UNIT – 2, STANDARD DISTRIBUTIONS

PART-A (2 marks)

1. **Determine the binomial distribution whose mean is 9 and whose standard deviation is $\frac{3}{2}$.**

Answer:

np = 9 and npq = $\frac{9}{4}$. q = $\frac{npq}{np} = \frac{1}{4} \Rightarrow p = 1 - q = \frac{3}{4}$

np = 9 $\Rightarrow$ n= $9 \times \frac{4}{3} = 12$

$\therefore$P[X=r] = 12 $C_r \cdot \left[\frac{3}{4}\right]^r \left[\frac{1}{4}\right]^{12-r}, r = 0,1,2, \ldots..12$

2. **A die is thrown 3 times. If getting a '6' is considered a success, find the probability of atleast two successes.**

Answer:

P=1/6; q= 5/6; n=3.

P[atleast two successes] = P(2) + P(3)

$$= 3C_2 \cdot \left[\frac{1}{6}\right]^2 \frac{5}{6} + 3C_3 \cdot \left[\frac{1}{6}\right]^3 = \frac{2}{27}$$

3. **Find the MGF of binomial distribution.**

Answer:

$M_x(t) = \sum_{r=0}^{n} nC_r \cdot (pe^t)^r \cdot q^{n-r}$

$= (q + pe^t)^n$

4. **For a random variable X, $M_x(t) = \frac{1}{81}(e^t + 2)^4$, find P[X$\leq 2$].**

Answer:

$$M_x(t) = \left(\frac{2}{3} + \frac{1}{3}e^t\right)^4.$$

For Binomial distribution, $M_x(t) = (q + pe^t)$

∴ n=4, q=2/3, p=1/3

∴ P[X≤2] = P(0) + P(1) + P(2)

$$= \left(\frac{2}{3}\right)^4 + 4C_1\frac{1}{3}\left(\frac{2}{3}\right)^3 + 4C_1\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$$

$$= \frac{1}{81}[16 + 32 + 24] = \frac{72}{81}$$

= 0.8889

5. **The mean and variance of a binomial variance are 4 and 4/3 respectively, find P [X≥ 1] .**

Answer:

np = 4, npq = $\frac{4}{3} \Rightarrow q = \frac{1}{3}$ and $p = \frac{2}{3}$ $\therefore n = 4 \times \frac{3}{2} = 6.$

P[X≥ 1] = 1- P[X< 1] = 1 -P[X= 0]

= 1- $\left(\frac{1}{3}\right)^6$ = 0.9986

6. **If 6 of 18 new buildings in a city violate the building code, what is the probability that a building inspector, who randomly selects 4 of the new buildings for inspection, will catch none of the buildings that violate the buildings code?**

Answer:

P= probability that a building violates building code.

⇨ P= $\frac{6}{18} = \frac{1}{3} \therefore q = \frac{2}{3}$ here n = 4,

Required probability = q^4 = $\left(\frac{2}{3}\right)^4$ = 0.1975

7. **For a binomial distribution, mean is 6 and standard deviation is $\sqrt{2}$. Find the first two terms of the distribution.**

Answer:

np = 6, npq = 2; $q = \frac{2}{3} \Rightarrow q = \frac{1}{3}$ $\therefore p = \frac{2}{3}$. Here n = 9.

The first two terms are $\left(\frac{1}{3}\right)^9, 9C_1\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^8$

8. **A certain rare blood can be found in only 0.05% of people. If the population of a randomly selected group is 3000, what is the probability that atleast 2 people in the group have this rare blood type ?**

Answer:

P=0.05% => p=0.0005; n = 3000; $\lambda = np$

$\Rightarrow$ λ=3000 x $\frac{5}{10000}$= 1.5

$P[X \geq 2] =$ 1- P(X<2) = 1- P(X=1)

= 1- $e^{-1.5}\left(1 + \frac{1.5}{1!}\right) = 0.4422$

9. **It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that 2 of 100 books bound by this bindery will have defective bindings.**

