



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Higher order linear differential equations with constant coefficients

Type : II

$$R(x) = \cos ax \text{ (or) } \sin ax.$$

\* Replace  $D^2 = -(a)^2$

$\frac{1}{P} = \text{Integrate.}$

Example : 1.

Solve  $(D^2 + 4)y = \cos 2x$

Soln: Given,

$$(D^2 + 4)y = \cos 2x$$

The A.E is  $m^2 + 4 = 0$

$$m^2 = -4$$

$$m^2 = (-1) \cdot 4$$

$$m^2 = i^2 \cdot 2^2$$

$$m = \pm 2i$$

$$\alpha + \beta i = 0 + 2i \Rightarrow \alpha = 0; \beta = 2$$

$$C.F = e^{\alpha x} [A \cos \beta x + B \sin \beta x]$$

$$a=2 \\ D^2 = -(a)^2 = -4$$

$$P.I = \frac{1}{D^2 + 4} \cos 2x$$

$$P.I = \frac{1}{0} \cdot \cos 2x.$$

$$P.I = \frac{x}{2D} \cdot \cos 2x$$

$$= \frac{x}{2} \int \cos 2x \cdot dx$$

$$= \frac{x}{2} \cdot \frac{\sin 2x}{2}$$

$$P.I = \frac{x \sin 2x}{4}$$

$$y = C.F + P.I$$



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Type : LI

$R(x) = x^n$

- $(1+x)^{-1} = 1 - x + x^2 + \dots$
- $(1-x)^{-1} = 1 + x + x^2 + \dots$
- $(1+x)^1 = 1 + x + x^2 + \dots$
- $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$
- $\frac{1}{D} = \int f(x) \cdot dx$
- $D = \text{differentiate}$

Example: 1

Find the particular integral of  $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

Soln:  $D^2y + Dy = x^2 + 2x + 4$

$(D^2 + D)y = x^2 + 2x + 4$

The A.E is  $m^2 + m = 0$

$m(m+1) = 0$

$m = 0 \quad m = -1$

C.F =  $Ae^{0x} + Be^{-x}$

P.I =  $\frac{1}{f(D)} \cdot R(x)$

$= \frac{1}{D(D+1)} \cdot x^2 + 2x + 4$

$= \frac{1}{D(D+1)} \cdot x^2 + 2x + 4$

$= \frac{1}{D} (D+1)^{-1} (x^2 + 2x + 4)$

$= \frac{1}{D} [1 - D + D^2 - \dots] (x^2 + 2x + 4)$



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$$\begin{aligned} &= \frac{1}{D} [x^2 + 2x + 4 - D(x^2 + 2x + 4) + D^2(x^2 - 2x + 4)] \dots \\ &= \frac{1}{D} [x^2 + 2x + 4 - 2x - 2 + 2] \\ &= \frac{1}{D} [x^2 + 4] \\ &= \int (x^2 + 4) dx \\ \text{P.I.} &= \frac{x^3}{3} + 4x \\ y &= C.F. + P.I. \\ y &= (Ae^{0x} + Be^{-x}) + \frac{x^3}{3} + 4x \end{aligned}$$