



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

First Shifting property:

If $\mathcal{L}\{f(t)\} = F(s)$ then

$$(i) \mathcal{L}[e^{-at}f(t)] = \left\{ \mathcal{L}[f(t)] \right\}_{s \rightarrow s+a} = F(s+a)$$

$$(ii) \mathcal{L}[e^{at}f(t)] = \left\{ \mathcal{L}[f(t)] \right\}_{s \rightarrow s-a} = F(s-a)$$

Proof:

(i) We know that,

$$\mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

$$\mathcal{L}[e^{-at}f(t)] = \int_0^{\infty} e^{-st} [e^{-at}f(t)] dt$$

$$= \int_0^{\infty} e^{-(s+a)t} f(t) dt$$

$$= F(s+a)$$

$$(ii) \mathcal{L}[e^{at}f(t)] = \int_0^{\infty} e^{-st} [e^{at}f(t)] dt$$

$$= \int_0^{\infty} e^{-(s-a)t} f(t) dt$$

$$= F(s-a)$$

Second Shifting Property:

If $\mathcal{L}\{f(t)\} = F(s)$ and $g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$

Then $\mathcal{L}[g(t)] = e^{-as} F(s)$.

Proof:

$$\mathcal{L}[g(t)] = \int_0^{\infty} e^{-st} g(t) dt$$

$$= \int_a^{\infty} e^{-st} g(t) dt + \int_0^a e^{-st} g(t) dt$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp;

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

$$\mathcal{L}[g(t)] = 0 + \int_a^{\infty} e^{-st} f(t-a) dt$$

$$= \int_a^{\infty} e^{-st} f(t-a) dt$$

$$\text{Put } t-a = u \Rightarrow dt = du$$

$$\text{When } t = a \Rightarrow u = 0$$

$$t \rightarrow \infty \Rightarrow u \rightarrow \infty$$

$$\mathcal{L}[g(t)] = \int_0^{\infty} e^{-s(u+a)} f(u) du$$

$$= \int_0^{\infty} e^{-us} e^{-as} f(u) du$$

$$= e^{-as} \int_0^{\infty} e^{-us} f(u) du$$

$$= e^{-as} \int_0^{\infty} e^{-st} f(t) dt \quad \text{Replace } u \rightarrow t$$

$$\mathcal{L}[g(t)] = e^{-as} F(s)$$

Laplace transforms of derivatives:

If $\mathcal{L}[f(t)] = F(s)$ then

$$\mathcal{L}[f'(t)] = sF(s) - f(0)$$

Proof:

$$\mathcal{L}[f'(t)] = \int_0^{\infty} e^{-st} f'(t) dt$$

Integrating by parts we get,

$$= \left[e^{-st} f(t) \right]_0^{\infty} - \int_0^{\infty} f(t) (-se^{-st}) dt$$



DEPARTMENT OF MATHEMATICS

$$L\{f'(t)\} = p \left[e^{-\infty} f(\infty) - e^0 f(0) \right] + s \int_0^{\infty} e^{-st} f(t) dt$$

$$L\{f'(t)\} = -f(0) + s L\{f(t)\}$$

$$= s F(s) - f(0)$$

Corollary:

Let $f''(t) = s^2 F(s) - s f(0) - f'(0)$

Let $L\{g'(t)\} = s G(s) - g(0)$

We know that,

$L\{f'(t)\} = s L\{f(t)\} - f(0)$

Replace $f(t) \rightarrow f'(t)$ & $f'(t) \rightarrow f''(t)$ & $f(0) \rightarrow f'(0)$

$\Rightarrow L\{f''(t)\} = s L\{f'(t)\} - f'(0)$

$= s [s L\{f(t)\} - f(0)] - f'(0)$

$= s^2 L\{f(t)\} - s f(0) - f'(0)$

$= s^2 F(s) - s f(0) - f'(0)$

Laplace Transform of integrals:

If $L\{f(t)\} = F(s)$ then $L\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s}$

Proof:

Let $g(t) = \int_0^t f(t) dt$ and $g(0) = 0$

then $g'(t) = f(t)$

WKT $L\{g'(t)\} = s L\{g(t)\} - g(0)$

$= s L\{g(t)\}$

$\Rightarrow L\{g(t)\} = \frac{1}{s} L\{g'(t)\}$



DEPARTMENT OF MATHEMATICS

$$\Rightarrow \mathcal{L} \left[\int_0^t f(t) dt \right] = \frac{1}{s} \mathcal{L} [f(t)] \quad \left\{ \begin{array}{l} \because g(t) = \int_0^t f(t) \\ g'(t) = f(t) \end{array} \right.$$

$$\Rightarrow \mathcal{L} \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s}$$

Derivative of Laplace Transform (or) Laplace transform of $t f(t)$:

If $\mathcal{L} [f(t)] = F(s)$ then

$$\mathcal{L} [t f(t)] = -\frac{d}{ds} F(s)$$

Proof:

We know that,

$$\mathcal{L} [f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \int_0^{\infty} e^{-st} f(t) dt$$

$$= \int_0^{\infty} \frac{\partial}{\partial s} (e^{-st}) f(t) dt$$

$$= \int_0^{\infty} -t e^{-st} f(t) dt$$

$$= -\int_0^{\infty} e^{-st} t f(t) dt$$

$$= -\mathcal{L} [t f(t)]$$

$$\Rightarrow \mathcal{L} [t f(t)] = -\frac{d}{ds} [F(s)]$$

In general,

$$\mathcal{L} [t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)]$$



DEPARTMENT OF MATHEMATICS

Problems :

Change of Scale property :

① Find $L[\sinh 3t]$ by using 'change of scale' property

Soln:

$$L[\sinh ht] = \frac{1}{s^2 - 1} = F(s)$$

$$L[\sinh 3t] = \frac{1}{3} F\left(\frac{s}{3}\right)$$

$$= \frac{1}{3} \frac{1}{\left(\frac{s}{3}\right)^2 - 1}$$

$$= \frac{1}{3} \left(\frac{9}{s^2 - 9}\right)$$

$$= \frac{3}{s^2 - 9}$$

② Find $L(\cos 5t)$ using change of scale property?

Soln:

$$L(\cos t) = \frac{s}{s^2 + 1} = F(s)$$

$$L(\cos 5t) = \frac{1}{5} F\left(\frac{s}{5}\right)$$

$$= \frac{1}{5} \left[\frac{s/5}{\left(\frac{s}{5}\right)^2 + 1} \right]$$

$$= \frac{1}{5} \left[\frac{5s}{s^2 + 25} \right]$$

$$= \frac{s}{s^2 + 25}$$