



## Conformal mapping :

A mapping  $w = f(z)$  is said to be conformal at  $z = z_0$  if  $f'(z_0) \neq 0$ .

The point at which the mapping  $w = f(z)$  is not conformal.

i.e.,  $f'(z) = 0$  is called a critical point of the mapping.

If the transformation  $w = f(z)$  is conformal at a point, the inverse transformation  $z = f^{-1}(w)$  is also conformal at the corresponding point.

Some standard transformations :

1. Translation :

The transformation  $w = c + z$ , where  $c$  is a complex constant, represents a translation.

$$\text{Let } z = x + iy$$

$$w = u + iv$$

$$c = a + ib$$

$$\text{Given } w = z + c$$

$$u + iv = x + iy + a + ib$$

$$u + iv = (x + a) + i(y + b)$$

Equating the real and imaginary parts, we get

$$u = x + a, \quad v = y + b$$



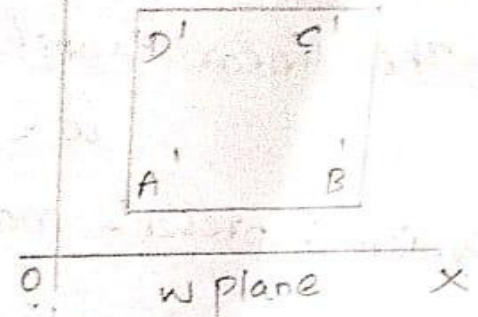
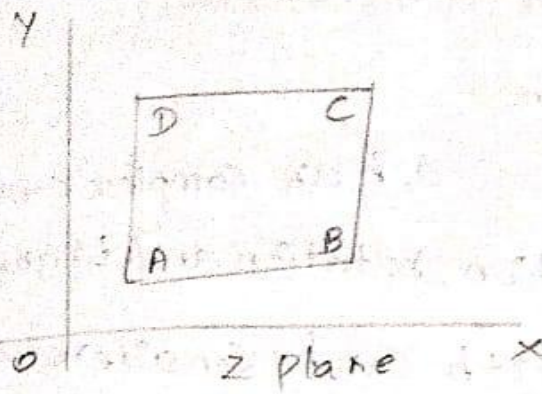
## SNS COLLEGE OF TECHNOLOGY

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### 2. Magnification :-

The transformation  $w = cz$ , where  $c$  is a real constant, represents magnification.

$$u + iv = c(x + iy)$$

$$u + iv = (cx + icy)$$

$$u = cx, v = cy$$

The image of the Point  $(x, y)$  is the point  $(cx, cy)$ .

### X Magnification and Rotation :-

The transformation  $w = cz$ , where  $c$  is a complex constant represents both magnification and rotation.

$$\text{Let } z = re^{i\theta}$$

$$w = Re^{i\phi} \quad \text{and} \quad c = ae^{i\alpha}$$

$$Re^{i\phi} = (ae^{i\alpha})(re^{i\theta}) = are^{i(\alpha+\theta)}$$

The transformation equations are  $R = ar$ ,  $\phi = \theta + \alpha$ .

The point  $(r, \theta)$  in  $z$ -plane is mapped on to the point  $(ar, \theta + \alpha)$ .



## 4. Magnification, Rotation and Translation.

The linear transformation

$$w = az + b \quad a, b \text{ are complex constants.}$$

which represents magnification, rotation and translation.

The transformation  $w = az + b$  can be considered as combination of two simple transformations.

$$w_1 = az \quad \text{and} \quad w = w_1 + b$$

$w_1 = az$  represents magnification by  $|a|$  and rotation through  $\arg(a)$ .

$w = w_1 + b$  represents translation by a vector representing  $b$ .

