SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 1 – DISCRETE FOURIER TRANSFORM

TOPIC – CIRCULAR CONVOLUTION

EMPATHY

1

- Convolution of Periodic signal using Linear convolution is repetitive

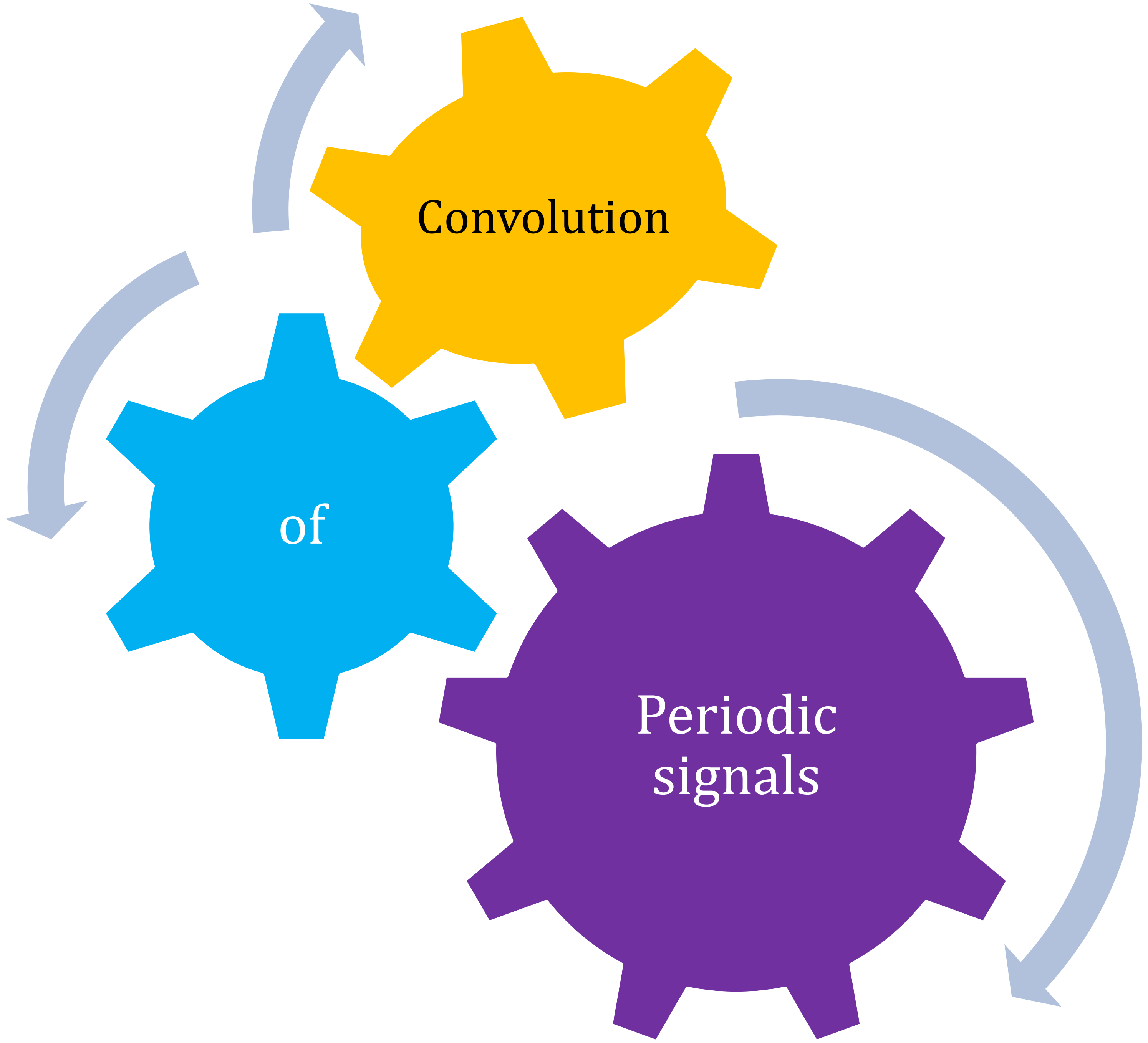
2

- Convolution takes lot of time

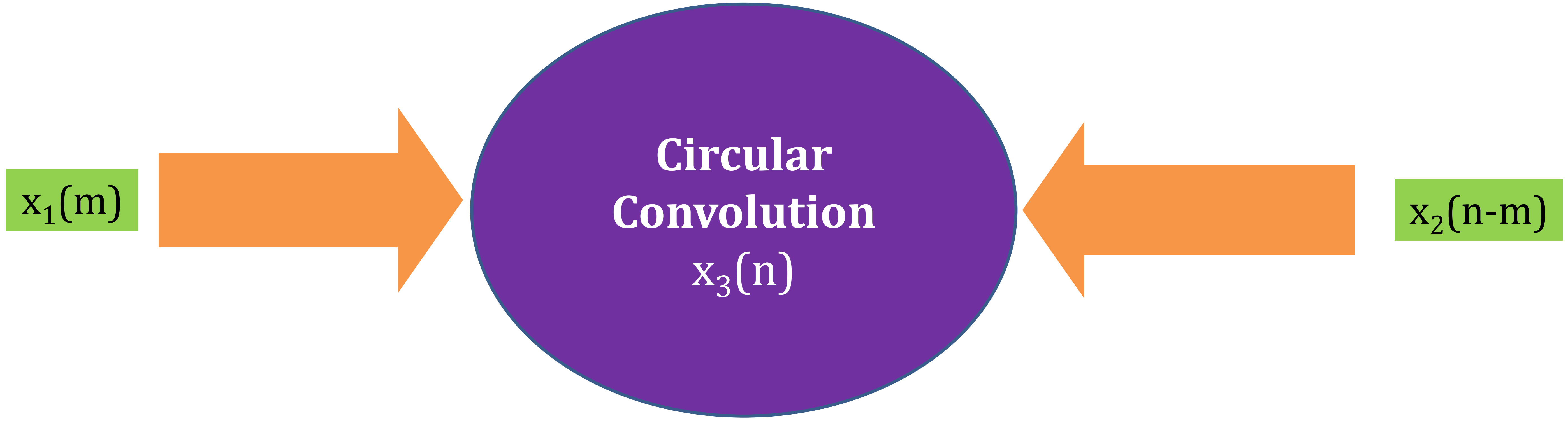
3

- Utilization of DFT for Convolution

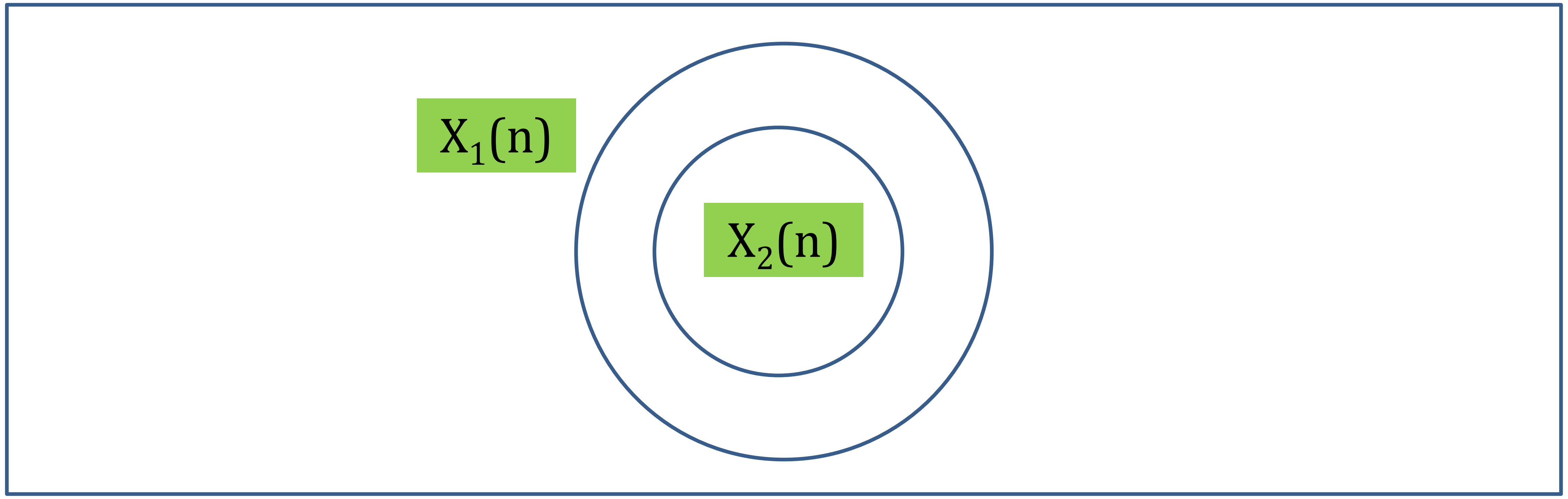
DEFINE



METHOD



NEED FOR CONVOLUTION



- To find the output of a system with input and impulse response of the system linear convolution is used
- Circular Convolution can be applied for periodic signals and to obtain linear Convolution through Circular padding of zeros are done

CIRCULAR CONVOLUTION

- The circular convolution of two periodic discrete time sequences $X_1(n)$ and $X_2(n)$ with periodicity of N sample is defined as

