Note :

Every SSS process of order 2 is a WSS process and not conversely.

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1. The process $x(t)$ whose probability under certain conditions is given by,

$$P[x(t)=n] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n-1}}, & n=1,2,\dots \\ \dfrac{at}{1+at}, & n=0 \end{cases}$$

Show that it is not stationary [Evolutionary]

Soln. :

The probability distribution of $\{x(t)\}$ is

$x(t)=n$	0	1	2	3	.....
$P[x(t)=n]$	$\dfrac{at}{1+at}$	$\dfrac{1}{(1+at)^2}$	$\dfrac{at}{(1+at)^3}$	$\dfrac{(at)^2}{(1+at)^4}$	

Mean :

$$E[x(t)=n] = \sum_{n=0}^{\infty} n\,p(n)$$

$$= 0\left(\frac{at}{1+at}\right) + 1\left(\frac{1}{(1+at)^2}\right) + 2\left(\frac{at}{(1+at)^3}\right) + \dots\dots$$

$$= \frac{1}{(1+at)^2}\left[1 + 2\,\frac{at}{1+at} + 3\left(\frac{at}{1+at}\right)^2 + \dots\dots\right]$$

$$= \frac{1}{(1+at)^2}\left[1 + 2\alpha + 3\alpha^2 + \dots\dots\right] \qquad \text{where } \alpha = \frac{at}{1+at}$$

$$= \frac{1}{(1+at)^2}\left[1-\alpha\right]^{-2}$$

$$= \frac{1}{(1+at)^2}\left[1 - \frac{at}{1+at}\right]^{-2}$$

$$= \frac{1}{(1+at)^2}\left[\frac{1+at-at}{1+at}\right]^{-2}$$

$$= \frac{1}{(1+at)^2}(1+at)^2$$

$$E[x(t)] = 1$$

and $E[x^2(t)=n^2] = \sum_{n=0}^{\infty} n^2 P(n)$

$$= \sum_{n=0}^{\infty}[n(n+1)-n]\,P(n)$$

$$= \sum_{n=0}^{\infty} n(n+1)P(n) - \sum_{n=0}^{\infty} n\,P(n)$$

$$= \left[0 + 1(2)\frac{\cancel{at}}{(1+at)^{\cancel{3}}} + 2(3)\frac{at}{(1+at)^3} + 3(4)\frac{(at)^2}{(1+at)^4} + \cdots\right] - 1$$

$$= \left[0 + 1(2)\frac{1}{(1+at)^2} + 2(3)\frac{at}{(1+at)^3} + 3(4)\frac{(at)^2}{(1+at)^4} + \cdots\right] - 1$$

$$= \left[\frac{2}{(1+at)^2} + \frac{6at}{(1+at)^3} + \frac{12(at)^2}{(1+at)^4} + \cdots\right] - 1$$

$$= \frac{2}{(1+at)^2}\left[1 + 3\frac{at}{1+at} + 6\frac{(at)^2}{(1+at)^2} + \cdots\right] - 1$$

$$= \frac{2}{(1+at)^2}\left[1 + 3\alpha + 6\alpha^2 + \cdots\right] - 1$$

where $\alpha = \frac{at}{1+at}$

$$= \frac{2}{(1+at)^2}[1-\alpha]^{-3} - 1$$

$$= \frac{2}{(1+at)^2}\left[1-\frac{at}{1+at}\right]^{-3}-1$$

$$= \frac{2}{(1+at)^2}\left[\frac{1+at-at}{1+at}\right]^{-3}-1$$

$$= \frac{2}{(1+at)^2}(1+at)^3-1$$

$$= 2(1+at)-1$$

$$= 2+2at-1$$

$$E[x^2(t)] = 1+2at$$

$$Var[x(t)] = E[x^2(t)] - \{E[x(t)]\}^2$$

$$= 1+2at-1$$

$V[x(t)] = 2at$ is not a constant.

$\therefore$ The given function is not a Stationary Process.

Note:

$E(a) = a$; $V(a) = 0$

Formula:

1). $\cos(A+B) = \cos A \cos B - \sin A \sin B$

2). $\cos(A-B) = \cos A \cos B + \sin A \sin B$

3). $\sin(A+B) = \sin A \cos B + \cos A \sin B$

4). $\sin(A-B) = \sin A \cos B - \cos A \sin B$

5]. $\cos A \cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

6]. $\sin A \sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$

7]. $\cos A \sin B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

8]. $\sin A \cos B = \frac{1}{2}[\sin(A+B)-\sin(A-B)$