



DEPARTMENT OF MATHEMATICS

Consider,

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = f(x)$$

→ ①

where $a_0, a_1, a_2, \dots, a_n$ are constants and $f(x)$ is a function of x is called a homogeneous linear differential equation of order n with variable coefficients.

For this type, substitute

$$z = \log x \quad (\text{or}) \quad x = e^z$$

Then $x \frac{dy}{dx} = D'y$

$$x^2 \frac{d^2 y}{dx^2} = D'(D'-1)y$$



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DEPARTMENT OF MATHEMATICS

$$x^3 \frac{d^3 y}{dx^3} = D'(D'-1)(D'-2)y \text{ and so on}$$

Problems:

- ① Transform the equation to constant coefficient equations:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$$

Soln:

Replace $\log x = z$

$$x \frac{dy}{dx} = D'y$$

$$x^2 \frac{d^2 y}{dx^2} = D'(D'-1)y$$

Given equation becomes,

$$D'(D'-1)y + D'y + y = z \sin z$$

$$(D'^2 - D')y + D'y + y = z \sin z$$

$$(D'^2 - \cancel{D'} + \cancel{D'} + 1)y = z \sin z$$

$$\boxed{(D'^2 + 1)y = z \sin z}$$