$$y(n) = x(n) * h(n) = \sum_{k=0}^{N-1} x(k).h(n-k) \qquad x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2[((n-m))_N]$$

- If x (n) and h(n) two finite duration signals with length M and P respectively then the length of y(n) = x(n) * h(n) is **N=M+P-1** samples

CIRCULAR CONVOLUTION

- If x (n) and h(n) two periodic signals with period N then the length of

  $y(n) = x(n) \circledast h(n)$ is also N

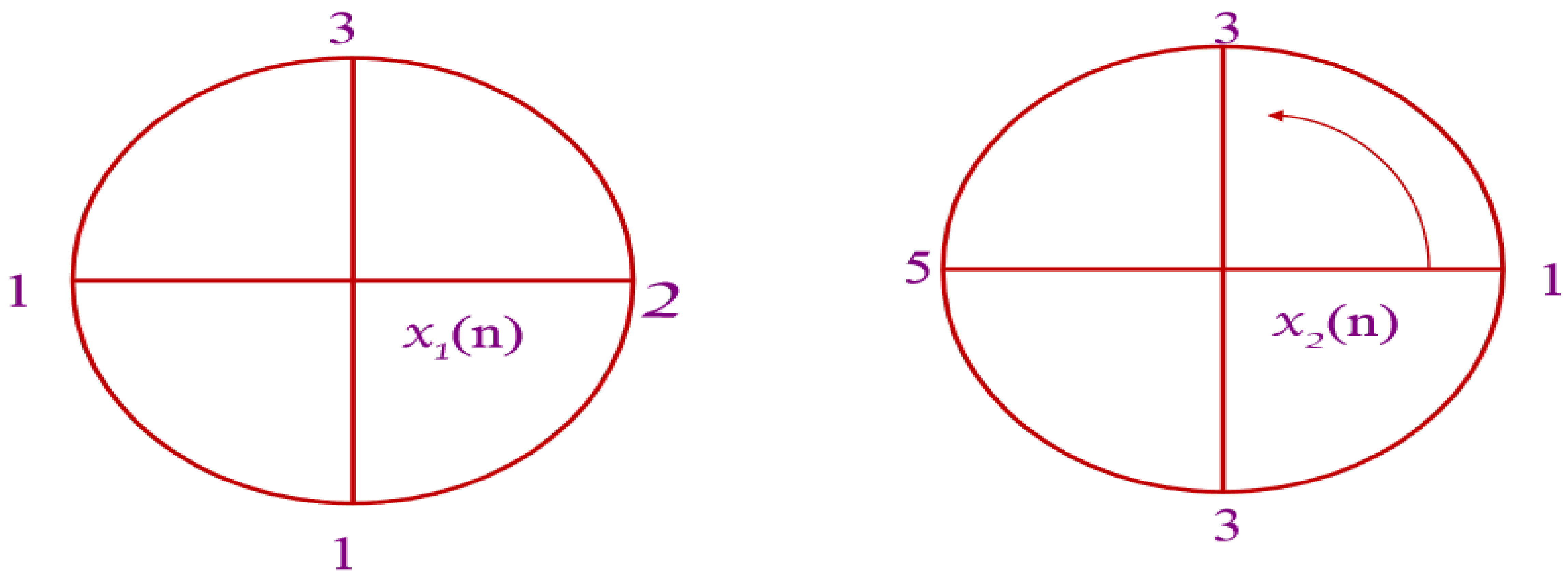
- The convolution of two periodic signal is also periodic and is circular convolution.

$$\mathbf{y(n) = x(n) \circledast h(n)}$$

- Linear Convolution can be obtained by circular convolution by changing the length of both signals x(n) and h(n) to N by zero padding

