



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Unit - III

Ordinary Differential Equations

Complex Differentiation

Introduction:

If x and y are real numbers then $Z = x + iy$ is called a complex number where x is called real part of z , y is called the imaginary part of z and the value of i is $\sqrt{-1}$.

The complex number $x - iy$ is called as the complex conjugate of z and it is denoted by $\bar{z}$.

$$\text{ie., } \bar{z} = x - iy$$

Note ::

$$1) |z| = \sqrt{x^2 + y^2}$$

$$2) |z|^2 = z\bar{z} \quad 3) z\bar{z} = x^2 + y^2 = r^2$$

$$4) |\bar{z}| = |z|$$

$$5) \text{Real Part of } z = \frac{z + \bar{z}}{2}$$

$$6) \text{Imaginary Part of } z = \frac{z - \bar{z}}{2}$$

7) $z = re^{i\theta}$ is called Polar form of z .

8) Amplitude of $z = \theta = \tan^{-1}(y/x)$.

Functions of Complex Variable ::

$w = f(z) = u(x, y) + iv(x, y)$ where $u(x, y)$ and $v(x, y)$ are real variables.



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A function is said to be analytic at a point if its derivative exists not only at that point but also in some neighbourhood of that point.

Single Valued function :-

If for each value of z in R there will be only one value of w , then w is called a single valued function of z .

Example :- $w = z^2$, $w = \frac{1}{z}$

z :	1	2	-2	3
w :	1	4	4	9

z :	1	2	-2	3
w :	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$

Multiple-Valued function :-

If there is more than one value of w corresponding to a given value of z , then w is called a multiple-valued function.

Example :- $w = z^{1/2}$

z :	4	9	1
w :	-2, 2	-3, 3	1, -1

Analytic function :-

A function $f(z)$ is said to be analytic at a point $z = a$ in a region R if

- $f(z)$ is differentiable at $z = a$
- $f(z)$ is differentiable at all points for some neighbourhood of $z = a$.

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$$(ii) \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta} \quad \text{and} \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

Sufficient Conditions ::

If the function $w = f(z) = u(r, \theta) + iv(r, \theta)$ is analytic in a region R of the z -plane, then

(i) $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial \theta}$, $\frac{\partial v}{\partial r}$ and $\frac{\partial v}{\partial \theta}$ exists and all are continuous.

Problems ::

1) Prove that $w = z^2$ is analytic.

Sol :: We know that $z = x + iy$

$$w = z^2 = (x + iy)^2 = x^2 + 2ixy - y^2$$

$$u + iv = (x^2 - y^2) + i(2xy)$$

$$u = x^2 - y^2$$

$$v = 2xy$$

$$u_x = 2x$$

$$v_x = 2y$$

$$u_y = -2y$$

$$v_y = 2x$$

$$u_x = v_y \quad \text{and} \quad u_y = -v_x$$

It satisfies the Cauchy-Riemann equations.

$w = z^2$ is analytic.