Classify the States:

Irreduciable:

If for every $i, j$ and for some $n$ such that $P_{ij}^{n} > 0$, then every state can be reached from every other state, then the markov chain is irreduciable.

Periodic State:

Let $P_{ij}^{(m)} > 0$ for all $m$. Let $i$ be a return state.

Then $d_i = GCD\{m : P_{ii}^{(m)} > 0\}$

where GCD stands for the greatest common divisor.

$\Rightarrow$ If $d_i > 1$, then the state 'i' is called periodic

$\Rightarrow$ If $d_i = 1$, then the state 'i' is called aperiodic.

Non-Null Persistant:

If a markov chain is finite and irreducible then all the states are non-null persistant.

Ergodic:

A non-null persistant & aperiodic state is said to be ergodic.

Non Ergodic:

A non-null persistant & periodic state is said to be Non ergodic.

1. Three boys A, B, C are throwing a ball to each other. A always throw a ball to B, B always throw a ball to C, but C is just as likely to throw the ball to B as to A. Find TPM & classify the states, draw diagram.

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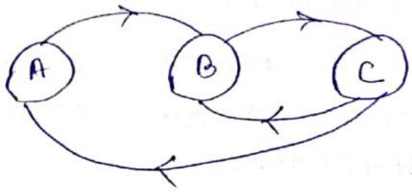
Soln.

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \end{array}$$




$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$P^5 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{1}{2} \end{bmatrix}$$

$\therefore\ d_i = GCD\{m,\ P_{ii}^{(m)} > 0\}$

$d_1 = GCD\{3, 5\} = 1$ $\qquad \because P_{11}^{(3)} > 0$, $P_{11}^{(5)} > 0$

$d_2 = GCD\{2, 3, 4, 5\} = 1$

$d_3 = GCD\{2, 3, 4, 5\} = 1$

$\therefore\ d_i = 1 \Rightarrow$ aperiodic

Now, $P_{11}^{(3)} > 0 \quad P_{12}^{(1)} > 0 \quad P_{13}^{(2)} > 0$

$P_{21}^{(2)} > 0 \quad P_{22}^{(2)} > 0 \quad P_{23}^{(1)} > 0$

$P_{31}^{(1)} > 0 \quad P_{32}^{(1)} > 0 \quad P_{33}^{(2)} > 0$

$\therefore$ The chain is irreducible.

Since we have 3 states, the chain is finite and irreducible.

$\therefore$ All the states are non-null persistent.

Since all the states are aperiodic & non-null persistent,

$\therefore$ It is ergodic.

2\. Let $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$, classify the states of the markov chain.

Soln.

Given $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$\therefore P_{11}^{(2)} > 0, \quad P_{22}^{(2)} > 0$ and $P_{33}^{(2)} > 0$

It is irreducible.

Since we've 3 states, the chain is finite and irreducible.
All the states are Non-null Persistant.

$$P^3 = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}; \quad P^4 = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix};$$

$$P^5 = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$\therefore d_i = GCD\{m, P_{ii}^{(m)} > 0\}$

$d_1 = GCD\{2, 4\} = 2$

$d_2 = GCD\{2, 4\} = 2$

$d_3 = GCD\{2, 4\} = 2$

$\therefore d_i = 2$ It is Periodic

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