CIRCULAR CONVOLUTION

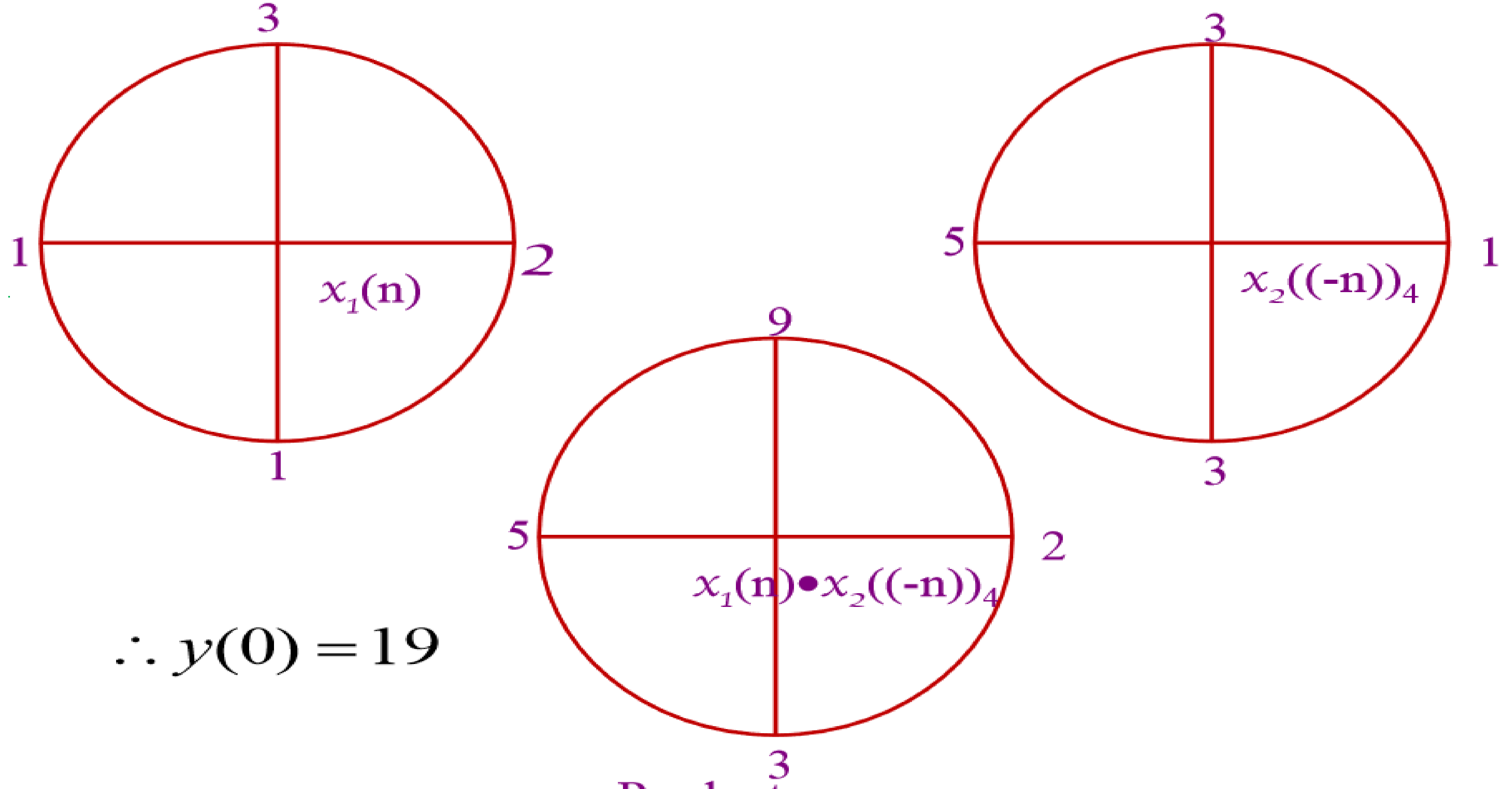
- Compute the circular convolution using time domain approach for the following sequence: $X_1(n) = \{2,3,1,1\}$ and $X_2(n) = \{1,3,5,3\}$