Answer:

λ = np $\Rightarrow \lambda$ =100 x 5/100 = 5

$\therefore$ P[X=2] = $\frac{5^2 e^{-5}}{2!}$ = 0.084

10. **If X is a poisson variate such that P(X=2) = 9P(X=4) + 90P(X=6), find the variance .**

Answer:

$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{9e^{-\lambda}\lambda^4}{4!} + \frac{90e^{-\lambda}\lambda^6}{6!} \Longrightarrow \lambda^4 + 3\lambda^2 - 4 = 0$

$\Longrightarrow (\lambda^2 + 4)(\lambda^2 - 1) = 0$

$\therefore \lambda^2 = 1 \Longrightarrow$ variance $= \lambda = 1$.

11. **The moment generating function of a random variable X is given by $M_x(t) = e^{3(e^t-1)}$. Find P(X=1)**

Answer:

$M_x(t) = e^{\lambda(e^t-1)} = e^{3(e^t-1)} \Longrightarrow \lambda = 3$

$P(X = 1) = \lambda e^{-\lambda} \Longrightarrow P(X=1) = 3e^{-3}$.

12. **State the conditions under which the position distribution is a limiting case of the Binomial distribution.**

Answer:

i) $n \to \infty$

ii) $p \to 0$

iii) $np = \lambda$, a constant

13. **Show that the sum of 2 independent poisson variates is a poisson variates.**

Answer:

Let $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$

Then $M_x(t) = e^{\lambda_1(e^t-1)}$; $M_y(t) = e^{\lambda_2(e^t-1)}$

$M_{x+y}(t) = M_x(t)M_y(t) = e^{(e^t-1)(\lambda_1+\lambda_2)}$

$\Longrightarrow X + Y$ is also a poisson variate

14. **In a book of 520 pages, 390 typo-graphical errors occur. Assuming poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.**

Answer:

$$\lambda = \frac{390}{520} = 0.75$$

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.75}(0.75)^x}{x!}$, x = 0.1.2,…

Required probability = [P(X $= 0$)]^5 = $(e^{-0.75})^5$ = $e^{-3.75}$

15. **If X is a poisson variate such that P(X=2)= 2/3 P(X=1) evaluate P(X=3).**

Answer:

$$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1!} \Longrightarrow \lambda = \frac{4}{3}$$

$$\therefore P[X=3] = \frac{e^{-\lambda}\left(\frac{4}{3}\right)^3}{3!}$$

16. **If for a poisson variate X, E(X^2) $=$ 6, What is E(X)?**

Answer:

$\lambda^2+\lambda = 6 \Longrightarrow \lambda^2+\lambda - 6$

$= 0 \Longrightarrow (\lambda+3)(\lambda-2) = 0 \Longrightarrow \lambda = 2, -3$

But $\lambda>0$, $\lambda=2$. E(X) = $\lambda = 2$

17. **If X is a poisson variate with mean λ , show that E(X^2) $= \lambda\, E(X+1)$.**

Answer:

E(X^2) $=\lambda^2+\lambda$

E(X+1) = E(X)+1 = λ + 1

$\therefore$ E(X^2) $= \lambda(\lambda+1) = \lambda E(X+1)$

18. **The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours ?**

Answer:

Let X be the R.V which represents the time to repair the machine.

