$$P.I_2 = \frac{1}{D^2-3D+2} 2e^x$$

$$= \frac{1}{1-3+2} 2e^x$$

$$= \frac{1}{0} 2e^{2x}$$

$$= \frac{x}{2D-3} 2e^x$$

$$= \frac{x}{2-3} 2e^x$$

$$= \frac{x}{-1} 2e^x = -\frac{x}{1} 2e^x$$

$$y = C.F + P.I_1 + P.I_2$$

$$y = (Ae^x + Be^{2x}) + \left(\frac{-1}{20}\right)\left[6\sin(2x+3) + 2\cos(2x+3) - x2e^x\right]$$

3) $(D^2+1)y = \sin^2 x.$

Sol:. $(D^2+1)y = \frac{1-\cos 2x}{2}$

$$(D^2+1)y = \frac{1e^{0x}}{2} - \frac{\cos 2x}{2}$$

The auxiliary equation is $m^2+1=0$

$$m^2 = -1$$

$$m = \pm i$$

$\alpha + i\beta$

$0 + i(1)$

$\alpha = 0, \beta = 1$

The roots are imaginary.

$$C.F = e^{0x}(A\cos x + B\sin x)$$

$$C.F = A\cos x + B\sin x.$$

$P.I_1 = \frac{1}{D^2+1} \cdot \frac{1}{2} e^{0x}$

$D \to 0 \Rightarrow = \frac{1}{1} \cdot \frac{1}{2} e^{0x}$

$P.I_1 = \frac{1}{2}$

$P.I_2 = \frac{1}{D^2+1}\left(-\frac{\cos 2x}{2}\right)$ $\qquad D^2 \to -(a^2) = -4$

$= \frac{1}{-4+1}\left(\frac{-\cos 2x}{2}\right)$

$= \frac{1}{3}\left(\frac{\cos 2x}{2}\right)$

$= \frac{\cos 2x}{6}$

$y = C.F + P.I_1 + P.I_2$

$= A\cos x + B\sin x + \frac{1}{2} + \frac{\cos 2x}{6}$

4) Solve : $(D^2+4D+2) = \sin 3x$

Sol:- The auxiliary equation is $m^2+4m+2=0$

$m = -2, -2$

The roots are real and equal.

$C.F = (A+Bx)e^{-2x}$ $\qquad D^2 = -9$

$P.I = \frac{1}{D^2+4D+2} \sin 3x$

$= \frac{1}{-9+4D+2} \sin 3x$

$= \frac{1}{4D-7} \sin 3x$

$$= \frac{1}{4D-7} \times \frac{4D+7}{4D+7} \sin 3x$$

$$= \frac{4D+7}{16D^2-49} \sin 3x$$

$$= \frac{4D+7}{16(-9)-49} \sin 3x$$

$$= \frac{4D+7}{-193} \sin 3x$$

$$= \frac{-1}{193}\left[4D \sin 3x (3) + 7 \sin 3x\right]$$

$$P.I_2 = \frac{-1}{193}\left[12 \cos 3x + 7 \sin 3x\right]$$

$$y = C.F + P.I$$

$$y = (A+Bx)e^{-2x} - \frac{1}{193}\left[12\cos 3x + 7 \sin 3x\right]$$

5) $(D^2+4)y = \cos 2x.$

Sol:. The auxiliary equation is

$$m^2+4=0 \qquad \alpha + i\beta = 0+2i$$

$$m^2 = -4 \qquad \alpha = 0, \beta = 2$$

$$m = \pm 2i$$

The roots are imaginary.