$$y(n) = \sum_{k=0}^{N-1} x_1(k).x_2(n-k) = \sum_{k=0}^{3} x_1(k).x_2(n-k)$$

CIRCULAR CONVOLUTION

$$y(0)=\sum_{k=0}^{3} x_1(k).x_2(-k)$$

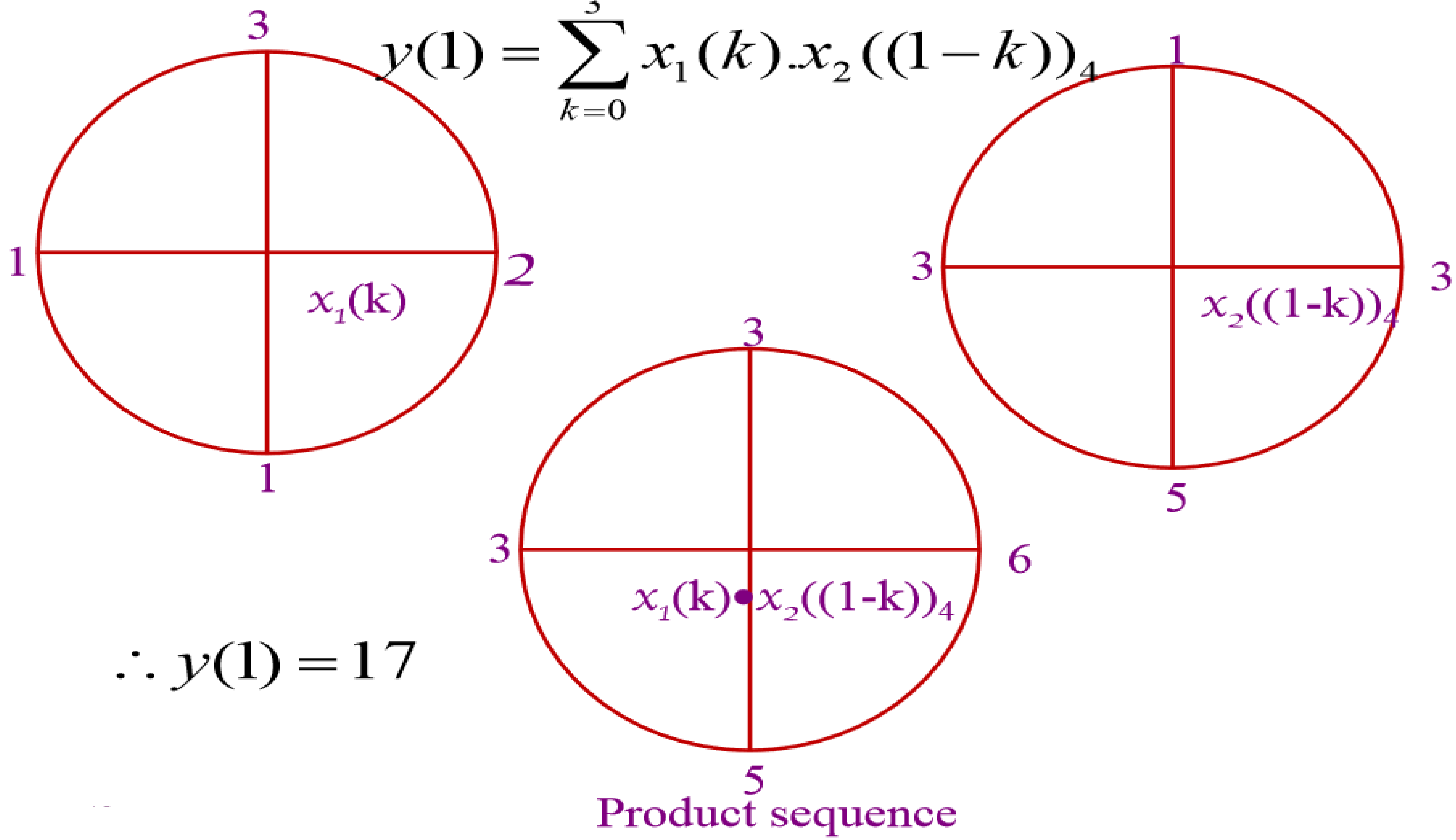


Product sequence

$$\therefore y(0)=19$$

CIRCULAR CONVOLUTION

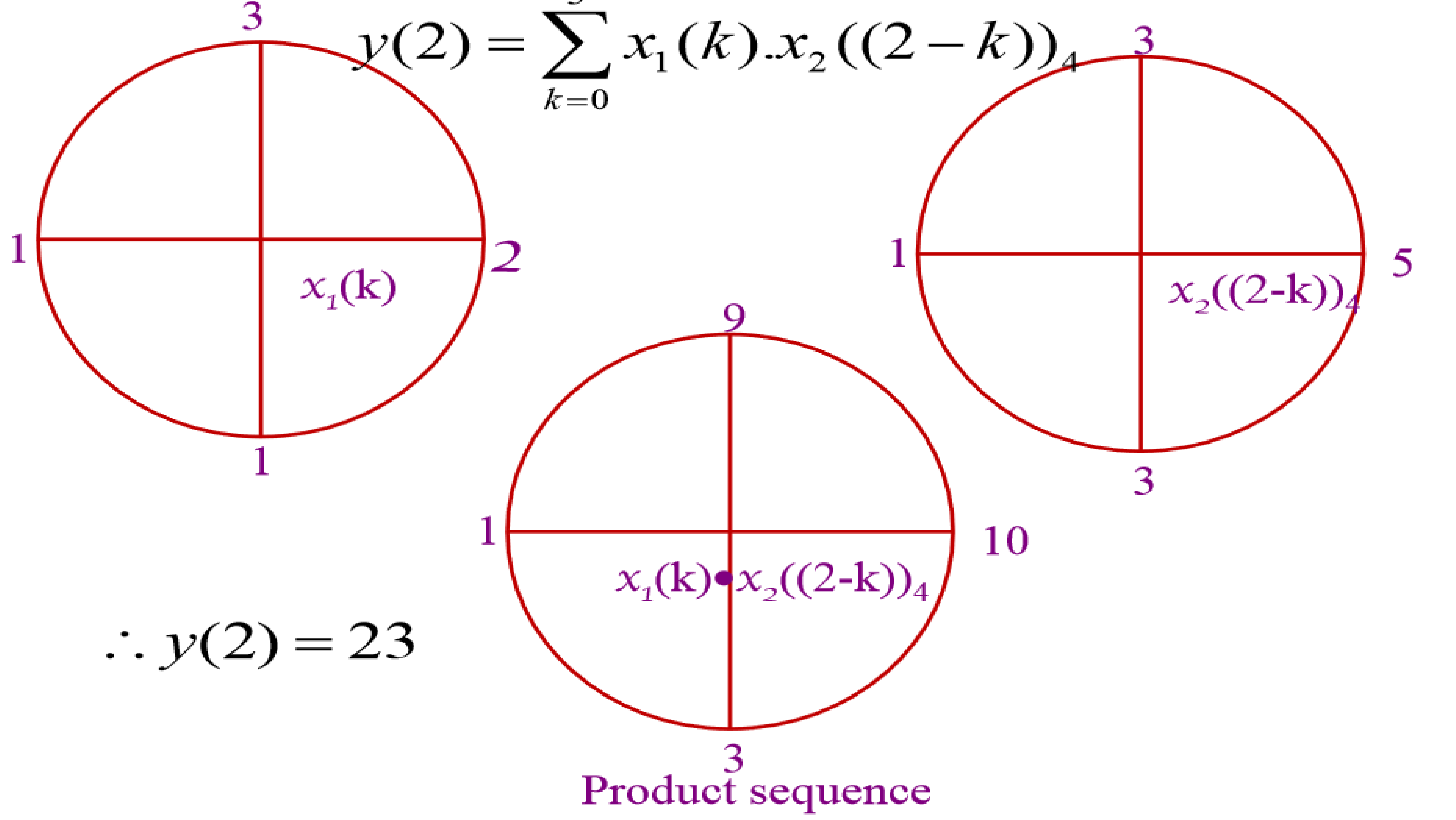
$$y(1) = \sum_{k=0}^{3} x_1(k).x_2((1-k))_4$$



