SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 641 035

DEPARTMENT OF MATHEMATICS

23MAT203-PROBABILITY AND RANDOM PROCESSES

UNIT 3

CLASSIFICATION OF RANDOM PROCESSES

Two marks

1. **Define Random processes and give an example of a random process.**

A Random process is a collection of R.V $\{X(s,t)\}$ that are functions of a real variable namely time t where $s \in S$ and $t \in T$

Example:

$X(t) = A\cos(\omega t + \theta)$ where θ is uniformly distributed in $(0, 2\pi)$ where A and ω are constants.

2. State the four classifications of Random processes.

The Random processes is classified into four types

(i)Discrete random sequence

If both T and S are discrete then Random processes is called a discrete Random sequence.

(ii)Discrete random processes

If T is continuous and S is discrete then Random processes is called a Discrete Random processes.

(iii)Continuous random sequence

If T is discrete and S is continuous then Random processes is called a Continuous Random sequence.

(iv)Continuous random processes

If T &S are continuous then Random processes is called a continuous Random processes.

3. Define stationary Random processes.

If certain probability distributions or averages do not depend on t, then the random process $\{X(t)\}$ is called stationary.

4. Define first order stationary Random processes.

A random processes $\{X(t)\}$ is said to be a first order SSS process if $f(x_1, t_1 + \delta) = f(x_1, t_1)$ (i.e.) the first order density of a stationary process $\{X(t)\}$ is independent of time t

5. Define second order stationary Random processes

A RP $\{X(t)\}$ is said to be second order SSS if $f(x_1, x_2, t_1, t_2) = f(x_1, x_2, t_1 + h, t_2 + h)$ where $f(x_1, x_2, t_1, t_2)$ is the joint PDF of $\{X(t_1), X(t_2)\}$.

6. Define strict sense stationary Random processes

Sol: A RP $\{X(t)\}$ is called a SSS process if the joint distribution $X(t_1)X(t_{21})X(t_3)........X(t_n)$ is the same as that of $X(t_1 + h)X(t_2 + h)X(t_3 + h)........X(t_n + h)$ for all $t_1, t_2, t_3........t_n$ and h >0 and for $n \geq 1$.

7. Define wide sense stationary Random processes

A RP $\{X(t)\}$ is called WSS if $E\{X(t)\}$ is constant and $E[X(t)X(t+\tau)] = R_{xx}(\tau)$ (i.e.) ACF is a function of τ only.

8. Define jointly strict sense stationary Random processes

Sol: Two real valued Random Processes $\{X(t)\}$ and $\{Y(t)\}$ are said to be jointly stationary in the strict sense if the joint distribution of the $\{X(t)\}$ and $\{Y(t)\}$ are invariant under translation of time.

9. Define jointly wide sense stationary Random processes

Sol: Two real valued Random Processes $\{X(t)\}$ and $\{Y(t)\}$ are said to be jointly stationary in the wide sense if each process is individually a WSS process and $R_{XY}(t_1, t_2)$ is a function of t_1, t_2 only.

10. **Define Evolutionary Random processes and give an example**.

Sol: A Random processes that is not stationary in any sense is called an Evolutionary process. Example: Poisson process.

11. **When is a random process said to be ergodic? Give an example**

Answer: A R.P {X(t)} is ergodic if its ensembled averages equal to appropriate time averages. Example: $X(t) = A\cos(\omega t + \theta)$ where θ is uniformly distributed in $(0, 2\pi)$ is mean ergodic.

12. **Define Markov Process.**

Sol: If for $t_1 < t_2 < t_3 < t_4 .... < t_n < t$ then

$$P(X(t) \le x / X(t_1) = x_1, X(t_2) = x_2, ...X(t_n) = x_n) = P(X(t) \le x / X(t_n) = x_n)$$

Then the process $\{X(t)\}$ is called a Markov process.

13. **Define Markov chain.**

Sol: A Discrete parameter Markov process is called Markov chain.

14. **Define one step transition probability.**

Sol: The one step probability $P\left[X_n = a_j / X_{n-1} = a_i\right]$ is called the one step probability from the state a_i to a_j at the n^{th} step and is denoted by $P_{ij}(n-1, n)$

19. **Consider a Markov chain with two states and transition probability matr**

$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$**.Find the stationary probabilities of the chain.**

Sol: $(\pi_1, \pi_2)\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = (\pi_1, \pi_2) \qquad \pi_1 + \pi_2 = 1$

$$\frac{3}{4}\pi_1 + \frac{\pi_2}{4} = \pi_1 \Rightarrow \frac{\pi_1}{4} - \frac{\pi_2}{2} = 0. \qquad \therefore \pi_1 = 2\pi_2$$

$$\therefore \pi_1 = \frac{2}{3}, \pi_2 = \frac{1}{3}.$$

If the input $x(t)$ and its output $y(t)$ are related by $y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)\, du$, then the system is linear time invariant system