$$C.F = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$$

$$= e^{0x}(A \cos 2x + B \sin 2x)$$

$$C.F = A \cos 2x + B \sin 2x$$

$$P.I = \frac{1}{D^2+4} \cos 2x \qquad D^2 = -4$$

$$= \frac{1}{-4+4} \cos 2x$$

$$= \frac{1}{0} \cos 2x$$

$$= \frac{x}{2D+0} \cos 2x$$

$$= \frac{x}{2D}\left[\frac{2D}{2D}\right] \cos 2x$$

$$= \frac{x.2D \cos 2x}{4D^2}$$

$$= \frac{x.2D}{4(-4)} \cos 2x$$

$$= \frac{x}{-8}(-\sin 2x)\,2$$

$$= \frac{x}{-4}(-\sin 2x)$$

$$= \frac{x \sin 2x}{4}$$

$$P.I = \frac{x \sin 2x}{4}$$

$$y = C.F + P.I$$

$$= A\cos 2x + B\sin 2x + \frac{x \sin 2x}{4}$$

$$\boxed{y = A\cos 2x + B\sin 2x + \frac{x \sin 2x}{4}}$$

6) Solve: $(D^2+2D+1) = \sin 2x \cos 2x$

Soln. The auxiliary equation is

$$m^2+2m+1=0$$

$$m=-1,-1.$$

$$\sin 2x = 2\sin x \cos x$$

The roots are real and equal.

$$\sin 2x \cos x = \frac{\sin 2x}{2}$$

$$C.F = (A+Bx)e^{-x}$$

$$D^2+2D+1 = \frac{\sin 2(2x)}{2}$$

$$D^2+2D+1 = \frac{\sin 4x}{2}$$

$$P.I = \frac{1}{2}\frac{1}{D^2+2D+1}\sin 4x$$

$$= \frac{1}{2}\left[\frac{1}{-16+2D+1}\right]\sin 4x$$

$$= \frac{1}{2}\left[\frac{1}{2D-15}\right]\left[\frac{2D+15}{2D+15}\right]\sin 4x$$

$$= \frac{1}{2}\left[\frac{2D+15}{4D^2-225}\right]\sin 4x$$

$$= \frac{1}{2}\left[\frac{2\cos 4x(4)+15\sin 4x}{-64-225}\right]$$

$$= \frac{1}{2}\left[\frac{8\cos 4x+15\sin 4x}{-289}\right]$$

$$= \frac{8\cos 4x+15\sin 4x}{-578}$$

$$y = C.F + P.I$$

$$= (A+Bx)e^{-x} - \frac{(8\cos 4x+15\sin 4x)}{578}$$

Type: I Problems

3) $(D^2+4D+8)y = e^{2x}+4$

Sol: The auxiliary equation is $m^2+4m+8=0$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{16-4(1)(8)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-32}}{2} = \frac{-4 \pm \sqrt{-16}}{2}$$

$$= -\frac{4}{2} \pm \frac{4i}{2}$$

$$m = -2 \pm 2i$$

The roots are imaginary.

$$C.F = e^{-2x}\left[A\cos 2x + B\sin 2x\right]$$

$$P.I_1 = \frac{1}{D^2+4D+8} e^{2x} \qquad D \to 2$$

$$P.I_1 = \frac{1}{4+8+8} e^{2x}$$

$$= \frac{1}{20} e^{2x}$$

$$P.I_2 = \frac{1}{D^2+4D+8} 4e^{0x} \qquad D \to 0$$

$$= \frac{4}{8} e^{0x} = \frac{1}{2}$$

$$y = C.F + P.I_1 + P.I_2$$

$$y = e^{-2x}\left[A\cos 2x + B\sin 2x\right] + \frac{1}{20}e^{2x} + \frac{1}{2}$$

4) Solve: $(4D^2-4D+1)y = 4.$

Sol: The auxiliary equation is

$$4m^2-4m+1=0$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16-4(4)(1)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{16-16}}{8} = \frac{4}{8} = \frac{1}{2}$$

The roots are real and equal

$$C.F = (A+Bx)e^{1/2x}$$

$$P.I = \frac{1}{4D^2-4D+1} 4e^{0x} \qquad D \to 0$$

$$= \frac{1}{1} 4e^{0x}$$

$$y = C.F + P.I$$

$$= (A+Bx)e^{1/2x} + 4$$

$$\boxed{y = (A+Bx)e^{\frac{1}{2}x} + 4}$$

5) $(D^2-7D+12)y = e^{5x}$

Sol: The auxiliary equation is

$$m^2-7m+12=0$$

$$m = 3, 4$$

The roots are real and different.

