



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Euler type

An equation of the form $x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = f(x)$ (1)

$a_1, a_2, \dots, a_n \Rightarrow$ Constants

$f(x) \Rightarrow$ function of x .

$$x = e^z, \quad z = \log x$$

$$x \frac{dy}{dx} = \frac{dy}{dz} = D'y$$

$$x^2 \frac{d^2 y}{dx^2} = D'(D'-1)y$$

$$x^3 \frac{d^3 y}{dx^3} = D'(D'-1)(D'-2)y$$

$$x^4 \frac{d^4 y}{dx^4} = D'(D'-1)(D'-2)(D'-3)y$$

Problems:

1) Solve: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 9y = 10 + \frac{5}{x^2}$

Sol: $(x^2 D^2 + xD - 9)y = 10 + \frac{5}{x^2}$

$$x = e^z, \quad z = \log x$$

$$xD = D', \quad x^2 D^2 = D'(D'-1) \rightarrow (2)$$

$$\frac{1}{x} = (e^z)^{-1} = \frac{1}{e^z} = e^{-z}$$

Sub (2) in (1) eqn, we get

$$[D'(D'-1) + D' - 9]y = 10 + 5e^{-2z}$$

$$(D'^2 - 9)y = 10 + 5e^{-2z}$$

The auxiliary eqn is $m^2 - 9 = 0$
 $m = \pm 3$

$$C.F = Ae^{3z} + Be^{-3z}$$



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$$\begin{aligned}
 P.I &= \frac{1}{D^2 - 9} (10 + 5e^{-2z}) \\
 &= \frac{1}{D^2 - 9} 10e^{0z} + \frac{5}{D^2 - 9} e^{-2z} \\
 &= \frac{1}{0 - 9} 10e^{0z} + \frac{5}{(-2)^2 - 9} e^{-2z}
 \end{aligned}$$

$$P.I = -\frac{10}{9} - e^{-2z}$$

$$y = C.F + P.I$$

$$\begin{aligned}
 y &= Ae^{3z} + Be^{-3z} - \frac{10}{9} e^{-2z} \\
 &= Ae^{3 \log x} + Be^{-3 \log x} - \frac{10}{9} e^{-2 \log x}
 \end{aligned}$$

$$= Ax^3 + Bx^{-3} - \frac{10}{9} x^{-2}$$

$$y = Ax^3 + \frac{B}{x^3} - \frac{10}{9} - \frac{1}{x^2}$$

2) Solve: $x^2 y'' - 3xy' - 5y = \cos(\log x)$

sol: $(x^2 D^2 - 3xD - 5)y = \cos(\log x) \rightarrow (1)$

$x = e^z, z = \log x$

$x D = D', x^2 D^2 = D'(D' - 1) \rightarrow (2)$

Sub (2) in (1), we get

$$(D'(D' - 1) - 3D' - 5)y = \cos z$$

$$(D'^2 - 4D' - 5)y = \cos z$$

The auxiliary equ is $m^2 - 4m - 5 = 0$
 $m = 5, m = -1.$

$$C.F = Ae^{-z} + Be^{5z}$$

$$P.I = \frac{1}{D'^2 - 4D' - 5} \cos z$$



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$$= \frac{1}{-1-4D^2-5} \cos z$$

$$= \frac{1}{-4D^2-6} \cos z$$

$$= \frac{-4D^2+6}{(-4D^2+6)(-4D^2-6)} \cos z$$

$$= \frac{-(4D^2-6)}{16D^4-36} \cos z$$

$$= \frac{-(-4\sin z - 6\cos z)}{-52}$$

$$= \frac{-1}{52} (4\sin z + 6\cos z)$$

$$P.I = \frac{-1}{26} (2\sin z + 3\cos z)$$

$$y = C.F + P.I$$

$$y = Ae^{-z} + Be^{5z} - \frac{1}{26} (2\sin z + 3\cos z)$$

$$= Ae^{-\log x} + Be^{5\log x} - \frac{1}{26} (2\sin \log x + 3\cos \log x)$$

$$y = \frac{A}{x} + Bx^5 - \frac{1}{26} (2\sin(\log x) + 3\cos(\log x))$$

$$3) \text{ Solve } \therefore (x^2 D^2 + xD - 9)y = x^2 \log x$$

$$\text{Ans: } Ax^3 + \frac{B}{x^3} - \frac{x^2}{25} (5\log x + 4)$$

$$4) \text{ Solve: } (x^2 D^2 - 3xD)y = x + 11$$

$$\text{Ans: } y = A + Bx^4 - \frac{x}{3} - \frac{11}{4} \log x$$

$$5) \text{ Solve: } x^2 \frac{d^2 y}{dx^2} - 7x \frac{dy}{dx} + 12y = x^2$$

$$\text{Ans: } Ax^2 + Bx^6 - \frac{x^2 \log x}{4}$$

$$, x = e^z$$

$$\begin{aligned} \log x &= z \\ z &= \log x \end{aligned}$$