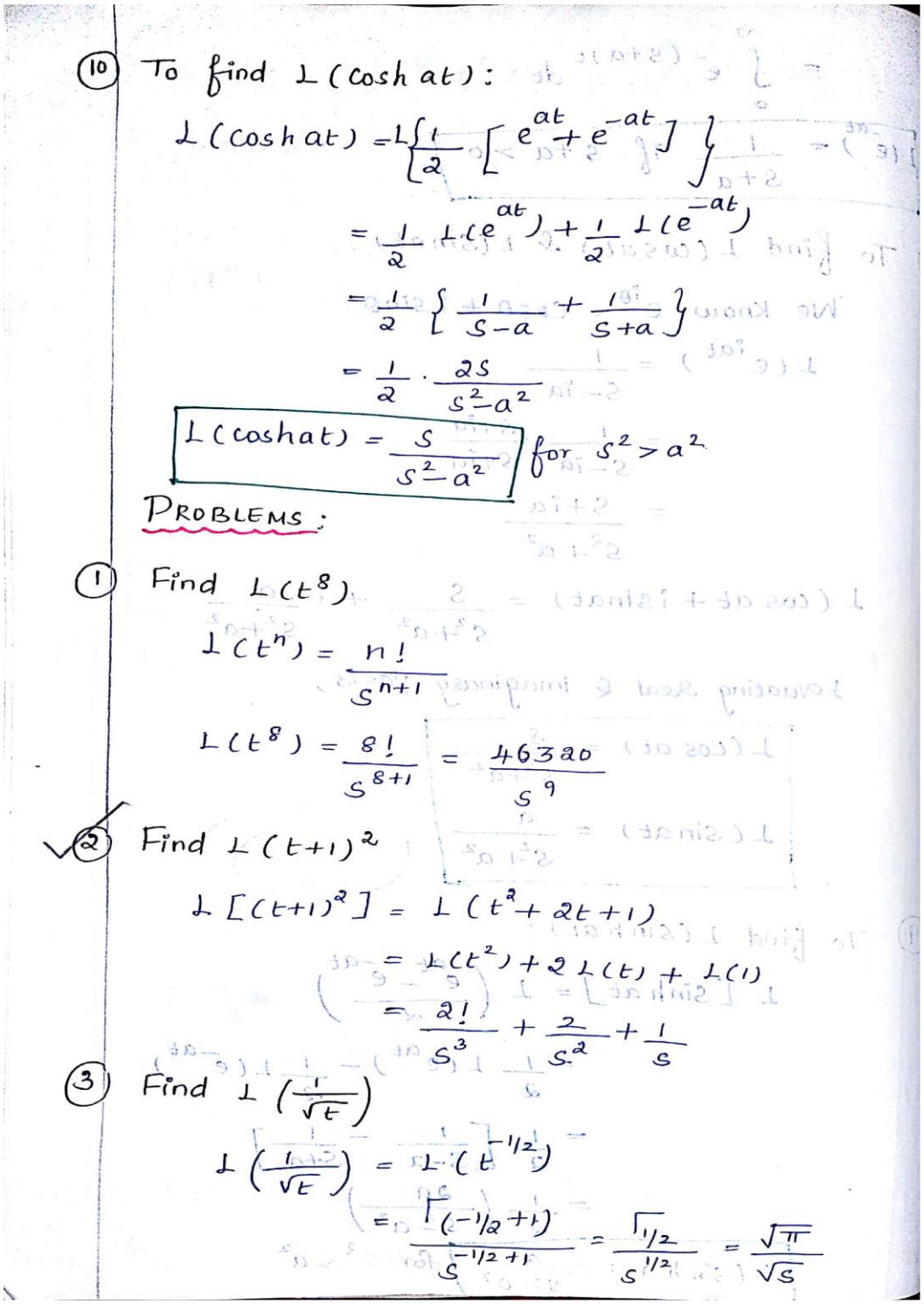


(10) To find $\mathcal{L}(\cosh at)$:

$$\mathcal{L}(\cosh at) = \mathcal{L}\left\{\frac{1}{2}\left[e^{at} + e^{-at}\right]\right\}$$

$$= \frac{1}{2}\mathcal{L}(e^{at}) + \frac{1}{2}\mathcal{L}(e^{-at})$$

$$= \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\}$$

$$= \frac{1}{2}\cdot\frac{2s}{s^2-a^2}$$

$$\boxed{\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}} \quad \text{for } s^2 > a^2$$

