DEPARTMENT OF AEROSPACE ENGINEERING

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UNIT IV – FINITE VOLUME TECHNIQUES

Treatment of Derivatives in FDM-Like Finite Volume Techniques

Treatment of Derivatives

When derivatives are needed for the definition of viscous terms, these are commonly calculated using Gauss' theorem. For instance, for the cell-centered formulation shown in Fig to define a derivative in the vertex a, an integration over the shaded volume gives

$$\left(\frac{\partial\phi}{\partial x}\right)_a \approx \frac{1}{\Omega_a}\int_{\Omega_a}\frac{\partial\phi}{\partial x}dxdy = \frac{1}{\Omega_a}\int_{S_a}\phi\, dy$$

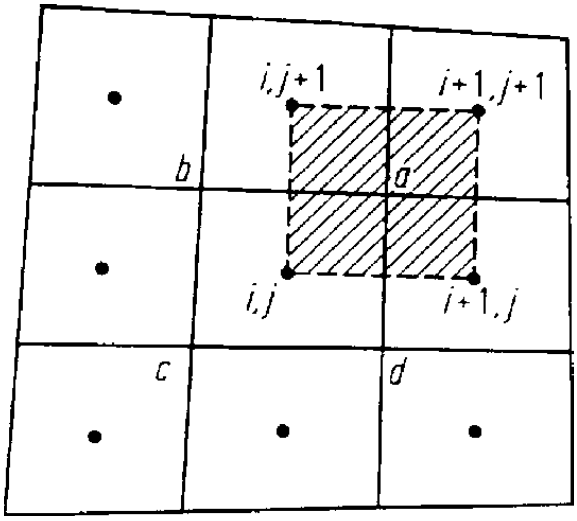
Thus

$$\left(\frac{\partial\phi}{\partial x}\right)_a \approx \frac{1}{\Omega_a}\left[\phi_{i+1,j+1}\frac{y_{i,j+1}-y_{i+1,j}}{2} + \phi_{i,j+1}\frac{y_{i,j}-y_{i+1,j+1}}{2} + \phi_{i,j}\frac{y_{i+1,j}-y_{i,j+1}}{2} + \phi_{i+1,j}\frac{y_{i+1,j+1}-y_{i,j}}{2}\right]$$

with

$$\Omega_a \approx \frac{y_{i+1,j+1}-y_{i,j}}{2}(x_{i+1,j}-x_{i,j+1}) + \frac{y_{i,j+1}-y_{i+1,j}}{2}(x_{i+1,j+1}-x_{i,j})$$

A similar procedure can be used for the other vertices of the cell abcd. This allows a definition of the viscous terms on the boundary of the cell.



Definition of a derivative

FDM-Like Finite Volume Methods

In the finite difference method, the nodes are at the vertices of the grid. This is particularly attractive to data on boundaries. For instance, pressure extrapolation at solid boundaries is then not necessary. A cell-centred FVM is therefore less attractive. A cell-vertex FVM does not have this drawback, but on the other hand, the flux through a volume surface is continuous. This does not allow an upwind definition of a flux.

More freedom in the definition of flux, combined with nodes at the vertices of the grid, can be obtained by using an interweaving grid. The interweaving grid can be constructed by connecting the cell centers. The cells of this interweaving grid can now be considered as control volumes for the nodes inside them. Fluxes at volume faces can, for instance, be defined as averages of fluxes calculated with function values in adjacent nodes. The semi-discretization is then very close to a finite difference semi-discretization and can be called a conservative finite difference method. We prefer here to call a finite volume method of this type a vertex-based FVM or a vertex-centered FVM. The method has gained much popularity in recent years. The central type discretization obtained with it is the same as the discretization by a Galerkin-FEM. So, it is very easy to bring concepts from FEM into this type of FVM. Moreover, it is very easy to use upwinding in this type of FVM.