∴ It is periodic & Non-null persistant.

∴ It is Non-ergodic

Steady State Distribution :

If P is TPM & Markov chain $\pi = \pi_1 \pi_2 \dots \pi_n$ Steady State Distribution is i). $\pi P = \pi$

ii). $\sum_{i=1}^{n} \pi_i = 1$

1. A house wife buys three kinds of cereals A, B, C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However if she buys either B or C, the next week she is three times as likely to buy A as the other cereal. How often she buys each of the 3 cereals?

Soln.

TPM:

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} \end{array}.$$

Let $\pi = (\pi_1 \ \pi_2 \ \pi_3)$

i). $\pi P = \pi$

$$(\pi_1 \ \pi_2 \ \pi_3) \begin{pmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{pmatrix} = (\pi_1 \ \pi_2 \ \pi_3)$$

$$\left(0 + \frac{3\pi_2}{4} + \frac{3\pi_3}{4} \quad \pi_1 + 0 + \frac{\pi_3}{4} \quad 0 + \frac{\pi_2}{4} + 0\right) = (\pi_1 \ \pi_2 \ \pi_3)$$

$$\Rightarrow \frac{3\pi_2 + 3\pi_3}{4} = \pi_1 \ ; \quad \pi_1 + \frac{\pi_3}{4} = \pi_2 \ ; \quad \frac{\pi_2}{4} = \pi_3$$

$$4\pi_1 = 3\pi_2 + 3\pi_3 \ \to (1) \ ; \quad 4\pi_1 + \pi_3 = 4\pi_2 \ \to (2) \ ; \quad \pi_2 = 4\pi_3 \ \to (3)$$

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Subs. (3) in (2),

$$4\pi_1 + \pi_3 - 4(4\pi_3) = 0$$

$$4\pi_1 + \pi_3 - 16\pi_3 = 0$$

$$4\pi_1 - 15\pi_3 = 0$$

$$\pi_1 = \frac{15}{4}\pi_3$$

ii). $\pi_1 + \pi_2 + \pi_3 = 1$

$$\frac{15}{4}\pi_3 + 4\pi_3 + \pi_3 = 1$$

$$\frac{15\pi_3 + 16\pi_3 + 4\pi_3}{4} = 1$$

$$\frac{35\pi_3}{4} = 1$$

$$\pi_3 = \frac{4}{35}$$

$(3) \Rightarrow \pi_2 = 4\pi_3 = 4\left(\frac{4}{35}\right) = \frac{16}{35}$; $\pi_1 = \frac{15}{4}\left(\frac{4}{35}\right) = \frac{15}{35}$

$$\therefore \pi = \left(\frac{15}{35} \quad \frac{16}{35} \quad \frac{4}{35}\right)$$

2]. A man either drives a car or catches a train to his office each day. He never goes two days in row by train, but if he drives one day, then next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day drove to work if and only if a '6' appear. Find

i). the probability that he takes a train on the 3rd day

ii). the probability that he drives to work in the long run?

Soln.

Let T be train and C be car.

Let (T, C) be a travel pattern.

TPM:

$$P = \begin{array}{c c} & \begin{array}{cc} T & C \end{array} \\ \begin{array}{c} T \\ C \end{array} & \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix} \end{array}$$

prob. of travelling by car = P[getting 6 in the die] $= \frac{1}{6}$

prob. of travelling by train $= 1 - \frac{1}{6} = \frac{5}{6}$

$$\therefore P^{(1)} = \left(\frac{5}{6} \quad \frac{1}{6}\right)$$

$$P^{(2)} = P^{(1)} \cdot P = \left(\frac{5}{6} \quad \frac{1}{6}\right) \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix} = \left(\frac{1}{12} \quad \frac{11}{12}\right)$$

$$P^{(3)} = P^{(2)} \cdot P = \left(\frac{1}{12} \quad \frac{11}{12}\right) \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix} = \left(\frac{11}{24} \quad \frac{13}{24}\right)$$

i). P[the man travels by train on the 3^{rd} day]

$$= \frac{11}{24}$$

ii). Steady state distribution : $\pi = (\pi_1 \quad \pi_2)$

i). $\pi P = \pi$

$$(\pi_1 \quad \pi_2) \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix} = (\pi_1 \quad \pi_2)$$

$$\left(\frac{\pi_2}{2} \quad \pi_1 + \frac{\pi_2}{2}\right) = (\pi_1 \quad \pi_2)$$

$$\Rightarrow \frac{\pi_2}{2} = \pi_1 \qquad \pi_1 + \frac{\pi_2}{2} = \pi_2$$

$$\pi_2 = 2\pi_1 \qquad \Rightarrow 2\pi_1 + \pi_2 = 2\pi_2$$

$$2\pi_1 = \pi_2$$

ii). $\pi_1 + \pi_2 = 1 \Rightarrow \pi_1 + 2\pi_1 = 1 \Rightarrow 3\pi_1 = 1 \Rightarrow \pi_1 = \frac{1}{3}$

$\pi_2 = 2/3$

$$\therefore \pi = \left(\frac{1}{3} \quad \frac{2}{3}\right)$$

P[the man travels by car in the long run] $= \frac{2}{3}$