Q. Show that the process $x(t) = A\cos\lambda t + B\sin\lambda t$ is WSS, where A & B are random variables, if

i). $E[A] = E[B] = 0$

ii). $E[A^2] = E[B^2] = k$

iii). $E[AB] = 0$

Soln.

Given. $x(t) = A\cos\lambda t + B\sin\lambda t$

i). $E[x(t)] = E[A\cos\lambda t + B\sin\lambda t]$

$= \cos\lambda t\, E[A] + \sin\lambda t\, E[B]$

$= \cos\lambda t\,(0) + \sin\lambda t\,(0) \quad [\because E(A) = E(B) = 0]$

$= 0$

ii). $E[x(t)\,x(t+\tau)] = E[(A\cos\lambda t + B\sin\lambda t)(A\cos\lambda(t+\tau) + B\sin\lambda(t+\tau))]$

$= E[(A\cos\lambda t + B\sin\lambda t)(A\cos(\lambda t + \lambda\tau) + B\sin(\lambda t + \lambda\tau))]$

$= E[A^2\cos\lambda t\cos(\lambda t + \lambda\tau) + AB\cos\lambda t\sin(\lambda t + \lambda\tau) + BA\sin\lambda t\cos(\lambda t + \lambda\tau) + B^2\sin\lambda t\sin(\lambda t + \lambda\tau)]$

$= E[A^2\cos\lambda t\cos(\lambda t + \lambda\tau)] + E[AB\cos\lambda t\sin(\lambda t + \lambda\tau)] + E[BA\sin\lambda t\cos(\lambda t + \lambda\tau)] + E[B^2\sin\lambda t\sin(\lambda t + \lambda\tau)]$

$= \cos\lambda t\cos(\lambda t + \lambda\tau)E(A^2) + \cos\lambda t\sin(\lambda t + \lambda\tau)E(AB) + \sin\lambda t\cos(\lambda t + \lambda\tau)E(BA) + \sin\lambda t\sin(\lambda t + \lambda\tau)E(B^2)$

$= \cos\lambda t\cos(\lambda t + \lambda\tau)K + \cos\lambda t\sin(\lambda t + \lambda\tau)(0) + \sin\lambda t\cos(\lambda t + \lambda\tau)(0) + \sin\lambda t\sin(\lambda t + \lambda\tau)(K)$

$[\because E(A^2) = E(B^2) = K \text{ (say)}]$

$= K[\cos\lambda t\cos(\lambda t + \lambda\tau) + \sin\lambda t\sin(\lambda t + \lambda\tau)]$

$= K[\cos(\lambda t - (\lambda t + \lambda\tau))] \quad \because \cos A\cos B + \sin A\sin B = \cos(A - B)$

$= K[\cos(-\lambda\tau)]$

$= K\cos(\lambda\tau)$ which depends on τ

$= R_{xx}(\tau)$ $\quad \therefore x(t)$ is WSS.