$$\therefore y(1) = 17$$

CIRCULAR CONVOLUTION

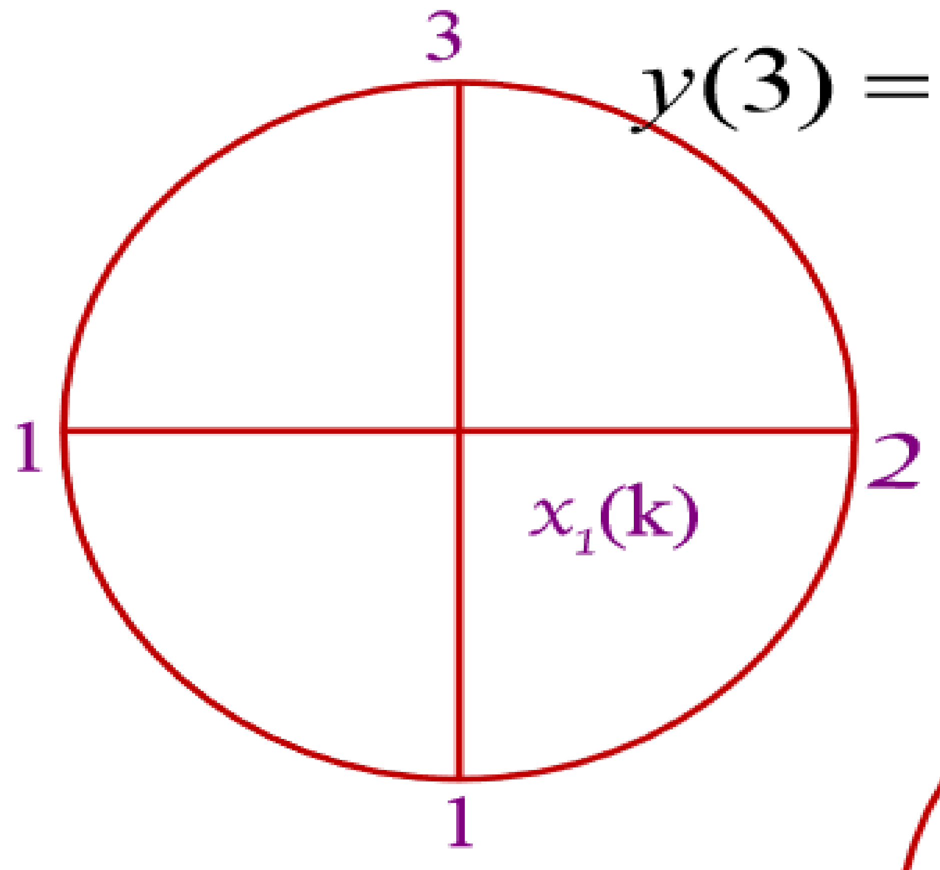
$$y(2) = \sum_{k=0}^{3} x_1(k).x_2((2-k))_4$$

