Legender's Linear differential equations:

An eqn which is of the form $(ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} (ax+b) \frac{dy}{dx} + a_n y) = R(x)$

is called L.L.D.E.

Rule:

Let $e^t = (ax+b)$ or $t = \log(ax+b)$.

$(ax+b)D = a\theta$.

$(ax+b)^2 D^2 = a^2(\theta^2 - \theta) \quad \theta(\theta-1)$.

Prob: 1

Solve $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$.

Sln:

$(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$.

$[(2x+3)^2 D^2 - (2x+3)D - 12]y = 6x$.

$e^t = 2x+3$ (or) $t = \log(2x+3)$. $\quad R(x) = 6x$

$x = \frac{e^t - 3}{2}$ $\quad R(x) = \cancel{6}^{3}\left(\frac{e^t - 3}{\cancel{2}}\right)$

$R(x) = 3e^t - 9$.

$(2x+3)D = 2\theta$

$(2x+3)^2 D^2 = 4(\theta^2 - \theta)$.

$4(\theta^2 - \theta) - 2\theta - 12)y = 3e^t - 9$.

$4\theta^2 - 6\theta - 12)y = 3e^t - 9$.

The A.E $4m^2 - 6m - 12 = 0$.

$2m^2 - 3m - 6 = 0$.

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2,\ b = -3,\ c = -6$

$m = \dfrac{3 \pm \sqrt{9+48}}{4}$

$m = \dfrac{3 \pm \sqrt{57}}{4}$

$m_1 = \dfrac{3+\sqrt{57}}{4},\quad m_2 = \dfrac{3-\sqrt{57}}{4}$

The $C.F = Ae^{\frac{3+\sqrt{57}}{4}x} + Be^{\frac{3-\sqrt{57}}{4}x}$.

$P.I = \dfrac{1}{4\theta^2 - 6\theta - 12}\, 3e^t - 9$.

$P.I_1 = 3\, \dfrac{1}{4\theta^2 - 6\theta - 12}\, e^t \qquad D = a = 1.$

$= \dfrac{3}{-14}\, e^t.$

$P.I_2 = \dfrac{1}{4\theta^2 - 6\theta - 12}\, 9e^{0t}.$

$= \dfrac{1}{+12}\, 9e^{0t}.$

$P.I = P.I_1 + P.I_2.$

$P.I = \dfrac{3}{-14}\, e^t + \dfrac{9}{12}\, e^{0t}.$

$P.I = \dfrac{3}{-14}\, e^{\log(2x+3)} + \dfrac{9}{12}.$

$P.I = \dfrac{9}{12} - \dfrac{3}{14}(2x+3).$

$y = C.F + P.I$

$= Ae^{\frac{3+\sqrt{57}}{4}x} + Be^{\frac{3-\sqrt{57}}{4}x} + \dfrac{9}{12} - \dfrac{3}{14}(2x+3).$