Conformal mapping :

A mapping $w = f(z)$ is said to be conformal at $z = z_0$ if $f'(z_0) \neq 0$.

The point at which the mapping $w = f(z)$ is not conformal.

ie.,) $f'(z) = 0$ is called a critical point of the mapping.

If the transformation $w = f(z)$ is conformal at a point the inverse transformation $z = f^{-1}(w)$ is also conformal at the corresponding point.

Some standard transformations :

1. Translation :

The transformation $w = c + z$, where c is a complex constant, represents a translation.

Let $z = x + iy$

$w = u + iv$

$c = a + ib$

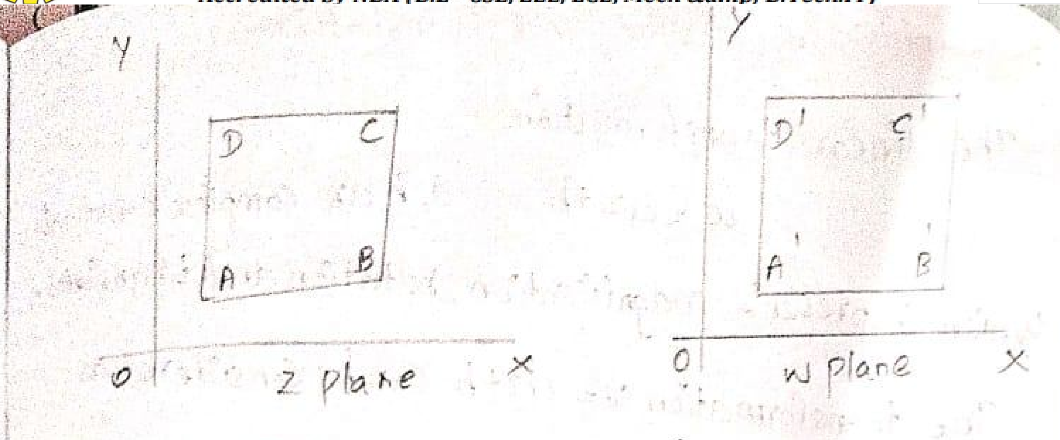
Given $w = z + c$

$u + iv = x + iy + a + ib$

$u + iv = (x + a) + i(y + b)$

Equating the real and imaginary parts, we get

$u = x + a, \quad v = y + b$



2. Magnification:-

The transformation $w = cz$, where c is a real constant, represents magnification.

$$u + iv = c(x + iy)$$

$$u + iv = (cx + icy)$$

$$u = cx, \quad v = cy$$

The image of the Point (x,y) is the point (cx, cy).

3. Magnification and Rotation:-

The transformation $w = cz$, where c is a complex constant represents both magnification and rotation.

Let $z = re^{i\theta}$

$w = Re^{i\phi}$ and $c = ae^{i\alpha}$

$$Re^{i\phi} = (ae^{i\alpha})(re^{i\theta}) = are^{i(\alpha+\theta)}$$

The transformation equations are $R = ar$, $\phi = \theta + \alpha$.

The point (r,θ) in z-plane is mapped on to the point $(ar, \theta + \alpha)$.