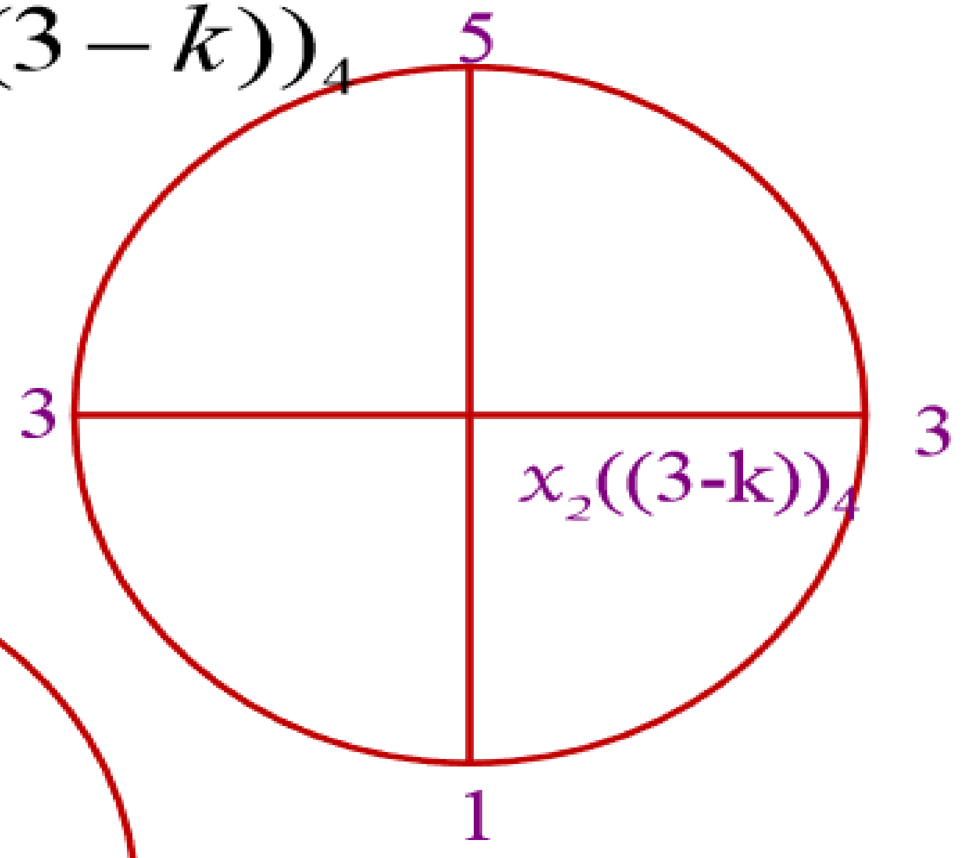


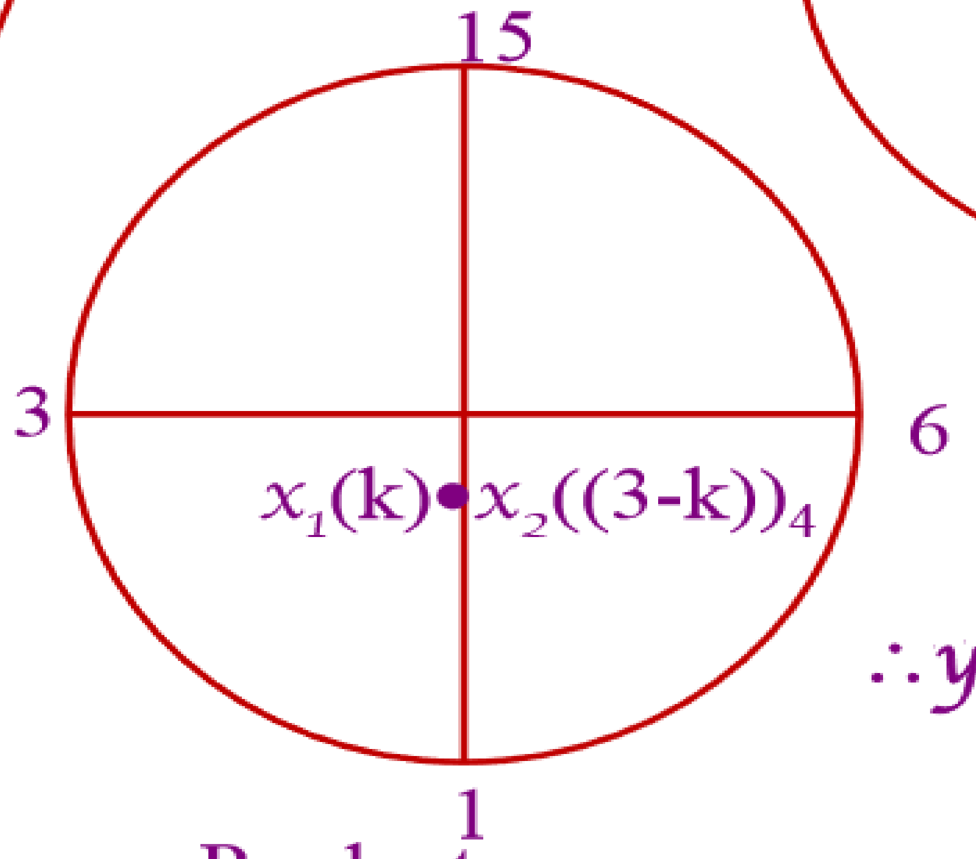
$$\therefore y(2) = 23$$

CIRCULAR CONVOLUTION

$$y(3) = \sum_{k=0}^{3} x_1(k).x_2((3-k))_4$$







Product sequence

$$\therefore y(3) = 25$$

$$\therefore y(n) = \{19,17,23,25\}$$

CIRCULAR CONVOLUTION – MATRIX APPROACH

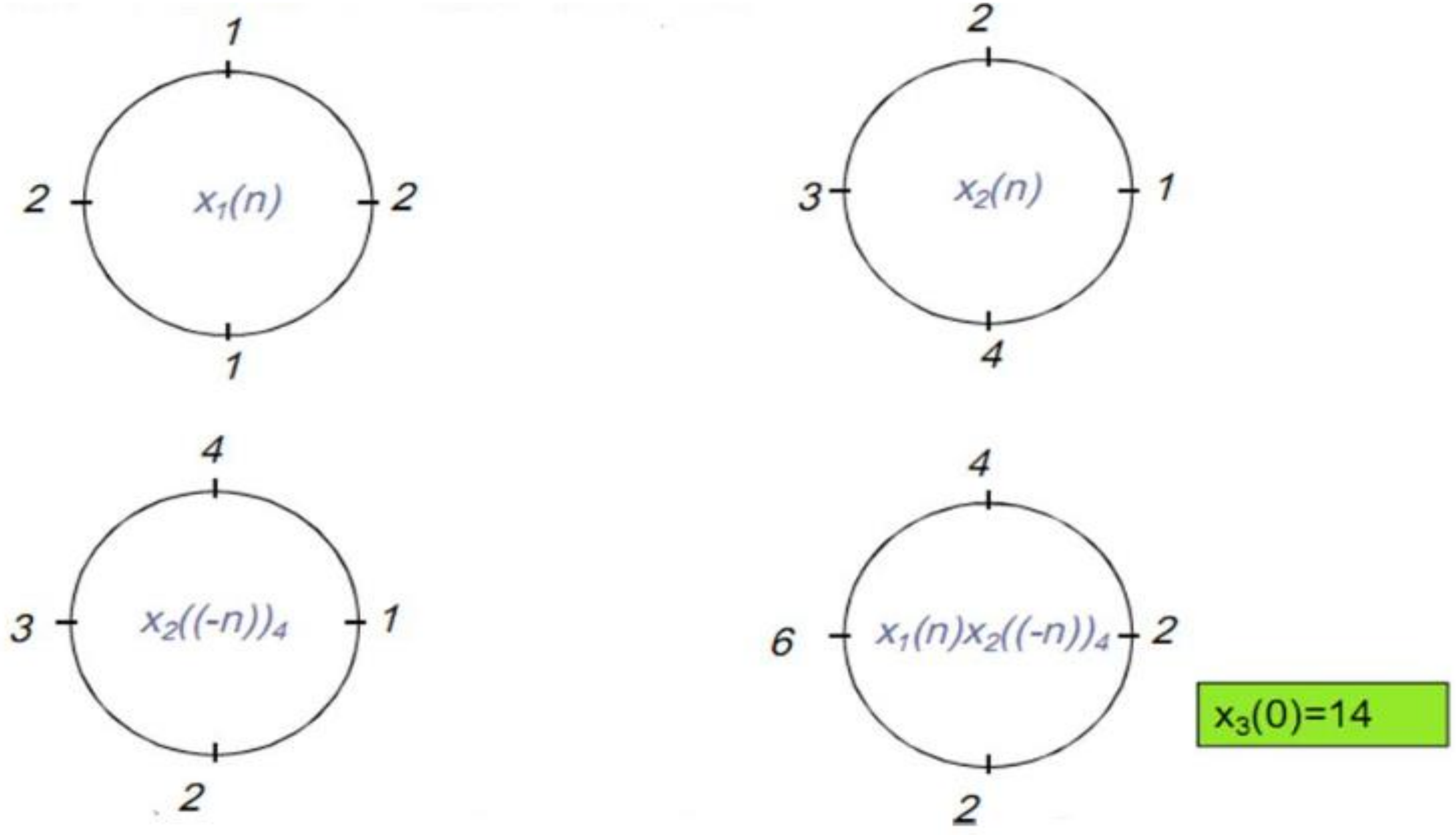
$$y[n] = \begin{bmatrix} h(0) & h(N-1) & .. & h(1) \\ h(1) & h(0) & .. & h(2) \\ \vdots & \vdots & .. & \vdots \\ h(N-1) & h(N-2) & .. & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

