SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT - V

SECOND ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS

Second order linear differential equation with Constant Coefficients

Consider a second order linear differential equation is,

$$(a_0 D^2 + a_1 D + a_2) y = R(x)$$

To find complementary function :

The auxiliary equation is,

$$a_0 m^2 + a_1 m + a_2 = 0.$$

Nature of roots	Complementary function (C.F)
① m_1 & m_2 are real and different	$C.F = Ae^{m_1 x} + Be^{m_2 x}$
② m_1 and m_2 are real and equal	$C.F = (Ax+B) e^{mx}$
③ m_1 and m_2 are Complex, let $m_1 = \alpha + i\beta$ $m_2 = \alpha - i\beta$	$C.F = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$

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Problems to find Complementary function :

① Solve : $(D^2 - 5D + 6)y = 0$

Soln: The A.E is

$$m^2 - 5m + 6 = 0$$

$$m = 2, 3$$

The roots are real & different.

$$C.F = Ae^{m_1 x} + Be^{m_2 x}$$

$$C.F = Ae^{2x} + Be^{3x}$$

$$\therefore \boxed{y = Ae^{2x} + Be^{3x}}$$

② Solve: $\dfrac{d^2 y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 0$

Soln:

Given : $(D^2 - 6D + 9)y = 0$

The A.E is

$$m^2 - 6m + 9 = 0$$

$$m = 3, 3$$

The roots are real and equal.

$$C.F = (Ax + B)e^{mx}$$

$$= (Ax + B)e^{3x}$$

$$\therefore \boxed{y = (Ax + B)e^{3x}}$$

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Rules to find particular integral (P.I) :

$$P.I = \frac{1}{f(D)} R(x)$$

Failure Case : When $f(D) = 0$.

$$\Rightarrow \quad P.I = \frac{x}{f'(D)} R(x)$$

Again failure, When $f'(D) = 0$

$$P.I = \frac{x^2}{f''(D)} R(x)$$

and so on.

Type I : $R(x) = e^{ax}$

Replace $D \rightarrow a$

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① Solve $(D^2 + 1)y = e^{-x}$

Soln: The A.E is $m^2 + 1 = 0 \Rightarrow m^2 = -1$

$$m = \pm i = 0 \pm i$$

$$\alpha = 0, \ \beta = 1$$

$$C.F = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$$

$$= e^{0x}(A \cos x + B \sin x)$$

$$C.F = A \cos x + B \sin x$$

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$$P.I = \frac{e^{-x}}{D^2+1}$$

$e = e$

Here $a = -1$

$D \to a \to -1$

$$= \frac{e^{-x}}{(-1)^2+1} = \frac{e^{-x}}{2}$$

$$y = C.F + P.I$$

$$\boxed{y = A\cos x + B\sin x + \frac{e^{-x}}{2}}$$

② Solve: $(D^2 + 4D + 4)y = 11e^{-2x}$

Soln: The A.E is

$$m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0$$

$$m = -2, -2$$

The roots are real and equal.

$$C.F = (Ax+B)e^{mx}$$

$$C.F = (Ax+B)e^{-2x}$$

$$P.I = \frac{11e^{-2x}}{D^2+4D+4}$$

$e^{ax} = e^{-2x}$

$a = -2$

$D \to a \to -2$

$$= \frac{11e^{-2x}}{(-2)^2+4(-2)+4}$$

$$= \frac{11e^{-2x}}{4-8+4} = \frac{11e^{-2x}}{0} \quad (\text{failure case})$$

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$$= \frac{11 \, x \, e^{-2x}}{2D+4}$$

$$= \frac{11 \, x \, e^{-2x}}{2(-2)+4} = \frac{11 \, x \, e^{-2x}}{-4+4} = \frac{11 \, x \, e^{-2x}}{0} \quad (failure)$$

$$P.I = \frac{11 \, x^2 \, e^{-2x}}{2}$$

$$\therefore y = c.F + P.I$$

$$\boxed{y = (Ax+B)e^{-2x} + \frac{11 \, x^2 \, e^{-2x}}{2}}$$

(3) Solve : $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = 4$

Soln:

$$(D^2 + 4D + 3)y = 4$$

The A.E is,

$$m^2 + 4m + 3 = 0$$

$$m = -1, -3$$

$$C.F = Ae^{-x} + Be^{-3x}$$

$$P.I = \frac{4}{D^2+4D+3}$$

$$= \frac{4 \cdot e^{0x}}{D^2+4D+3} = \frac{4e^{0x}}{3}$$

$e^{0x} = e^{ax}$

$a = 0$

$D \to a \to 0$

$$P.I = 4/3$$

$$y = c.F + P.I$$

$$\boxed{y = Ae^{-x} + Be^{-3x} + 4/3}$$