



Solution of System of Simultaneous linear first order differential equations with Constant coefficients.

1) Solve : $\frac{dx}{dt} - y = t$; $\frac{dy}{dt} + x = \sin t$.

The given equation can be written as

$$Dx - y = t \rightarrow (1)$$

$$x + Dy = \sin t \rightarrow (2)$$

$$(1) \times D \rightarrow D^2x - Dy = D(t) = 1 \rightarrow (3)$$

$$x + Dy = \sin t \rightarrow (4)$$

$$(D^2 + 1)x = 1 + \sin t$$

The auxiliary equ is $m^2 + 1 = 0$
 $m^2 = -1$
 $m = \pm i$

Complementary function C.F = $A \cos t + B \sin t$

$$P.I = \frac{1}{D^2 + 1} (1 + \sin t)$$

$$= \frac{1}{D^2 + 1} (1) + \frac{1}{D^2 + 1} \sin t$$

$$= \frac{1}{D^2 + 1} e^{0t} + \frac{1}{-1 + 1} \sin t$$

$$= \frac{1}{D} e^{0t} + \frac{1}{0} \sin t$$

$$D^2 \rightarrow -1$$

$$\frac{1}{D} (\sin t) = -\cos t$$



$$= 1 - \frac{t}{2D} \sin t$$

$$= 1 + \frac{t}{2} \frac{1}{D} (\sin t)$$

$$= 1 + \frac{t}{2} (-\cos t)$$

$$P.I = 1 - \frac{t \cos t}{2}$$

~~$$y = C.F + P.I$$~~
$$x = C.F + P.I$$

$$x = A \cos t + B \sin t - \frac{t \cos t}{2} + 1 \rightarrow (5)$$

$$\frac{dx}{dt} = -A \sin t + B \cos t - \frac{1}{2} (-t \sin t + \cos t)$$

$$= -A \sin t + B \cos t + \frac{t}{2} \sin t - \frac{1}{2} \cos t \rightarrow (6)$$

Sub (6) in (5), we get

$$y = Dx - t = \frac{dx}{dt} - t$$

$$y = -A \sin t + B \cos t + \frac{t}{2} \sin t - \frac{1}{2} \cos t - t$$

$$x = A \cos t + B \sin t - \frac{t \cos t}{2} + 1$$

2) Solve: $\frac{dx}{dt} - y = t$, $\frac{dy}{dt} + x = t^2$

Sol: The given equation can be written as

$$Dx - y = t \rightarrow (1)$$

$$x + Dy = t^2 \rightarrow (2)$$



$$\textcircled{1} \times D \Rightarrow D^2 x - Dy = D(t) = 1 \rightarrow \textcircled{3}$$
$$x + py = t^2 \rightarrow \textcircled{4}$$

$$\textcircled{3} + \textcircled{4} \Rightarrow (D^2 + 1)x = t^2 + 1$$

The auxiliary eqn is $m^2 + 1 = 0$
 $m^2 = -1$

$$m = \pm i$$

$$C.F = A \cos t + B \sin t$$

$$P.I = \frac{1}{D^2 + 1} (t^2 + 1)$$

$$= (1 + D^2)^{-1} (t^2 + 1)$$

$$= [1 - D^2 + D^4 - \dots] (t^2 + 1)$$

$$= t^2 + 1 - 2$$

$$= t^2 - 1$$

$$x = C.F + P.I$$

$$= A \cos t + B \sin t + t^2 - 1 \rightarrow \textcircled{5}$$

$$\frac{dx}{dt} = -A \sin t + B \cos t + 2t \rightarrow \textcircled{6}$$

Sub $\textcircled{6}$ in $\textcircled{1}$, we get

$$y = \frac{dx}{dt} - t$$

$$y = -A \sin t + B \cos t + 2t - t$$

$$x = A \cos t + B \sin t + t^2 - 1$$



3) Solve: $3(1-D)x + 4y = 3t + 1$, $3(D+1)y + 2x = e^t$

Sol:

$$3(1-D)x + 4y = 3t + 1 \rightarrow (1)$$

$$2x + 3(D+1)y = e^t \rightarrow (2)$$

$$(1) \times 2 \rightarrow 6(1-D)x + 8y = 6t + 2 \rightarrow (3)$$

$$(2) \times 3(1-D) \rightarrow 6(1-D)x + 9(1-D^2)y = 3(1-D)e^t \rightarrow (4)$$

$$[8 - 9(1-D^2)]y = 6t + 2 - 3e^t + 3e^t$$

$$= 6t + 2$$

$$(8 - 9 + 9D^2)y = 6t + 2$$

$$(9D^2 - 1)y = 6t + 2 \rightarrow (5)$$

The auxiliary eqn is $9m^2 - 1 = 0$

$$9m^2 = 1$$

$$m^2 = \frac{1}{9}$$

$$m = \pm \frac{1}{3}$$

$$\text{C.F} = Ae^{\frac{1}{3}t} + Be^{-\frac{1}{3}t}$$

$$\text{P.I} = \frac{1}{9D^2 - 1} (6t + 2)$$

$$= -\frac{1}{1 - 9D^2} (6t + 2)$$

$$= -(1 - 9D^2)^{-1} (6t + 2)$$

$$= -(1 + 9D^2 + \dots) (6t + 2)$$

$$= -(6t + 2)$$



$$y = C.F + P.I$$

$$= Ae^{\frac{1}{3}t} + Be^{-\frac{1}{3}t} - (6t + 2) \rightarrow (6)$$

$$\frac{dy}{dt} = \frac{A}{3}e^{\frac{1}{3}t} - \frac{B}{3}e^{-\frac{1}{3}t} - 6 \rightarrow (7)$$

$$2x = e^t - 3 \frac{dy}{dt} - 3y \rightarrow (8)$$

$$x = \frac{1}{2} \left(e^t - 3 \left(\frac{A}{3}e^{\frac{1}{3}t} - \frac{B}{3}e^{-\frac{1}{3}t} - 6 \right) \right)$$

$$- 3 \left(Ae^{\frac{1}{3}t} + Be^{-\frac{1}{3}t} - 6t - 2 \right)$$
$$= \frac{1}{2} \left(e^t - Ae^{\frac{1}{3}t} + Be^{-\frac{1}{3}t} + 18 - 3Ae^{\frac{1}{3}t} - 3Be^{-\frac{1}{3}t} + 18t + 6 \right)$$

$$= \frac{1}{2} \left(e^t - 4Ae^{\frac{1}{3}t} - 2Be^{-\frac{1}{3}t} + 18t + 24 \right)$$

$$x = \frac{e^t}{2} - 2Ae^{\frac{1}{3}t} - Be^{-\frac{1}{3}t} + 9t + 12$$

$$y = Ae^{\frac{1}{3}t} + Be^{-\frac{1}{3}t} - 6t + 2.$$