P[X$\geq 10/x \geq 9$] = P(X≥ 1) (by memory less property)

$= \int_1^\infty \frac{1}{2} e^{-\frac{x}{2}} dx$ = 0.6065

19. **The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{3}$. What is the probability that the repair time exceeds 3 hours ?**

Answer:

X- represent the time to repair the machine

P.d.f of X, f(x) = $\frac{1}{3}e^{-\frac{x}{3}}$, x>0

P(x>3) = $\int_3^\infty \frac{1}{3}e^{-\frac{x}{3}}dx = e^{-1}$ = 0.3679

20. **Find the MGF of an exponential distribution with parameter λ.**

Answer:

$M_x(t) = \lambda \int_0^\infty e^{tx}e^{-\lambda x}dx = \lambda \int_0^\infty e^{-(\lambda - x)x}dx$

$= \frac{\lambda}{\lambda - t} = \left(1 - \frac{t}{\lambda}\right)^{-1}$

21. **Write down the MGF of gamma distribution and hence find its mean and variance.**

Answer:

$M_x(t) = (1-t)^{-\lambda} = 1 + \lambda t + \frac{\lambda^2+\lambda}{2!}t^2 + \cdots$

$Mean = \lambda$; E(X^2) = $\lambda^2 + \lambda \Rightarrow var(X) = \lambda$.

22. **Mention any four properties of normal distribution ?**

Answer:

(1) The curve is bell shaped

(2) Mean, Median, Mode coincide.

(3) All odd central moments vanish

(4) X-axis is an asymptote to the normal curve

23. **If X is normal variate with mean 30 and S.D 5, find P[26 $<$ X $<$ 40]**

Answer:

P [26 < X < 40] = P [-0.8 ≤ Z ≤ 2] where Z = $\frac{X-30}{5}$

= P [0 ≤ Z ≤ 0.8] + P[0 ≤ Z ≤ 2]

= 0.2881 + 0.4772 = 0.7653

24.**If X is a normal variate with mean 30 and SD is 5, find P [$|X - 30|$≤5].**

Answer:

P [$|X - 30|$≤ 5] = P [25 ≤ X ≤ 35] = P [-1 ≤ Z ≤ 1]

= 2P (2 ≤ Z ≤ 1) = 2(0.3413) = 0.6826

25.**X is normally distributed R.V with mean 12 and SD 4. Find P [X ≤ 20].**

Answer:

P [X ≤ 20] = P [Z ≤ 2] where Z = $\frac{X-12}{4}$

= P [-∞ ≤ Z 0] + P [0 ≤ Z ≤ 2]

= 0.5 + 0.4772 = 0.9772

26.**For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50is 48. Find the mean and s.d of the distribution.**

Answer:

Mean A + μ_1' ⇒ Mean = 10 + 40 = 50

μ_1'($about\ the\ point\ X = 50$) = 48 ⇒ μ_4= 48

Since mean is 50, 3σ^4 = 48

σ = 2.

27.**If X is normally distributed with mean 8 and s.d 4 , find P (10 ≤ X ≤ 15).**

Answer:

P (10 ≤ X ≤ 15) = P [0.5 ≤ X ≤ 1.75]

= P [0.5 ≤ X ≤ 1.75] – P [0 ≤ X ≤ 0.5]

= 0.2684

28.**X is a normal variate with mean 1 and variance 4 , Y is another normal variate independent of X with mean 2 and variance 3 , what is the distribution of X + 2Y ?**

Answer:

E [X + 2Y] = E (X) + 2E (Y) = 1 +4 = 5

V[X+2Y] = V (X) + 4V(Y) = 4+4(3) = 16

X + 2Y ~N(5,16) by additive property.

29.**If X is a normal R.V with mean zero and variance $\boldsymbol{\sigma^2}$, Find the p.d.f of Y=$\boldsymbol{e^x}$.**

Answer:

$$f_y(Y) = f_x(x)\left|\frac{dx}{dy}\right| = \frac{1}{y}\, f_x(\log y)$$

$$= \frac{1}{\sigma y\sqrt{2\pi}} \exp[-(\log y - \mu)^2 / 2\sigma^2]$$

30.**If X has an exponential distribution with parameter 1 , find the pdf of y = $\boldsymbol{\sqrt{x}}$.**

Answer:

$$f_y(Y) = f_x(x)\left|\frac{dx}{dy}\right| = 2ye^{-y^2}, y > 0$$

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