1). Show that the random process $x(t) = A\cos(\omega t+\theta)$ where A & ω are constant ; θ is uniformly distributed in $(-\pi, \pi)$ is WSS.

Soln.

Given $x(t) = A\cos(\omega t+\theta)$

To prove : WSS.

i). $E[x(t)] =$ constant

ii). $R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$

Since θ is uniformly distributed in $(-\pi, \pi)$.

$$F(\theta) = \frac{1}{b-a}$$

$$= \frac{1}{\pi+\pi}$$

$$F(\theta) = \frac{1}{2\pi}$$

i). $E[x(t)] = \int x(t)\, F(\theta)\, d\theta$

$$= \int_{-\pi}^{\pi} A \cos(\omega t+\theta)\frac{1}{2\pi}\, d\theta$$

$$= \frac{A}{2\pi}\left[\sin(\omega t+\theta)\right]_{-\pi}^{\pi}$$

$$= \frac{A}{2\pi}\left[\sin(\pi+\omega t) - \sin(-\pi+\omega t)\right]$$

$$= \frac{A}{2\pi}\left[-\sin\omega t + \sin\omega t\right]$$

$\sin(180+\theta) = -\sin\theta$

$\sin(180-\theta) = \sin\theta$

$E[x(t)] = 0$

ii). $E[x(t)\,x(t+\tau)]$

$= E[A\cos(\omega t+\theta)\; A\cos(\omega(t+\tau)+\theta)]$

$= A^2\, E[\cos(\omega t+\theta)\,\cos(\omega t+\omega\tau+\theta)]$

$= \frac{A^2}{2} E[\cos(\omega t+\theta+\omega t+\omega\tau+\theta) + \cos(\omega t+\theta-\omega t-\omega\tau-\theta)]$

$= \frac{A^2}{2} E[\cos(2\omega t+\omega\tau+2\theta) + \cos(-\omega\tau)]$

$= \frac{A^2}{2} \{E[\cos(2\omega t+\omega\tau+2\theta)] + E[\cos(\omega\tau)]\}$

$= \frac{A^2}{2}\cos\omega\tau + \frac{A^2}{2} E[\cos(2\omega t+\omega\tau+2\theta)]$ $\rightarrow (1)$

$E[\cos(2\omega t+\omega\tau+2\theta)]$

$= \int_{-\pi}^{\pi} \cos(2\omega t+\omega\tau+2\theta)\frac{1}{2\pi}\, d\theta$

$= \frac{1}{2\pi}\left[\frac{\sin(2\omega t+\omega\tau+2\theta)}{2}\right]_{-\pi}^{\pi}$

$= \frac{1}{4\pi}[\sin(2\pi+2\omega t+\omega\tau) - \sin(-2\pi+2\omega t+\omega\tau)]$

$= \frac{1}{4\pi}[\sin(2\pi+2\omega t+\omega\tau) + \sin(2\pi-(2\omega t+\omega\tau))]$

$= \frac{1}{4\pi}[\sin(2\omega t+\omega\tau) - \sin(2\omega t+\omega\tau)]$

$= \frac{1}{4\pi}(0)$ $\quad\because \sin(360+\theta)=\sin\theta$, $\sin(360-\theta) = -\sin\theta$

$= 0$

$(1) \Rightarrow E[x(t)\,x(t+\tau)] = \frac{A^2}{2}\cos\omega\tau + 0 = \frac{A^2}{2}\cos\omega\tau$

$= R_{xx}(\tau)$ which depends on τ

$\therefore x(t)$ is WSS.