PROBLEMS:

(1) Find $\mathcal{L}(t^8)$

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(t^8) = \frac{8!}{s^{8+1}} = \frac{40320}{s^9}$$

(2) Find $\mathcal{L}(t+1)^2$

$$\mathcal{L}[(t+1)^2] = \mathcal{L}(t^2+2t+1)$$

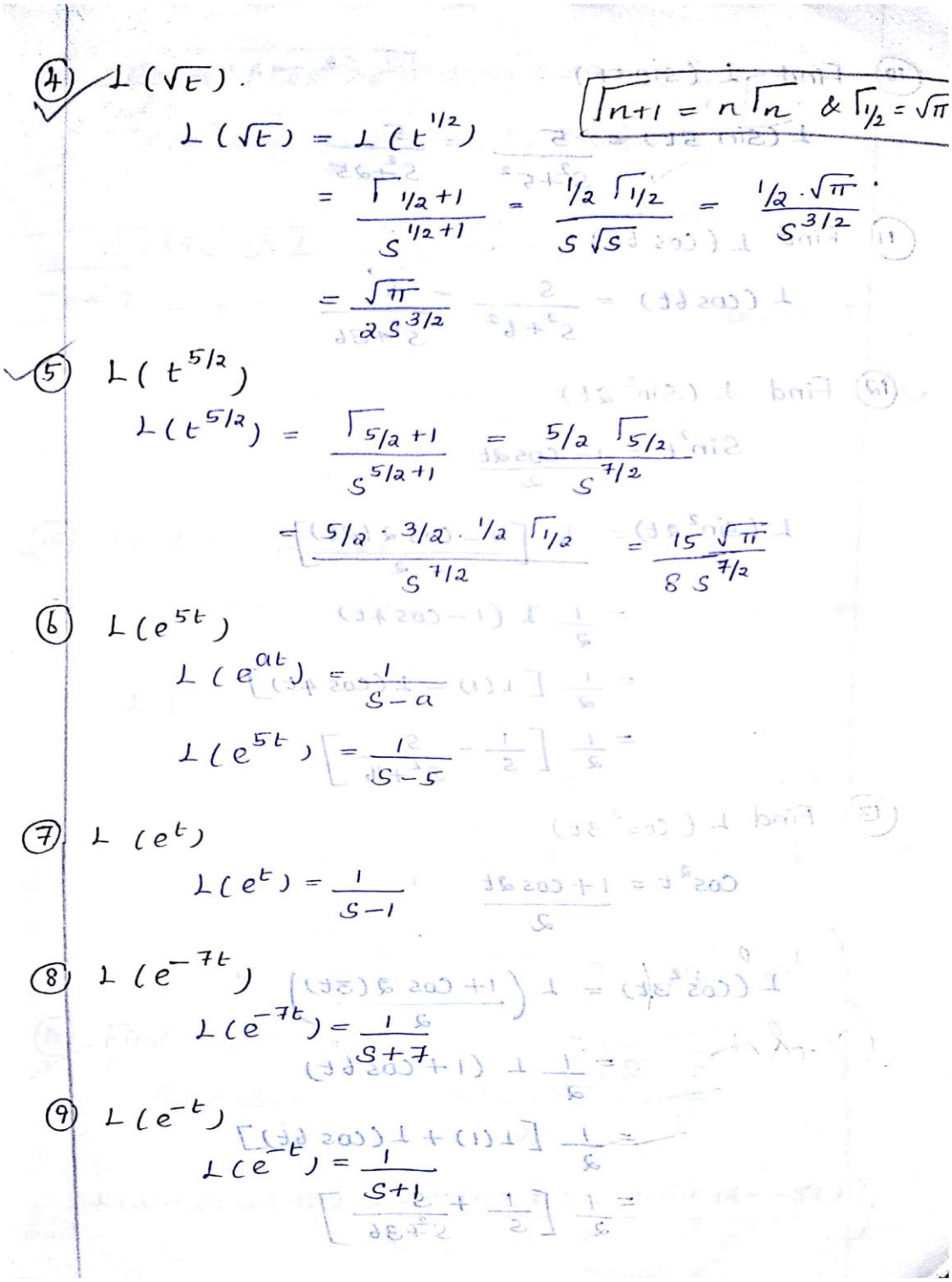
$$= \mathcal{L}(t^2) + 2\mathcal{L}(t) + \mathcal{L}(1)$$

$$= \frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

(3) Find $\mathcal{L}\left(\frac{1}{\sqrt{t}}\right)$

$$\mathcal{L}\left(\frac{1}{\sqrt{t}}\right) = \mathcal{L}(t^{-1/2})$$

$$= \frac{\Gamma(-1/2+1)}{s^{-1/2+1}} = \frac{\Gamma_{1/2}}{s^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{s}}$$



(4) $L(\sqrt{t})$.

