UNIT – 1, PROBABILITY AND RANDOM VARIABLES

PART-A (2 marks)

1. **Define Probability:**

If there are n equally likely mutually exclusive and exhaustive outcomes and m of them are favourable to an event A, Then the probability of the happening of A is P(A)= $\frac{\text{No of favourable cases}}{\text{Total no of exhaustive cases}}$

2. **Define i) Discrete random variable**

ii) Continuous random variable

i) Let X be a random variable, if the number of possible values of X is finite or count ably finite, then X is called a discrete random variable.

ii) A random variable X is called the continuous random variable, if x takes all its possible values in an interval.

3. **Define probability mass function (PMF):**

Let X be the discrete random variable taking the values ,X_1 , X_2 ………. Then the number P (X_i) = P(X =X_i) is called the probability mass function of X and it satisfies the following conditions.

i) P (X_i) $\geq$ 0 for all;

ii) $\sum_{i=1}^{\infty}$ P (X_i) = 1

4. **Define probability Density function (PDF):**

Let x be a continuous random variable. The Function f(x) is called the probability density function (PDF) of the random variable x if it satisfies.

i) f(x) $\geq$ 0

ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

5. **Define cumulative distribution function (CDF):**

Let x be a random variable. The cumulative distribution function, denoted by F(X)and is given by F (X)=P(X≤x).

6. **If x is a discrete R.V having the p.m.f**

X:	-1	0	1
P(X):	k	2k	3k

Find P(x≥ $\mathbf{0}$)

Answer:$6k = 1 \Rightarrow k = \frac{1}{6}$

$P[x\geq 0] = 2k + 3k \Rightarrow P[x\geq 0] = \frac{1}{6}$

7. **The random variable x has the p.m.f. P (x)=$\frac{x}{15}$, x=1,2,3,4,5 and = 0 else where. Find P [$\frac{1}{2} < x < \frac{5}{2} / x > 1$] .**

Answer:

$P[\frac{1}{2} < x < \frac{5}{2} / x > 1] = \frac{P[x=2]}{P(x>1)} = \frac{P[x=2]}{1-P(x\leq 1)} = \frac{2/15}{1-1/15} = \frac{1}{7}$

8. **If the probability distribution of X is given as :**

X	**1**	**2**	**3**	**4**
P(X)	**0.4**	**0.3**	**0.2**	**0.1**

Find P [$\frac{1}{2} < x < \frac{7}{2} / x > 1$] .

Answer :

$P[\frac{1}{2} < x < \frac{7}{2} / x > 1] = \frac{P[1<x<7/2]}{P(x>1)} = \frac{P(x=2)+P(x=3)}{1-P(x=1)} = \frac{0.5}{0.6} = \frac{5}{6}$

9. **A.R.V. X has the probability function**

X	-2	-1	0	1
P(X)	0.4	k	0.2	0.3

Find k and the mean value of X

Answer:

k=0.1 Mean $=\sum xP(x) = \frac{1}{10}$ [-8-1+0+3] = -0.6

10. **If the p.d.f of a R.V. X is** $f(x) = \frac{x}{2}$ **in** $0 \leq x \leq 2$**, find**

P $[x > 1.5/x > 1]$**.**

Answer :

$$P[x > 1.5/x > 1] = \frac{P[x>1.5]}{P(x>1)} = \frac{\int_{1.5}^{2} \frac{x}{2}dx}{\int_{1}^{2} \frac{x}{2}dx} = \frac{4-2.25}{4-1} = 0.5833$$

11. **If the p.d.f of a R.V.X is given by** f**(x) = {1/4,-2<x<2.0, else where. Find P[|X|>1].**

Answer:

$P[|X|>1] = 1 - P[|X|<1] = 1 - \int_{-1}^{1} \frac{1}{4} dx = \frac{1}{2}$

12. **If** $f(x) = kx^2$**, 0<x<3 is to be density function, Find the value of k.**

Answer:

$\int_0^3 kx^2 dx = 1 \Rightarrow 9k = 1 \therefore k = \frac{1}{9}$

13. **If the c.d.f. of a R.V X is given by F(x) = 0 for x<0;=** $\frac{x^2}{16}$ **for** $0 \leq x < 4$ $and = 1\ for\ x \geq 4, find\ P(X > 1/X < 3)$.

