Method of variation of parameters :

The Second order linear differential equation is,

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Step 1 : Find Complimentary function. From this calculate y_1 and y_2 (coefficient of constant)

Step 2 : Find Wronskian

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$$P.I = Py_1 + Qy_2$$

where $P = -\int \frac{R}{W} y_2 \, dx$

$$Q = \int \frac{R}{W} y_1 \, dx$$

1. Find Wronskian for the following :

(i) $\frac{d^2y}{dx^2} + 4y = \operatorname{Cosec} x$

(ii) $y'' + y = \sec^2 x$

(iii) $(D^2 + 16)y = \sec 4x$

Soln :

(i) $\frac{d^2y}{dx^2} + 4y = \operatorname{cosec} x$

The A.E is

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i = 0 \pm 2i$$

$$\alpha = 0, \ \beta = 2$$

$$C.F = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$$

$$= e^{0x}(A \cos 2x + B \sin 2x)$$

$$C.F = A \cos 2x + B \sin 2x$$

$$y_1 = \cos 2x, \quad y_2 = \sin 2x$$

$$y_1' = -\sin 2x \, (2) \quad ; \quad y_2' = \cos 2x \cdot 2$$

$$= -2 \sin 2x \qquad\qquad = 2 \cos 2x$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2(\cos^2 2x + \sin^2 2x)$$

$$\boxed{W = 2}$$