e.g. $h(n) = \{2,3,1,1\}$ and $x(n) = \{1,3,5,3\}$

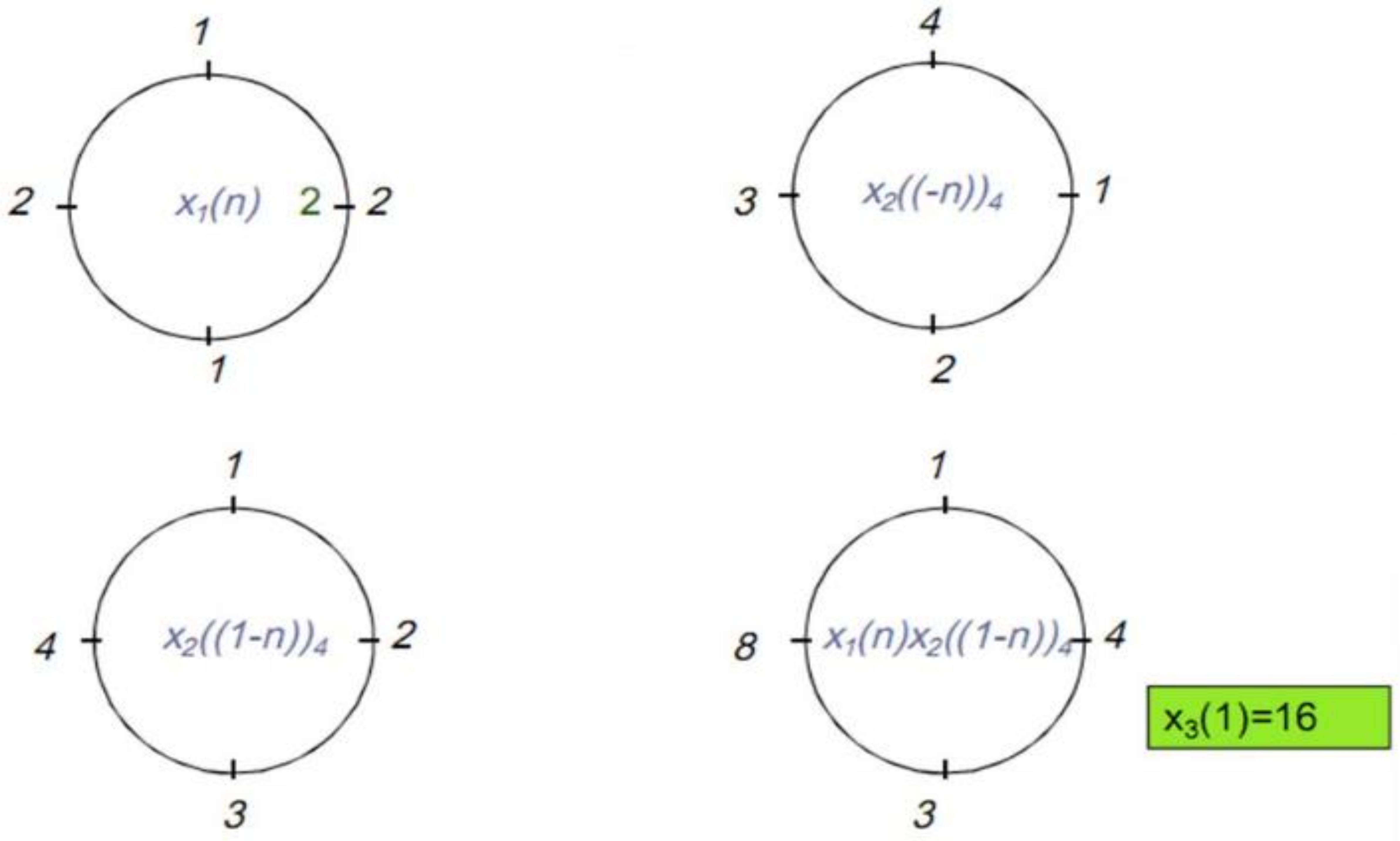
$$\therefore y[n] = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

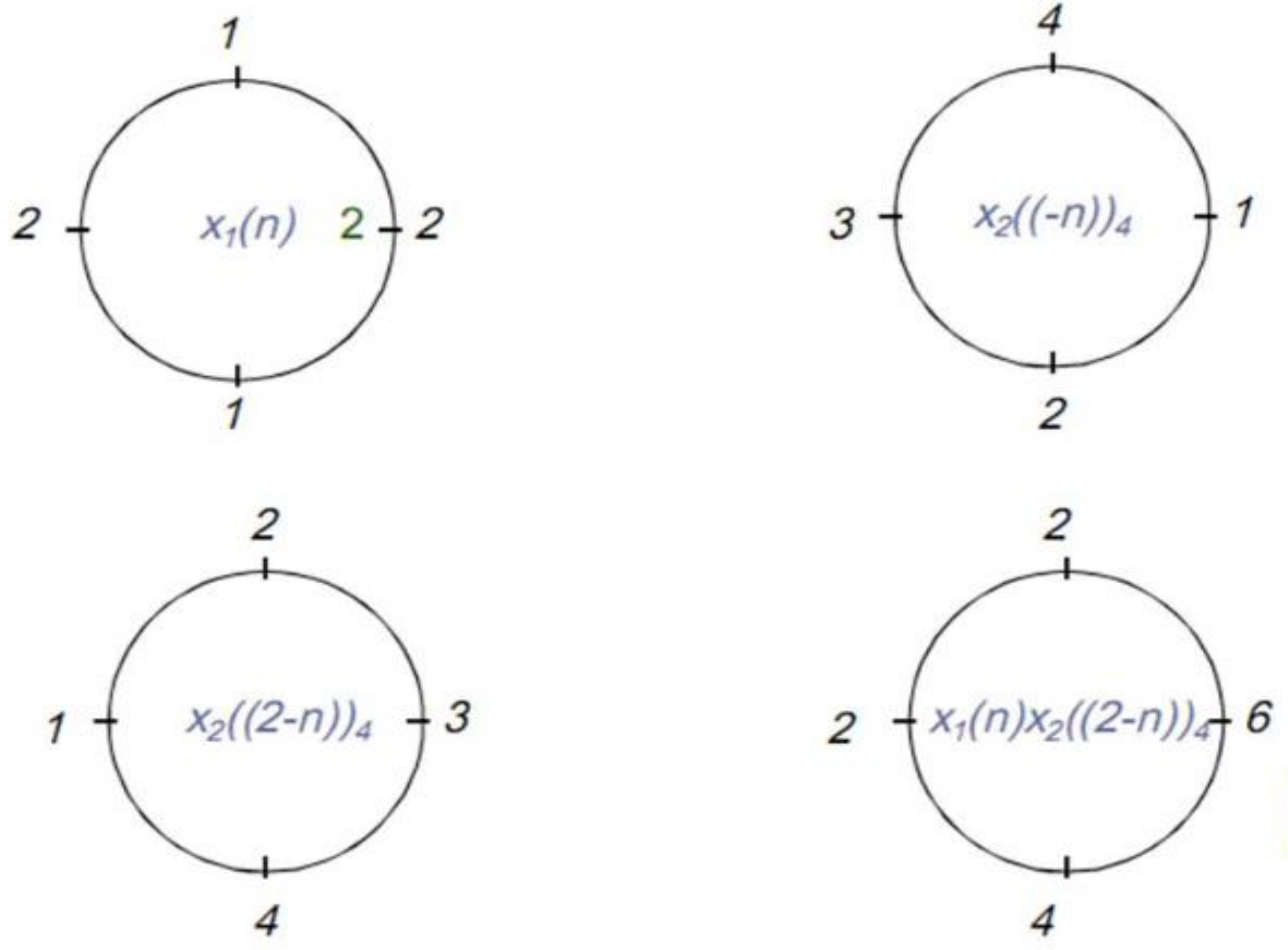
$$\therefore y[n] = \begin{bmatrix} 2+3+5+9 \\ 3+6+5+3 \\ 1+9+10+3 \\ 1+3+15+6 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \\ 23 \\ 25 \end{bmatrix}$$

