Homogeneous differential equations with variable coefficient.

Euler form or cauchy's form   Legender's form. (ax+b) form.

An equation of the form $(a_0x^nD^n + a_1x^{n-1}D^{n-1} + \dots + a_{n-1}xD + a_n)y = x$. where $a_0, a_1 \dots a_n$ are constant. and 'x' is a function is called euler homogeneous linear eqn.

Rule:

$x = e^t$ (or) $t = \log x$

$$x\frac{dy}{dx} = xDy = \theta y$$

$$x^2\frac{d^2y}{dx^2} = x^2D^2y = \theta(\theta-1)y.$$

Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$.

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}.$$

$$x^2D^2y + 4xDy + 2y = x^2 + \frac{1}{x^2}.$$

$$(x^2D^2 + 4xD + 2)y = x^2 + \frac{1}{x^2}.$$

$x = e^t$ (or) $t = \log x$ $\qquad R(x) = x^2 + \frac{1}{x^2}$

$x^2D^2y = \theta(\theta-1)$ $\qquad = (e^t)^2 + \frac{1}{e^{t^2}}$

$= \theta^2 - \theta$

$xDy = \theta.$ $\qquad R(x) = e^{2t} + e^{-2t}.$

$$(\theta^2 - \theta) + 4\theta + 2)y = e^{2t} + e^{-2t}.$$

$$(\theta^2 + 3\theta + 2)y = e^{2t} + e^{-2t}.$$

The A.E is $m^2 + 3m + 2 = 0$

$$(m+1)(m+2) = 0$$

$$m = -1,\ m = -2.$$

The C.F is $Ae^{-x} + Be^{-2x}$.

$$P.I_1 = \frac{1}{\theta^2 + 3\theta + 2} e^{2t} \quad [\theta = a = 2].$$

$$= \frac{1}{(2)^2 + 3(2) + 2}$$

$$P.I_1 = \frac{1}{12} e^{2t}.$$

$$P.I_2 = \frac{1}{\theta^2 + 3\theta + 2} e^{-2t} \quad [\theta = a = 2].$$

$$= \frac{1}{(-2)^2 + 3(-2) + 2} \qquad = \frac{t}{2\theta + 3} e^{-2t}$$

$$= \frac{t}{-1} e^{-2t} \Rightarrow -te^{-2t}.$$

$$P.I = P.I_1 + P.I_2.$$

$$= \frac{1}{2}e^{2t} - te^{-2t}.$$

$$= \frac{(e^t)^2}{12} - \frac{t}{(e^t)^2}.$$

$$P.I = \frac{x^2}{12} - \frac{\log x}{x^2}.$$

$$y = C.F + P.I$$

$$= Ae^{-x} + Be^{-2x} + \frac{x^2}{12} - \frac{\log x}{x^2}.$$