$C.F = Ae^{3x} + Be^{4x}$

$P.I = \frac{1}{D^2-7D+12} e^{5x}$ $\quad D \to 5$

$= \frac{1}{25-35+12} e^{5x}$

$= \frac{1}{37-35} e^{5x}$

$= \frac{1}{2} e^{5x}$

$y = C.F + P.I$

$y = Ae^{3x} + Be^{4x} + \frac{1}{2} e^{5x}$

Type - III :

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - \cdots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + \cdots$

1) Solve: $(D^2+2)y = x^2$

Sol:. The auxiliary equation is $m^2 + 2 = 0$.

$m^2 = -2$ $\quad \alpha = 0$

$m = \pm\sqrt{2}i$ $\quad \beta = \sqrt{2}$

$C.F = e^{0x}[A\cos\sqrt{2}x + B\sin\sqrt{2}x]$

$= A\cos\sqrt{2}x + B\sin\sqrt{2}x.$

$$P.I = \frac{1}{D^2+2} x^2$$

$$= \frac{1}{2\left(1+\frac{D^2}{2}\right)} x^2$$

$$= \frac{1}{2}\left(1+\frac{D^2}{2}\right)^{-1} x^2$$

$$= \frac{1}{2}\left[1-\frac{D^2}{2}+\left(\frac{D^2}{2}\right)^2+\cdots\right] x^2$$

$$= \frac{1}{2}\left[1-\frac{D^2}{2}\right] x^2$$

$$= \frac{1}{2}\left[x^2-\frac{D^2}{2}x^2\right]$$

$$= \frac{1}{2}x^2-\frac{1}{2}\left(\frac{D^2x^2}{2}\right)$$

$$= \frac{1}{2}x^2-\frac{1}{2}\left(\frac{2}{2}\right)$$

$$= \frac{1}{2}x^2-\frac{1}{2}$$

$$P.I = \frac{1}{2}(x^2-1)$$

$$y = C.F + P.I$$

$$= A\cos\sqrt{2}x + B\sin\sqrt{2}x + \frac{1}{2}(x^2-1).$$

2) Solve: $(D^2+3D+2)y = x^2$

Sol:- The auxiliary equation is $m^2+3m+2=0$

$m = 1$ and 2.

The roots are real and different

$C.F = Ae^{-x} + Be^{-2x}$

$$P.I = \frac{1}{D^2+3D+2} x^2$$

$$= \frac{1}{2\left(1+\frac{D^2+3D}{2}\right)} x^2$$

$$= \frac{1}{2}\left(1+\frac{D^2+3D}{2}\right)^{-1} x^2$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2+3D}{2}\right)+\left(\frac{D^2+3D}{2}\right)^2 - \cdots\right] x^2$$

$$= \frac{1}{2}\left[1-\frac{D^2}{2}-\frac{3D}{2}+\frac{9D^2}{4}\right] x^2$$

$$= \frac{1}{2}\left[x^2-\frac{1}{2}D^2(x^2)-\frac{3D(x^2)}{2}+\frac{9D^2(x^2)}{4}\right]$$

$$= \frac{1}{2}\left[x^2-\frac{1}{2}(2)-\frac{3}{2}(2x)+\frac{9}{4}(2)\right]$$

$$= \frac{1}{2}\left[x^2-1-3x+\frac{9}{2}\right]$$

$$= \frac{1}{2}\left[x^2-3x+\frac{9}{2}\right]$$

$y = C.F + P.I$

$$= Ae^{-x} + Be^{-2x} + \frac{1}{2}\left[x^2-3x+\frac{9}{2}\right]$$