SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 641 035

DEPARTMENT OF MATHEMATICS

23MAT203-PROBABILITY AND RANDOM PROCESSES

Unit 5

LINEAR SYSTEMS WITH RANDOM INPUTS

Two marks

14. Define a system and Define the linear system.

Answer:

A system is a functional relationship between the input X(t) and the output Y(t).i.e., $Y(t) = f[X(t)]$, $-\infty < t < \infty$.

A System is a functional relationship between the input X(t) and the output Y(t).

If $f[a_1X_1(t)+ a_2X_2(t)] = a_1 f[X_1(t)]+ a_2 f[X_2(t)]$, then f is called a linear system.

15. Define time invariant system.

Answer:

If $Y(t+h) = f[X(t+h)]$ where $Y(t) = f[X(t)]$,then f is called the time invariant system.

16. Check whether the following system is linear .y(t)=t x(t)

Answer:

Consider two input functions $x_1(t)$ and $x_2(t)$. The corresponding outputs are

$y_1(t)=t\, x_1(t)$ and $y_2(t)=t\, x_2(t)$

Consider $y_3(t)$ as the linear combinations of the two inputs.

$y_3(t)= t[a_1 x_1(t)+a_2 x_2(t)]= a_1t\, x_1(t)+a_2\, t\, x_2(t)$ ……………..(1)

consider the linear combinations of the two outputs.

$a_1y(t)+a_2 y_2(t)= a_1t\, x_1(t)+a_2\, t\, x_2(t)$ …………………(2)

From (1)and(2), (1)=(2)

The system y(t)=t x(t) is linear.

17.

Check whether the following system is linear .y(t)= $x^2(t)$

Answer:

Consider two input functions $x_1(t)$ and $x_2(t)$. The corresponding outputs are

$y_1(t)=x_1^2(t)$ and $y_2(t)=x_2^2(t)$

Consider $y_3(t)$ as the linear combinations of the two inputs.

$y_3(t)= [a_1 x_1(t)+a_2 x_2(t)]^2= a_1^2 x_1^2(t)+a_2^2 x_2^2(t)+2 a_1x_1(t)a_2x_2(t)$ ....(1)

consider the linear combinations of the two outputs.

$a_1y(t)+a_2 y_2(t)= a_1 x_1^2(t)+a_2 x_2^2(t)$ ....(2)

From (1) and (2), $(1) \neq (2)$

The system $y(t)=x^2(t)$ is not linear.

18. **Define the Linear Time Invariant System.**

Answer:

A linear system is said to be also time-invariant if the form of its impulse response $h(t,u)$ does not depend on the time that the impulse is applied.

For linear time invariant system, $h(t,u)=h(t-u)$

If a system is such that its Input X(t) and its Output Y(t) are related by a Convolution integral,

i.e., if $Y(t)=\int_{-\infty}^{\infty} h(u)\,X(t-u)\,du$, then the system is a

linear time-invariant system.

19. **Find the ACF of the random process {X(t)}, if its power spectral density is given by**

$$S(\omega) = \begin{cases} 1+\omega^2, & for\ |\omega| \le 1 \\ 0, & for\ |\omega| > 1 \end{cases}$$

Solution:

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega = \frac{1}{2\pi}\int_{-1}^{1}\{1+\omega^2\}e^{i\omega\tau}\,d\omega = \frac{1}{2\pi}\int_{-1}^{1}\{e^{i\omega\tau}+\omega^2 e^{i\omega\tau}\}d\omega = \frac{1}{2\pi}\left\{\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-1}^{1} + \int_{-1}^{1}\omega^2\cos\omega\tau\,d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-}^{1} + 2\int_{0}^{1}\omega^2\cos\omega\tau\,d\omega\right\} = \frac{1}{2\pi}\left[\frac{e^{i\tau}-e^{-i\tau}}{i\tau}\right] + 2\left[\omega^2\frac{\sin\omega\tau}{\tau} + \frac{2\omega\cos\omega\tau}{\tau^2} - \frac{2\sin\omega\tau}{\tau^3}\right]_0^1$$

$$= \frac{1}{2\pi}\left[\frac{2\sin\tau}{\tau} + \frac{2\sin\tau}{\tau} + \frac{4\cos\tau}{\tau^2} - \frac{4\sin\tau}{\tau^3}\right] = \frac{1}{2\pi}\left[\frac{2\tau^2\sin\tau + 2\tau^2\sin\tau + \tau 4\cos\tau - 4\sin\tau}{\tau^3}\right]$$

$$= \frac{2\{\tau^2\sin\tau + \tau\cos\tau - \sin\tau\}}{\pi\tau^3}$$

20. **Define Average power.**

$$\text{Average power } = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\,d\omega = R(0)$$

21. A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the system transfer function.

Solution:

The unit step function $U(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

$$h(t) = \begin{cases} 0, & t < 0 \\ e^{-\beta t}, & t \geq 0 \end{cases}$$

$$\therefore H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t}\, dt$$

$$= \int_0^{\infty} e^{-\beta t} e^{-i\omega t}\, dt$$

$$= \int_0^{\infty} e^{-(\beta + i\omega)t}\, dt$$

$$= \left[\frac{e^{-(\beta+i\omega)t}}{-(\beta+i\omega)} \right]_0^{\infty}$$

$$= -\frac{1}{\beta+i\omega} \left[e^{-(\beta+i\omega)t} \right]_0^{\infty}$$

$$= -\frac{1}{\beta+i\omega} [0-1]$$

$$= \frac{1}{\beta+i\omega}$$

22. State any properties of Linear time invariant system.

If the input $x(t)$ and its output $y(t)$ are related by $y(t) = \int_{-\infty}^{\infty} h(u)\, x(t-u)\, du$, then the system is linear time invariant system