SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23GET276 – VQAR II

II YEAR/ IV SEMESTER

UNIT 2 – QUANTITATIVE ABILITY IV

TOPIC – PERMUTATION AND COMBINATION

PERMUTATION AND COMBINATION

1. Factorial Notation:

Let n be a positive integer. Then, factorial n, denoted $n!$ is defined as:

$$n! = n(n - 1)(n - 2) \ldots 3.2.1.$$

Examples:

i. We define $0! = 1$.

ii. $4! = (4 \times 3 \times 2 \times 1) = 24$.

iii. $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

2. Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

i. All permutations (or arrangements) made with the letters *a*, *b*, *c* by taking two at a time are (***ab***, ***ba***, ***ac***, ***ca***, ***bc***, ***cb***).

ii. All permutations made with the letters *a*, *b*, *c* taking all at a time are:
(***abc***, ***acb***, ***bac***, ***bca***, ***cab***, ***cba***)

PERMUTATION AND COMBINATION

3. Number of Permutations:

Number of all permutations of n things, taken r at a time, is given by:

$$^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

Examples:

i. $^6P_2 = (6 \times 5) = 30.$

ii. $^7P_3 = (7 \times 6 \times 5) = 210.$

iii. **Cor. number of all permutations of n things, taken all at a time = $n!$.**

4. An Important Result:

If there are n subjects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r^{th} kind,
such that $(p_1 + p_2 + \ldots p_r) = n$.

Then, number of permutations of these n objects is $= \dfrac{n!}{(p_1!).(p_2)!\ldots\ldots(p_r!)}$

PERMUTATION AND COMBINATION

5. Combinations:

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a **combination**.

Examples:

1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

   Note: AB and BA represent the same selection.

2. All the combinations formed by *a*, *b*, *c* taking ***ab***, ***bc***, ***ca***.

3. The only combination that can be formed of three letters *a*, *b*, *c* taken all at a time is ***abc***.

4. Various groups of 2 out of four persons A, B, C, D are:

   AB, AC, AD, BC, BD, CD.

5. Note that *ab* *ba* are two different permutations but they represent the same combination.

6. Number of Combinations:

The number of all combinations of n things, taken r at a time is:

$$^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2)\ \dots\ \text{to } r \text{ factors}}{r!}.$$

Note:

i. $^nC_n = 1$ and $^nC_0 = 1$.

ii. $^nC_r = {}^nC_{(n-r)}$

Examples:

i. $^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$

ii. $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$

PERMUTATION AND COMBINATION

From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

A. 564

B. 645

C. 735

D. 756

E. None of these

Answer: Option **D**

Explanation:

We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only).

$\therefore$ Required number of ways $= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5)$

$$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1}\right) + ({}^7C_3 \times {}^6C_1) + ({}^7C_2)$$

$$= 525 + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6\right) + \left(\frac{7 \times 6}{2 \times 1}\right)$$

$$= (525 + 210 + 21)$$

$$= 756.$$

In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

A. 360

B. 480

C. 720

D. 5040

E. None of these

Answer: Option **C**

Explanation:

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LNDG (EAI).

Now, 5 (4 + 1 = 5) letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

∴ Required number of ways = (120 x 6) = 720.

PERMUTATION AND COMBINATION

In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

- A. 810
- B. 1440
- C. 2880
- D. 50400
- E. 5760

Answer: Option **D**

Explanation:

In the word 'CORPORATION', we treat the vowels OOAIO as one letter.

Thus, we have CRPRTN (OOAIO).

This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.

Number of ways arranging these letters = $\frac{7!}{2!}$ = 2520.

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged

in $\frac{5!}{3!}$ = 20 ways.

$\therefore$ Required number of ways = (2520 x 20) = 50400.

PERMUTATION AND COMBINATION

Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

A. 210

B. 1050

C. 25200

D. 21400

E. None of these

Answer: Option **C**

Explanation:

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$$= ({}^7C_3 \times {}^4C_2)$$

$$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right)$$

$$= 210.$$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves = 5!

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120.$$

∴ Required number of ways = (210 x 120) = 25200.

In how many ways can the letters of the word 'LEADER' be arranged?

A. 72

B. 144

C. 360

D. 720

E. None of these

Answer: Option **C**

Explanation:

The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

$$\therefore \text{ Required number of ways} = \frac{6!}{(1!)(2!)(1!)(1!)(1!)} = 360.$$

PERMUTATION AND COMBINATION

In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

A. 159

B. 194

C. 205

D. 209

E. None of these

Answer: Option **D**

Explanation:

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

∴ Required number of ways $= ({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4)$

$= ({}^6C_1 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2)$

$= (6 \times 4) + \left(\frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right) + \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4\right) + \left(\frac{6 \times 5}{2 \times 1}\right)$

$= (24 + 90 + 80 + 15)$

$= 209.$

PERMUTATION AND COMBINATION

In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?

A. 266

B. 5040

C. 11760

D. 86400

E. None of these

Answer: Option **C**

Explanation:

Required number of ways $= ({}^{8}C_{5} \times {}^{10}C_{6})$

$= ({}^{8}C_{3} \times {}^{10}C_{4})$

$$= \left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}\right)$$

$= 11760.$

PERMUTATION AND COMBINATION

A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

A. 32

B. 48

C. 64

D. 96

E. None of these

Answer: Option **C**

Explanation:

We may have(1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

$\therefore$ Required number of ways $= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$

$$= \left(3 \times \frac{6 \times 5}{2 \times 1}\right) + \left(\frac{3 \times 2}{2 \times 1} \times 6\right) + 1$$

$= (45 + 18 + 1)$

$= 64.$

In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

A. 63

B. 90

C. 126

D. 45

E. 135

Answer: Option **A**

Explanation:

Required number of ways = $({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \left(\frac{7 \times 6}{2 \times 1} \times 3\right) = 63.$

PERMUTATION AND COMBINATION

How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

A. 40

B. 400

C. 5040

D. 2520

Answer: Option **C**

Explanation:

'LOGARITHMS' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time.

$= {}^{10}P_4$

$= (10 \times 9 \times 8 \times 7)$

$= 5040.$

PERMUTATION AND COMBINATION

In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together?

A. 10080

B. 4989600

C. 120960

D. None of these

Answer: Option **C**

Explanation:

In the word 'MATHEMATICS', we treat the vowels AEAI as one letter.

Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

$\therefore$ Number of ways of arranging these letters $= \frac{8!}{(2!)(2!)} = 10080.$

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \frac{4!}{2!} = 12.$

$\therefore$ Required number of words $= (10080 \times 12) = 120960.$

PERMUTATION AND COMBINATION

In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

A. 120

B. 720

C. 4320

D. 2160

E. None of these

Answer: Option **B**

Explanation:

The word 'OPTICAL' contains 7 different letters.

When the vowels OIA are always together, they can be supposed to form one letter.

Then, we have to arrange the letters PTCL (OIA).

Now, 5 letters can be arranged in 5! = 120 ways.

The vowels (OIA) can be arranged among themselves in 3! = 6 ways.

∴ Required number of ways = (120 x 6) = 720.

THANK YOU