Inverse Laplace Transform:

If the Laplace transform of $f(t)$ is $F(s)$ i.e., $L[f(t)] = F(s)$. Then $f(t)$ is called an inverse Laplace transform of $F(s)$ and is written as $f(t) = L^{-1}[F(s)]$ where L^{-1} is called the inverse Laplace transform Operator.

Table of Inverse Laplace Transforms:

	$L[f(t)] = F(s)$	$L^{-1}[F(s)] = f(t)$
(1)	$L(1) = \frac{1}{s}$	$L^{-1}\left(\frac{1}{s}\right) = 1$
(2)	$L(t) = \frac{1}{s^2}$	$L^{-1}\left(\frac{1}{s^2}\right) = t$
(3)	$L(t^n) = \frac{n!}{s^{n+1}}$	$L^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$
(4)	$L(e^{at}) = \frac{1}{s-a}$	$L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$
(5)	$L(e^{-at}) = \frac{1}{s+a}$	$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$
(6)	$L(\sin at) = \frac{a}{s^2+a^2}$	$L^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$
(7)	$L\left(\frac{\sin at}{a}\right) = \frac{1}{s^2+a^2}$	$L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}$
(8)	$L(\cos at) = \frac{s}{s^2+a^2}$	$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$
(9)	$L(\sinh at) = \frac{a}{s^2-a^2}$	$L^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$
(10)	$L(\cosh at) = \frac{s}{s^2-a^2}$	$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$
(11)	$L[\delta(t)] = 1$	$L^{-1}(1) = \delta(t)$

Important Results in Laplace Transforms:

If a and b are constants while $F(s)$ and $G(s)$ are the Laplace transform of $f(t)$ & $g(t)$ respectively.

1. **Linearity property:**

$$\mathcal{L}^{-1}[aF(s)+bG(s)] = a\mathcal{L}^{-1}[F(s)] + b\mathcal{L}^{-1}[G(s)]$$

2. **First Shifting property:**

(i) $\mathcal{L}^{-1}[F(s+a)] = e^{-at}\mathcal{L}^{-1}[F(s)]$

(ii) $\mathcal{L}^{-1}[F(s-a)] = e^{at}\mathcal{L}^{-1}[F(s)]$

3. **Second Shifting property:**

If $\mathcal{L}^{-1}[F(s)] = f(t)$ then

$\mathcal{L}^{-1}[e^{-as}F(s)] = g(t)$ where

$$g(t) = \begin{cases} f(t-a), & t>a \\ 0, & t<a \end{cases}$$

4. **Change of Scale property:**

$$\mathcal{L}^{-1}[F(ks)] = \frac{1}{k} f\left(\frac{t}{k}\right)$$

5. **Multiplication by s:**

If $\mathcal{L}^{-1}[F(s)] = f(t)$ and $f(0) = 0$

Then $\mathcal{L}^{-1}[sF(s)] = \frac{d}{dt}\mathcal{L}^{-1}[F(s)]$

Note:

If $f(0) \neq 0$ then $\mathcal{L}^{-1}[sF(s)] = \frac{d}{dt}\mathcal{L}^{-1}[F(s)] + f(0)\delta(t)$.

Problem Identification:

If $L^{-1}\left[\frac{s}{\text{quadratic equation}}\right]$ then use result 5.

If $L^{-1}\left[\frac{s}{\text{linear equation}}\right]$ then we use the above note.

6. **Division by s:**

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,dt$$

7. **Inverse Laplace transform of derivatives:**

If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F'(s)] = -tL^{-1}[F(s)]$

Problem Identification:

If $L^{-1}\left[\frac{s + \text{any term}}{(\text{quadratic eqn})^2}\right]$ then we use the above result.

8. **Note:**

If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F(s)] = -\frac{1}{t}L^{-1}\left[\frac{d}{ds}F(s)\right]$

Problem Identification:

If L^{-1} [log function or cot function or tan fn] then we use the above result.