Perform circular convolution $X_1(n) = \{2,1,2,1\}$ and $X_2(n) = \{1,2,3,4\}$



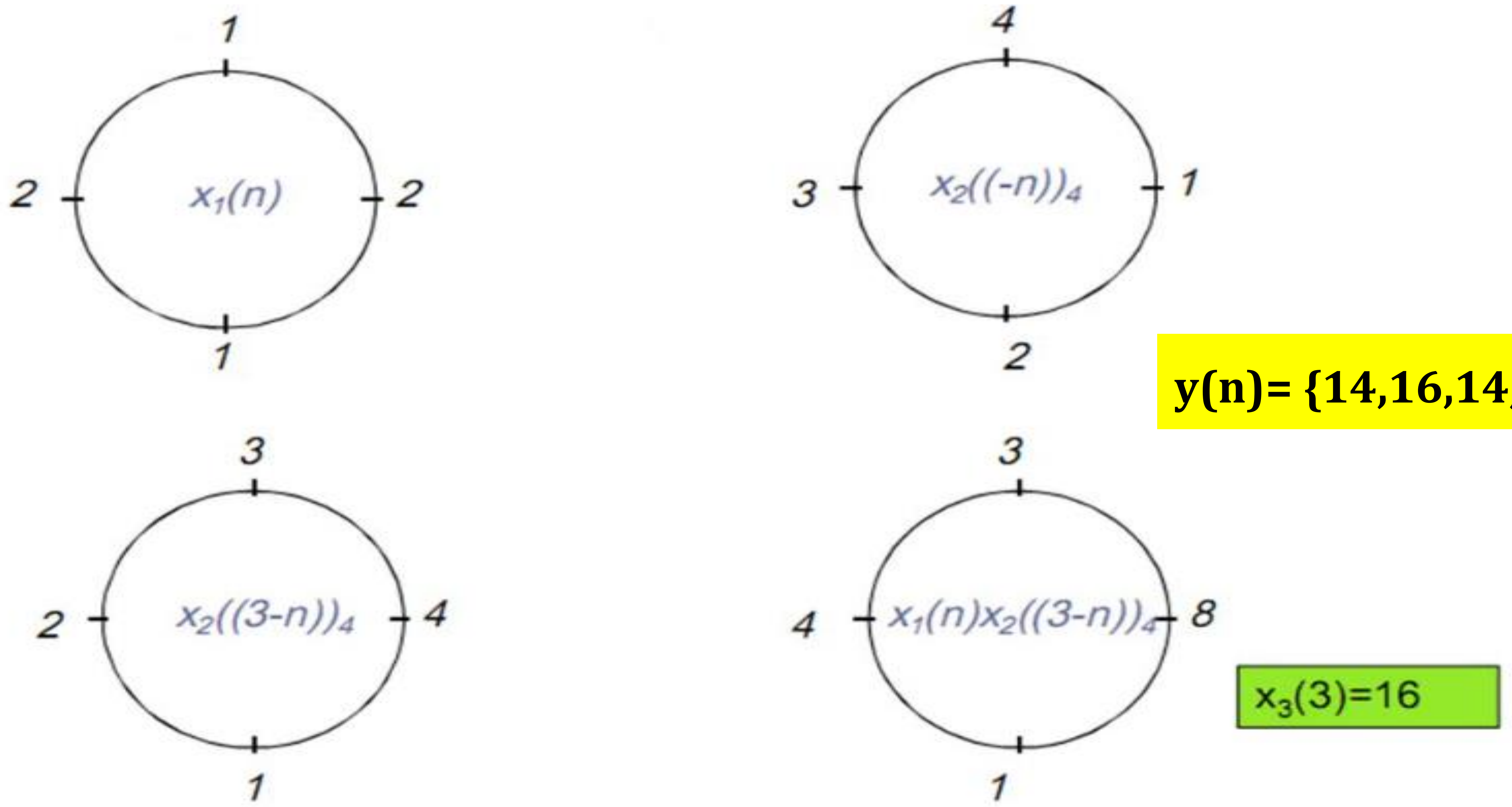
CIRCULAR CONVOLUTION





$x_3(2)=14$

CIRCULAR CONVOLUTION



y(n)= {14,16,14,16}

DIFFERENCE B/W LINEAR & CIRCULAR CONVOLUTION

S.No.	Linear Convolution	Circular Convolution
1	In case of convolution two signal sequences input signal x(n) and impulse response h(n) given by the same system, output y(n) is calculated	Multiplication of two DFT s is called as circular convolution.
2	Multiplication of two sequences in time domain is called as Linear convolution	Multiplication of two sequences in frequency domain is called as circular convolution.
3	Linear Convolution of two signals returns N-1 elements where N is sum of elements in both sequences.	Circular convolution returns same number of elements that of two signals.
4	$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$	$x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2[((n-m))_N]$

ASSESSMENT

1. Define Circular Convolution.
2. If x (n) and h(n) two finite duration signals with length M and P respectively then the length of y(n) = x(n) * h(n) is ------------------- samples
3. Mention some applications of Circular Convolution.
4. Determine circular convolution of $\mathbf{X_1(n) = \{2,1,2,1\}}$ **and** $\mathbf{X_2(n) = \{1,2,3,4\}}$
5. What is the difference between linear convolution and circular convolution.

THANK YOU