Markov Process:

It is the one in which the future value is independent of the past value given the present value.

Markovian:

A random process $X(t)$ is said to be markovian if

$$P[X(t_{n+1}) \le x_{n+1} / X(t_n) = x_n, X(t_{n-1}) = x_{n-1}, \ldots X(t_0) = t_0]$$

$$= P[X(t_{n+1}) \le x_{n+1} / X(t_n) = x_n]$$

where $t_0 \le t_1 \le t_2 \le \ldots \ldots \le t_n \le t_{n+1}$ and

$x_0, x_1, \ldots x_n, x_{n+1}$ are called the states of the process.

Eg:

The probability of raining today depends on the previous whether conditions existed for the last two days and not on the past weather condition.

Markov chain:

If $P[X_n = a_n / X_{n-1} = a_{n-1}, X_{n-2} = a_{n-2}, \ldots, X_0 = a_0]$

$= P[X_n = a_n / X_{n-1} = a_{n-1}]$, then the process X_n, $n = 0, 1, 2, \ldots$ is called a markov chain.

One-Step Transition Probability:

The conditional probability $P[X_n = a_j / X_{n-1} = a_i]$ is called the one-step transition probability from state a_i to state a_j at the n^{th} step and it is denoted by $P_{ij}(n-1, n)$

Homogeneous Markov chain:

If the one step transition probability does not depend on the step ie, $P_{ij}(n-1, n) = P_{ij}(m-1, m)$, then the markov chain is called the homogeneous markov chain.

Transition Probability Matrix [TPM]

When the markov chain is homogeneous, the one step transition probability is denoted by P_{ij}. The matrix P_{ij} satisfies the following conditions

i). $P_{ij} \geq 0$

ii). $\sum P_{ij} = 1, \forall i$

Result:

1]. $P(X_i = a / X_j = b) = P_{ba}^{i-j}$

2]. $P(X_n = j) = \sum_{i=0}^{j} P(X_n = j / X_0 = i) \cdot P(X_0 = i)$

Eg: $P(X_2 = 3 / X_0 = 1) = P_{13}^{2-0} = P_{13}^{(2)}$