Answer:

$$P(X > 1/X < 3) = \frac{P[1<X<3]}{P[0<X<3]} = \frac{F(3)-F(1)}{F(3)-F(0)} = \frac{8/16}{9/16} = \frac{8}{9}.$$

14. **The cumulative distribution of X is F(x) = $\frac{x^3+1}{9}$, $-1, < X < 2\ and \quad =$ 0, otherwise. Find P[0 <X<1].**

Answer:

P[0<X<1] = F(1) – F(0) = $\frac{2}{9} - \frac{1}{9} = \frac{1}{9}$

15. **A Continuous R.V X that can assume any value between x=2 and x=5 had the p.d.f f(x) = k(1+x). Find P(x<4).**

Answer:

$$\int_2^3 k(1+x)dx = 1 \Rightarrow \frac{27k}{2} = 1 \therefore k = \frac{2}{27}$$

P[X<4] = $\int_2^4 \frac{2}{27}(1+x)dx = \frac{16}{27}$

16. **The c.d.f of X is given by F (x) = $\begin{bmatrix} \mathbf{0}, \boldsymbol{x} > 0 \\ \boldsymbol{x^2}, \ \mathbf{0} \le \boldsymbol{x} \le \mathbf{1} \\ \mathbf{1}, \boldsymbol{x} > 1 \end{bmatrix}$ Find the p.d.f of x, and obtain P(X>0.75).**

Answer:

F(x) = $\frac{d}{dx}$F(x) = $\begin{bmatrix} 2x, 0 \le x \le 1 \\ 0, otherwise \end{bmatrix}$

P[x<0.75] = 1- P[X≤ 0.75] $= 1 - F(0.75) = 1 - (0.75)^2 = 0.4375$

17. **Check whether f(x)=$\frac{1}{4}xe^{-x/2}$for 0<x<∞ can be the p.d.f of X.**

Answer:

$= \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{x}{4} e^{-x/2}\, dx = \int_{0}^{\infty} te^{-1}\, dt$ where t = $\frac{x}{2}$

=$(-te^{-1} - e^{-1})_0^{\infty}$ = - [0-1]=1

$$\therefore f(\,x\,) is\ the\ p.d.f\ of\ X.$$

18. **A continuous R.V X has a p.d.f f(x) = $3x^2$, $0 \le x \le 1$. Find b such that P(X>b)= 0.05.**

Answer:

$$3\int_{b}^{1} x^2 dx = 0.05 \Rightarrow 1 - b^3 = 0.05 \Rightarrow b^3 = 0.95 \therefore b = (0.95)^{\frac{1}{3}}$$

19. **Let X be a random variable taking values -1, 0 and 1 such that P(X=-1) = 2P(X=0) = P(X=1). Find the mean of 2X-5.**

Answer:

$$\sum P(X = x) = 1 \Rightarrow 5P(X = 0) = 1 \therefore P(X = 0) = \frac{1}{5}$$

Probability distribution of X:

X	-1	0	1
P(X)	2/5	1/5	2/5

$$Mean = E(x) = \sum xp(x) = -1\left(\frac{2}{5}\right) + 0\left(\frac{1}{5}\right) + 1\left(\frac{2}{5}\right) = 0$$

E[2X-5] = 2E(X) – 5 = 2[0] – 5 = -5.

20. **Find the cumulative distribution function F(x) corresponding to the p.d.f.**

$\boldsymbol{F}$**(x) = $\frac{1}{\pi(1+x^2)}$, -$\infty < x < \infty$.**

Answer

F(x) $= \int_{-\infty}^{x} f(x)dx = \frac{1}{\pi}\int_{-\infty}^{x} \frac{dx}{1+x^2} = \frac{1}{\pi}[tan^{-1}x]$

$= \frac{1}{\pi}[\frac{\pi}{2} + tan^{-1}$ x]

21. **The diameter of an electric cable, say X is assumed to a continues R.V with p.d.f of given by f(x) = kx(1-x), 0≤ $x \leq 1$. Determine k and P$\left(x \leq \frac{1}{3}\right)$**

Answer:

$$\int_0^1 kx(1-x)dx = 1 \Rightarrow k\left[\frac{1}{2} - \frac{1}{3}\right] = 1 \quad \therefore k = 6$$

$$P\left[X \leq \frac{1}{3}\right] = 6\int_0^{1/3}(x - x^2)dx = 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1/3} = [(3x^2 - 2x^3)]_0^{1/3} = \frac{1}{3} - \frac{2}{27} = \frac{7}{27}$$

22. **A random variable X has the p.d.f f(x) given by f(x)=$\begin{cases} Cxe^{-x}, if\ x > 0 \\ 0, if\ x \leq 0 \end{cases}$.**

Find the value of C and C.D.F of X.

Answer:

$$C\int_0^\infty xe^{-x}dx = 1 \Rightarrow C[x(-e^{-x}]_0^\infty = 1$$

$$\therefore C[-0+1] = 1 \Rightarrow C = 1$$

$C.D.F : F(x) = \int_0^x f(x)dx = 1 - (1+x)e^{-x}$ for x ≥ 0.

23. **State the properties of cumulative distribution function.**

Answer:

i) F(-∞)=0 and F(∞) = 1.

ii) F(∞) is non – decreasing function of X.

iii) If F(∞) $is\ the\ p.d.f$ of X, then f(x)= $F'(x)$

iv) P[a≤ $X \leq b$] = F(b) – F(a)

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