PART-B

Unit II, Two Dimensional Random Variables

1. From the following distribution of (X,Y) find. (i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv)$P\left(X \leq \frac{1}{Y} \leq 3\right)$ (v) $P\left(Y \leq \frac{3}{X} \leq 1\right)$ (vi) $P(X + Y \leq 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

2. The joint probability function (X,Y) is given by $P(x,y) = k(2x+3y) \quad x = 0,1,2;\ y = 1,2,3$
   (i) Find the marginal distributions.
   (ii) Find the probability distributions of (X+Y)
   (iii) Find all conditional probability distributions.
3. The joint p.d.f of the random variable (X,Y) is given by
   $$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2,\ 0 < y < 1 \\ 0 & otherwise \end{cases}$$
   Find
   (i) Marginal density function s of X and Y
   (ii) Conditional density of X given Y
   (iii) $P\left(\frac{1}{4} < X < \frac{\frac{1}{2}}{Y} = \frac{1}{3}\right)$

4. The joint p.d.f of the two dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{8xy}{9} & : 1 \le x \le y \le 2 \\ 0 & : otherwise \end{cases}$$

Find

(i) Marginal densities of X and Y

(ii) The conditional density functions $f(x/y)$ and $f(y/x)$.

5. If the joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y) =$

$$\begin{cases} x^2 + \frac{xy}{3}: & 0 < x < 1; 0 < y < 2 \\ 0: & otherwise \end{cases}$$

Find (i) $P(X > {}^1/_2)$ (ii) $P(Y > 1)$ (iii) $P(Y < X)$

(iii) $P\left(Y < \frac{1}{2} \middle/ X < \frac{1}{2}\right)$ (v) $P(X + Y \ge 1)$

(vi) find the conditional density functions.

(vii) Check whether the conditional density functions are valid.

6. The joint p.d.f of the random variable (X,Y) is given by

$f(x,y) = kxye^{-(x^2+y^2)}$ $x > 0, y > 0$

(i) Find k (ii) Prove that X and Y are independent.

7. Given $f_{XY}(x,y) =$

$$\begin{cases} cx(x-y) & ,0 < x < 2, -x < y < x \\ 0 & otherwise \end{cases}$$

(i) Evaluate c

(ii) Find $f_X(x)$

(iii) $f_{\frac{Y}{X}}(y/x)$

(iv) $f_Y(y)$.

8. Two random variables X and Y have the following joint probability density functions

$$f(x,y) = \begin{cases} 2 - x - y: 0 \le x \le 1, 0 \le y \le 1 \\ 0 \quad : otherwise \end{cases}$$

   (i) Find the marginal density functions of X and Y
   (ii) Conditional density function
   (iii) Var X and Var Y
   (iv) Correlation coefficient between X and Y.
9. Given the joint p.d.f of X and Y is

$$f(x,y) = \begin{cases} 8xy: 0 < x < y < 1 \\ 0 \quad : otherwise \end{cases}$$

   Find the marginal and conditional p.d.f's X and Y.Are X and Y independent?
10. Let (X,Y) be the two dimensional random variable described by the joint p.d.f

$$f(x,y) = \begin{cases} 8xy: 0 \le x \le 1, 0 \le y \le x \\ 0 \quad : otherwise \end{cases}$$

   Find the Cov(X,Y).
11. The joint p.d.f of the random variable (X,Y) is $f(x,y) = 3(x+y): 0 \le x \le 1, 0 \le y \le 1, x + y \le 1.$ Find Cov(X,Y).
12. If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation co-efficient between x+y and x-y.
13. Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

   Find the two regression lines. Also find y when x=55.
14. In a correlation analysis the equations of the two regression lines are $3x + 12y = 9; and\ 3y + 9x = 46.$

Find (i) The value of the correlation coefficient (ii) Mean value of X and Y.

15. Find the correlation coefficient and the equation of the regression lines for the following values of X and Y.

X	1	2	3	4	5
Y	2	5	3	8	7

16. Find the most likely price in City A corresponding to the price of Rs.70 at City B from the following:

	City B	City A
Average Price	65	67
S.D. of Price	2.5	3.5

Correlation coefficient is 0.8.

17. The joint p.d.f of the random variable (X,Y) is given as

$$f(x,y) = \begin{cases} e^{-(x+y)} & ; x > 0, y > 0 \\ 0 & ; otherwise \end{cases}$$

Find the distribution of $\frac{1}{2}(X - Y)$.

18. The independent random variables X and Y follow exponential distribution with parameter $\lambda = 1$. Find the p.d.f of $U = X - Y$.

19. Let X and Y are normally distributed independent random variables with mean 0 and variance σ^2. Find the p.d.f's of $R = \sqrt{x^2 + y^2}$ $and\ \theta = tan^{-1}\left(\frac{y}{x}\right)$.

20. The joint p.d.f of a two dimensional random variable (X, Y) is given by $f(x, y) = x + y, 0 \le x, y \le 1$. Find the p.d.f of U=XY.

21. If X and Y are independent random variables , with p.d.f $f(x) = e^{-x}, x \ge 0$: $f(y) = e^{-y}, y \ge 0$. Show that $U = \frac{X}{X+Y}$ $and\ V = X + Y$ are independent.

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