



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Taylor's Series: $1 - \frac{z}{1} + \frac{(z)^2}{2!} + \dots =$

If $f(z)$ is analytic inside a circle C with centre at $z=a$, then $f(z)$ can be expressed as,

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \dots + \frac{(z-a)^n}{n!} f^n(a) + \dots$$

which is convergent at every point inside C . This is called Taylor's series of $f(z)$ about $z=a$.

Note:

The Taylor's series of $f(z)$ about the point $z=0$ is given by,

$$f(z) = f(0) + \frac{z}{1!} f'(0) + \frac{z^2}{2!} f''(0) + \dots + \frac{z^n}{n!} f^n(0) + \dots$$

1) Expand $f(z) = \log(1+z)$ as Taylor's series about $z=0$ if $|z| < 1$.

Soln:

$$f(z) = \log(1+z) \quad f(0) = \log 1 = 0$$

$$f'(z) = \frac{1}{1+z} \quad f'(0) = \frac{1}{1} = 1$$

$$f''(z) = \frac{-1}{(1+z)^2} \quad f''(0) = \frac{-1}{1} = -1$$

$$f'''(z) = \frac{2}{(1+z)^3} \quad f'''(0) = \frac{2}{1} = 2$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

Taylor's series about $z=0$ is given by

$$f(z) = f(0) + \frac{z}{1!} f'(0) + \frac{z^2}{2!} f''(0) + \frac{z^3}{3!} f'''(0) + \dots$$

$$= 0 + \frac{z}{1!} (1) + \frac{z^2}{2!} (-1) + \frac{z^3}{3!} (2) + \dots$$

$$= z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$$

2. Expand $f(z) = \cos z$ about $z = \pi/3$ in Taylor's series.

Soln:

$$f(z) = \cos z \quad f(\pi/3) = \cos \pi/3 = 1/2$$

$$f'(z) = -\sin z \quad f'(\pi/3) = -\sin \pi/3 = -\sqrt{3}/2$$

$$f''(z) = -\cos z \quad f''(\pi/3) = -\cos \pi/3 = -1/2$$

$$f'''(z) = \sin z \quad f'''(\pi/3) = \sin \pi/3 = \sqrt{3}/2$$

Taylor's series is,

$$f(z) = f(\pi/3) + \frac{(z-\pi/3)}{1!} f'(\pi/3) + \frac{(z-\pi/3)^2}{2!} f''(\pi/3) + \dots$$

$$= \frac{1}{2} + \frac{(z-\pi/3)}{1!} \left(-\frac{\sqrt{3}}{2}\right) + \frac{(z-\pi/3)^2}{2!} \left(-\frac{1}{2}\right) + \frac{(z-\pi/3)^3}{3!} \left(\frac{\sqrt{3}}{2}\right) + \dots$$

$$= \frac{1}{2} - \frac{(z-\pi/3)}{1!} \left(\frac{\sqrt{3}}{2}\right) - \frac{(z-\pi/3)^2}{2!} \left(\frac{1}{2}\right) + \frac{(z-\pi/3)^3}{3!} \left(\frac{\sqrt{3}}{2}\right) + \dots$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

③ Expand $f(z) = \sin z$ in a Taylor's Series about $z = \pi/4$.

Soln:

$$f(z) = \sin z$$

$$f'(z) = \cos z$$

$$f''(z) = -\sin z$$

$$f'''(z) = -\cos z$$

$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f'\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f''\left(\frac{\pi}{4}\right) = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$f'''\left(\frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

Taylor's Series formula for $z=a$

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \frac{(z-a)^3}{3!} f'''(a) + \dots$$

At $z = \pi/4$

$$f(z) = f\left(\frac{\pi}{4}\right) + \frac{(z-\frac{\pi}{4})}{1!} f'\left(\frac{\pi}{4}\right) + \frac{(z-\frac{\pi}{4})^2}{2!} f''\left(\frac{\pi}{4}\right) + \frac{(z-\frac{\pi}{4})^3}{3!} f'''\left(\frac{\pi}{4}\right) + \dots$$

$$f(z) = \frac{1}{\sqrt{2}} + \left(z-\frac{\pi}{4}\right) \frac{1}{\sqrt{2}} + \frac{\left(z-\frac{\pi}{4}\right)^2}{2} \cdot \left(\frac{-1}{\sqrt{2}}\right) + \frac{\left(z-\frac{\pi}{4}\right)^3}{6} \cdot \left(\frac{-1}{\sqrt{2}}\right) + \dots$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \left(z-\frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}} \left(z-\frac{\pi}{4}\right)^2 - \frac{1}{6\sqrt{2}} \left(z-\frac{\pi}{4}\right)^3 + \dots$$