## 5. Inversion and Reflection

The transformation  $w = \frac{1}{z}$  represents inversion with respect to the unit circle  $|z| = 1$ , followed by reflection in the real axis.

$$w = \frac{1}{z} \quad (\text{or}) \quad z = \frac{1}{w}$$

$$x + iy = \frac{1}{u + iv}$$

$$x + iy = \frac{u - iv}{u^2 + v^2}$$

$$x = \frac{u}{u^2 + v^2}, \quad y = \frac{-v}{u^2 + v^2} \rightarrow \textcircled{2}$$

↪ (1)



The general equation of circle in  $z$  plane is

$$x^2 + y^2 + z^2 + 2gx + 2fy + c = 0 \rightarrow (3)$$

sub (1), (2) in (3) equ, we get

$$\frac{u^2}{(u^2+v^2)^2} + \frac{v^2}{(u^2+v^2)^2} + 2g\left(\frac{u}{u^2+v^2}\right) + 2f\left(\frac{-v}{u^2+v^2}\right) + c = 0$$

$$C(u^2+v^2) + 2gu - 2fv + 1 = 0 \rightarrow (4)$$

which is the equation of circle in  $w$  plane.

Problems:

1) what is the region of the  $w$ -plane into which the rectangular region in the  $z$ -plane bounded by the lines  $x=0, y=0, x=1, y=2$  is mapped under the transformation  $w = z + (2-i)$ .

Sol: Given  $w = z + (2-i)$

$$\text{i.e., } u+iv = x+iy + (2-i) = (x+2) + i(y-1)$$

Equating the real and imaginary parts

$$u = x+2, \quad v = y-1$$

Given boundary lines are the transformal boundary lines are

$$x = 0$$

$$y = 0$$

$$x = 1$$

$$y = 2$$

$$u = 0+2 = 2$$

$$v = 0-1 = -1$$

$$u = 1+2 = 3$$

$$v = 2-1 = 1$$



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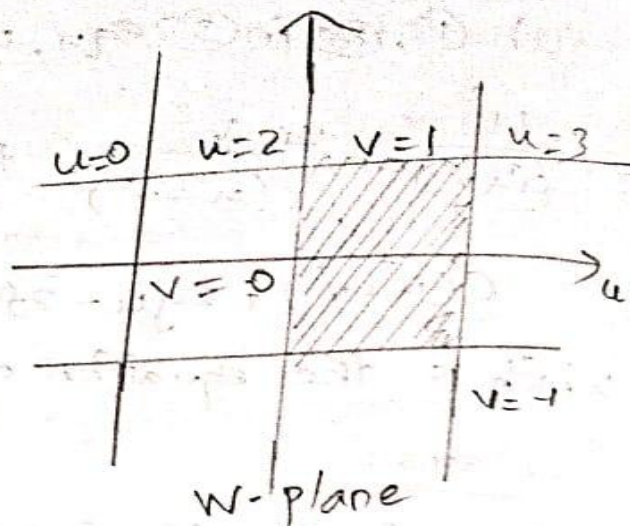
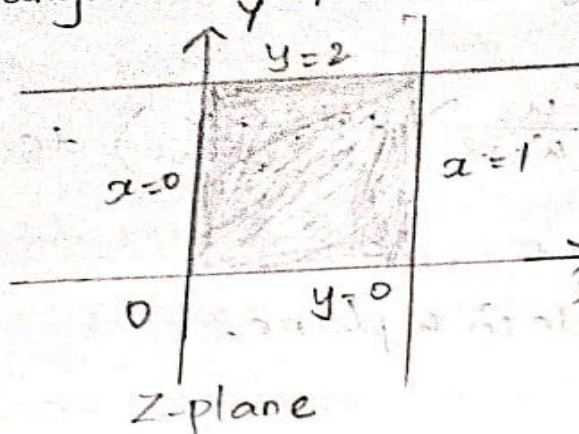
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Hence the lines  $x=0, y=0, x=1, y=2$  are mapped into the lines  $u=2, v=-1, u=3, v=1$  which form a rectangle in  $w$  plane.



2) Find the image of the circle  $|z|=1$  by the transformation  $w = z + 2 + 4i$ .

Sol: Given  $w = z + 2 + 4i$

$$u + iv = x + iy + 2 + 4i$$
$$= (x+2) + i(y+4)$$

Equating the real and imaginary parts,

$$u = x+2, v = y+4$$

$$x = u-2, y = v-4$$

Given:  $|z|=1$

$$x^2 + y^2 = 1$$

$$(u-2)^2 + (v-4)^2 = 1$$



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Hence the circle  $x^2 + y^2 = 1$  is mapped into  $(u-2)^2 + (v-4)^2 = 1$  in  $w$  plane which is also a circle with centre  $(2, 4)$  and radius 1.

