore the total number of energy states per unit volume per unit energy range

$$Z(E) = \frac{D(E)}{V} = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E_f^{\frac{1}{2}} = \frac{1}{2\pi^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} 8\pi^3 E_f^{\frac{1}{2}}$$

$$Z(E) = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E_f^{\frac{1}{2}}$$

Therefore the number of energy states in the energy interval E and $E + dE$ are

$$\boxed{Z(E)dE = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} E_f^{\frac{1}{2}} dE}$$



Important questions

1. a. Explain the salient features of classical free electron theory
- b. On the basis of classical free electron theory, derive the expressions for i) drift Velocity, ii) current density iii) mobility?
- c. What are drawbacks of classical free electron theory of materials?
2. a. Explain Fermi-Dirac distribution for electrons in a metal. Discuss its variation with temperature?
- b. Explain the terms 'Mean free path' 'Relaxation time' and 'Drift velocity' of an electron in a metal?
- c. Discuss the origin of electrical resistance in metals?
3. a. Derive the expression for electrical conductivity on the basis of quantum free electron theory?
- b. Explain i) Fermi energy?
- c. Evaluate the Fermi function for an energy KT above Fermi energy?