Unit V

1. Prove that, if the input to a time – invariant, stable linear system is a WSS process, then the output will also be a WSS process.

2. Prove that (i) $R_{XY}((\tau) = R_{XX}(\tau) * h(\tau)$ (ii) $R_{YY}((\tau) = R_{XY}(\tau) * h(-\tau)$ (iii) $S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$ (iv) $S_{YY}(\omega) = S_{XY}(\omega)H^*(\omega)$ (v) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^2$

3. X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary Random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$.

4. An LTI system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{YY}(\tau)$ corresponding to an input X(t).

5. Assume a random process X(t) is given as input to a system with transfer function $H(\omega)$ =1 for $-\omega_0 < \omega < \omega_0$.If the autocorrelation function of the input process $\frac{N_0}{2}\ \delta(r)$. Point out the autocorrelation function

of the output process.

6. Let X(t) be a stationary process with mean 0 and autocorrelation function $e^{-2|c|}$. If X(t) is the input to a linear system and Y(t) is the output process ,Calculate (i) E[Y(t)] (ii) $S_{YY}(\omega)$ and (iii) $R_{YY}(|r|)$, if the system function $H(\omega) = \frac{1}{\omega+2i}$.

7. A wide sense stationary random process {X(t)} with autocorrelation $R_{XX}(\tau) = Ae^{-a|\tau|}$, where A and a are real positive constants, is applied to the input of a linear transmission input system with impulse response $h(t) = e^{-bt}\, u(t)$ Where b is a real positive constant. Give the power spectral density of the output Y(t) of the system.

8. A linear system is described by the impulse response $h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}u(t)$. Assume an input process whose autocorrelation function is $B\delta(\tau)$.Point out the mean and autocorrelation function of the output function.

9. If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & elsewhere \end{cases}$$

Identify the auto correlation function of N(t).

10. Let X(t) be the input voltage to a circuit system and Y(t) be the output voltage.If X(t)is a stationary random process with mean 0 and autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha|\tau|}$.

Identify
(i) E[Y(t)]
(ii) $S_{XX}(\omega)$ and
The spectral density of Y(t) if the power transfer function

$$H(\omega) = \frac{R}{R + iL\omega}$$

11. A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t}, t \geq 0$. The auto correlation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process Y(t).

12. Find the power spectral density of a random telegraph signal.

13. If X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_y and power spectrum $S_{yy}(\omega)$ of the output if the system

transfer function is given by $H(\omega) = \frac{1}{\omega+2i}$.

14. If X(t) is the input and Y(t) is the output of the system. The autocorrelation of X(t) is $R_{XX}(\tau) = 3.\delta(\tau)$. Find the power spectral density, autocorrelation function and mean square value of the output Y(t) with $H(\omega) = \frac{1}{6+j\omega}$

15. Analyse the mean of the output of a linear system is given by $\mu_Y = H(0)\mu_X$, where X(t) is a WSS.