$\Gamma(n+1) = n\Gamma(n)$ & $\Gamma(1/2) = \sqrt{\pi}$

$$L(\sqrt{t}) = L(t^{1/2}) = \frac{\Gamma(1/2+1)}{s^{1/2+1}} = \frac{1/2\,\Gamma(1/2)}{s\sqrt{s}} = \frac{1/2\cdot\sqrt{\pi}}{s^{3/2}}$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}$$

(5) $L(t^{5/2})$

$$L(t^{5/2}) = \frac{\Gamma(5/2+1)}{s^{5/2+1}} = \frac{5/2\,\Gamma(5/2)}{s^{7/2}}$$

$$= \frac{5/2\cdot 3/2\cdot 1/2\,\Gamma(1/2)}{s^{7/2}} = \frac{15\sqrt{\pi}}{8s^{7/2}}$$

(6) $L(e^{5t})$

$$L(e^{at}) = \frac{1}{s-a}$$

$$L(e^{5t}) = \frac{1}{s-5}$$

(7) $L(e^{t})$

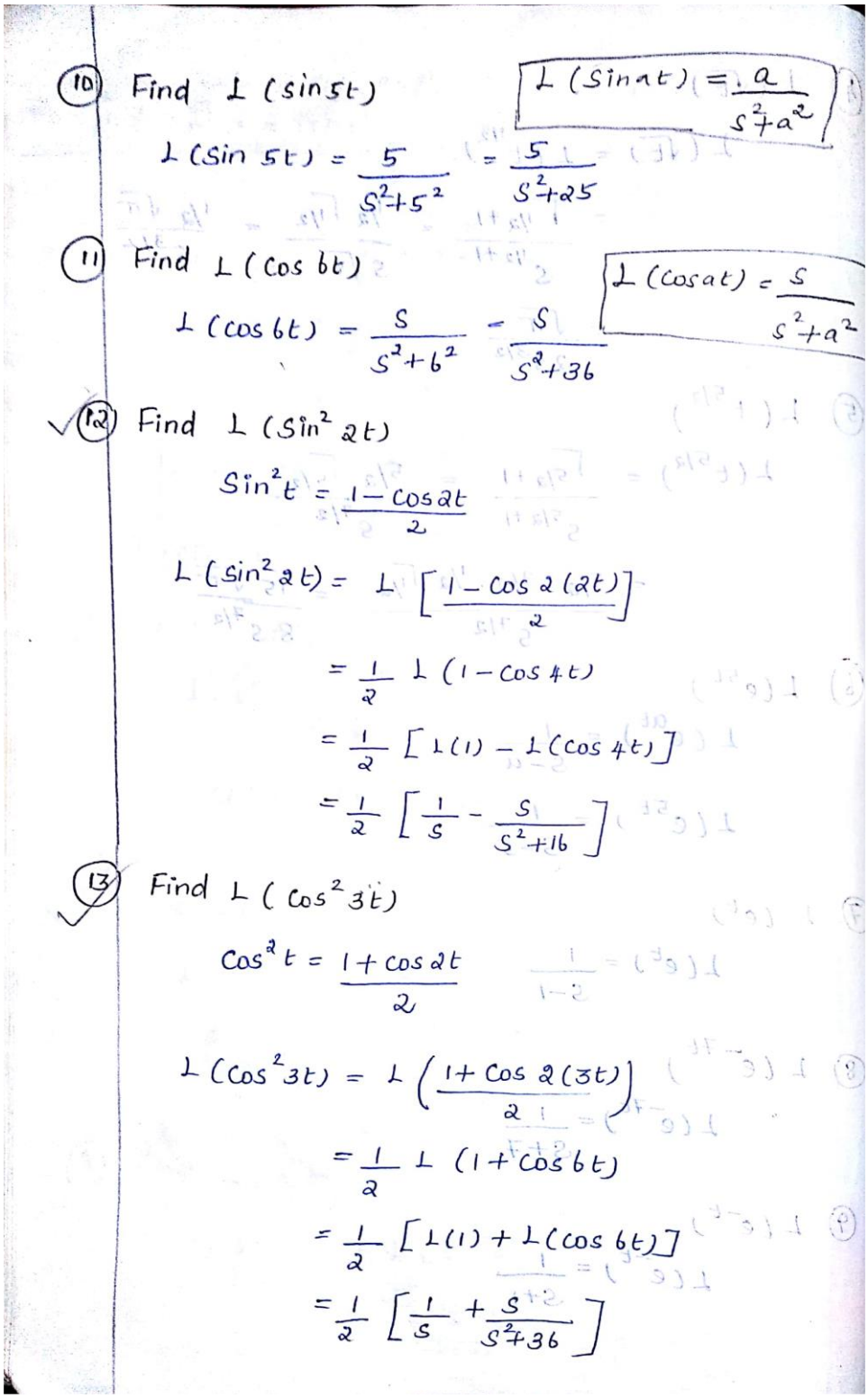
$$L(e^{t}) = \frac{1}{s-1}$$

(8) $L(e^{-7t})$

$$L(e^{-7t}) = \frac{1}{s+7}$$

(9) $L(e^{-t})$

$$L(e^{-t}) = \frac{1}{s+1}$$



(10) Find $\mathcal{L}(\sin 5t)$

$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$

$\mathcal{L}(\sin 5t) = \frac{5}{s^2+5^2} = \frac{5}{s^2+25}$

(11) Find $\mathcal{L}(\cos 6t)$

$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$

$\mathcal{L}(\cos 6t) = \frac{s}{s^2+6^2} = \frac{s}{s^2+36}$

(12) Find $\mathcal{L}(\sin^2 2t)$

$\sin^2 t = \frac{1-\cos 2t}{2}$

$\mathcal{L}(\sin^2 2t) = \mathcal{L}\left[\frac{1-\cos 2(2t)}{2}\right]$

$= \frac{1}{2}\mathcal{L}(1-\cos 4t)$

$= \frac{1}{2}[\mathcal{L}(1) - \mathcal{L}(\cos 4t)]$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+16}\right]$

(13) Find $\mathcal{L}(\cos^2 3t)$

$\cos^2 t = \frac{1+\cos 2t}{2}$

$\mathcal{L}(\cos^2 3t) = \mathcal{L}\left(\frac{1+\cos 2(3t)}{2}\right)$

$= \frac{1}{2}\mathcal{L}(1+\cos 6t)$

$= \frac{1}{2}[\mathcal{L}(1) + \mathcal{L}(\cos 6t)]$

$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+36}\right]$

(14) Find $L(\cos^3 2t)$

$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$

$L[\cos^3 2t] = L\left[\frac{\cos 3(2t) + 3\cos(2t)}{4}\right]$

$= \frac{1}{4}\{L(\cos 6t) + 3L(\cos 2t)\}$

$= \frac{1}{4}\left\{\frac{s}{s^2+36} + 3\cdot\frac{s}{s^2+4}\right\}$

$= \frac{1}{4}\left\{\frac{s}{s^2+36} + \frac{3s}{s^2+4}\right\}$

(15) Find $L(\sin^3 3t)$

$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$

$L(\sin^3 3t) = L\left[\frac{3\sin 3t - \sin 3(3t)}{4}\right]$

$= \frac{1}{4}\{3L(\sin 3t) - L(\sin 9t)\}$

$= \frac{1}{4}\left\{3\left(\frac{3}{s^2+3^2}\right) - \frac{9}{s^2+9^2}\right\}$

$= \frac{9}{4}\left\{\frac{1}{s^2+9} - \frac{1}{s^2+81}\right\}$

(16) Find $L(\sin 2t \cos 3t)$.

$\sin A\cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$

$L(\sin 2t\cos 3t) = L\left\{\frac{\sin(2t+5t) + \sin(2t-5t)}{2}\right\}$

$$= \frac{1}{2}\{L(\sin 5t) + L(\sin(-t))\}$$

$$= \frac{1}{2}\{L(\sin 5t) - L(\sin t)\}$$

$$= \frac{1}{2}\left\{\frac{5}{s^2+25} - \frac{1}{s^2+1}\right\}$$