



SNS COLLEGE OF TECHNOLOGY

Coimbatore-26
An Autonomous Institution



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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT II – TIME RESPONSE ANALYSIS

TOPIC 5- STEADY STATE ERRORS



OUTLINE



- REVIEW ABOUT PREVIOUS CLASS
- STEADY STATE ERROR
- CLASSIFICATION OF CONTROL SYSTEMS
- STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS
- STATIC ERROR CONSTANTS
- STATIC POSITION ERROR CONSTANT (K_p)
- ACTIVITY
- STATIC VELOCITY ERROR CONSTANT (K_v)
- STATIC ACCELERATION ERROR CONSTANT (K_A)
- EXAMPLE
- SUMMARY



STEADY STATE ERROR



- If the output of a control system at steady state does not exactly match with the input, the system is said to have steady state error
- Any physical control system inherently suffers steady-state error in response to certain types of inputs.
- A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input.



CLASSIFICATION OF CONTROL SYSTEMS



- Control systems may be classified according to their ability to follow
 - Step inputs,
 - Ramp inputs,
 - Parabolic inputs, and so on.
- The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system.



CLASSIFICATION OF CONTROL SYSTEMS



- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots (T_m s + 1)}{s^N (T_1 s + 1)(T_2 s + 1) \cdots (T_p s + 1)}$$

- It involves the term s^N in the denominator, representing N poles at the origin.
- A system is called type 0, type 1, type 2, ... , if $N=0$, $N=1$, $N=2$, ... , respectively.



CLASSIFICATION OF CONTROL SYSTEMS

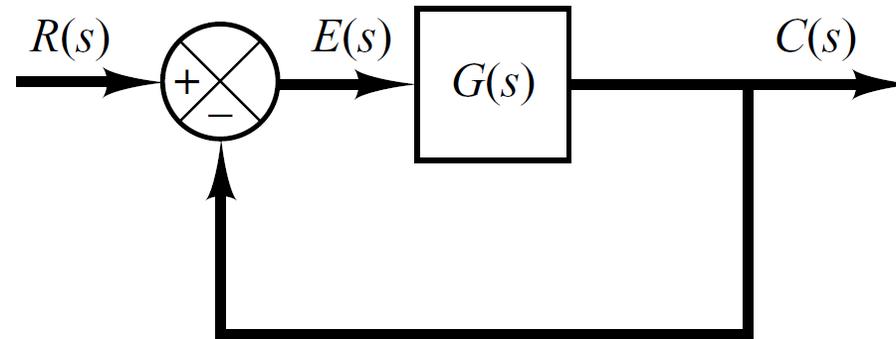


- As the type number is increased, accuracy is improved.
- However, increasing the type number aggravates the stability problem.
- A compromise between steady-state accuracy and relative stability is always necessary.



STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS

- Consider the system shown in following figure.



- The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \quad G(s) = \frac{K(T_a s + 1)(T_b s + 1) \cdots (T_m s + 1)}{s^N (T_1 s + 1)(T_2 s + 1) \cdots (T_p s + 1)}$$



STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS



- Steady state error is defined as the error between the input signal and the output signal when $t \rightarrow \infty$.

- The transfer function between the error signal $E(s)$ and the input signal $R(s)$ is

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)}$$

- The final-value theorem provides a convenient way to find the steady-state performance of a stable system.

- Since $E(s)$ is
$$E(s) = \frac{1}{1 + G(s)} R(s)$$

- The steady state error is

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)}$$



STATIC ERROR CONSTANTS



- The static error constants are figures of merit of control systems. The higher the constants, the smaller the steady-state error.
- In a given system, the output may be the **position, velocity, pressure, temperature, or the like.**
- Therefore, in what follows, we shall call the output “position,” the rate of change of the output “velocity,” and so on.
- This means that in a temperature control system “position” represents the output temperature, “velocity” represents the rate of change of the output temperature, and so on.



STATIC POSITION ERROR CONSTANT (K_p)



- The steady-state error of the system for a unit-step input is

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \frac{\cancel{s}}{1 + G(s)} \frac{1}{\cancel{s}} \\ &= \frac{1}{1 + G(0)} \end{aligned}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \rightarrow 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$



STATIC POSITION ERROR CONSTANT (K_p)



- For a **Type 0** system

$$K_p = \lim_{s \rightarrow 0} \frac{K(T_a s + 1)(T_b s + 1) \cdots}{(T_1 s + 1)(T_2 s + 1) \cdots} = K$$

- For **Type 1** or higher order systems

$$K_p = \lim_{s \rightarrow 0} \frac{K(T_a s + 1)(T_b s + 1) \cdots}{s^N (T_1 s + 1)(T_2 s + 1) \cdots} = \infty, \quad \text{for } N \geq 1$$

- For a unit step input the steady state error e_{ss} is

$$e_{ss} = \frac{1}{1 + K}, \quad \text{for type 0 systems}$$

$$e_{ss} = 0, \quad \text{for type 1 or higher systems}$$



ACTIVITY-PUZZLES



Can you Solve this?

IF

$4+4=4$

$25+25=10$

$16+16=8$

$9+9=6$

THEN

$1+1=?$

SHARE ONCE YOU SOLVED IT!!!



www.FunWithPuzzles.com

Solve this if you are a Genius

On which number the bus is parked?



23

347189

459437

561785

673033

785381



STATIC VELOCITY ERROR CONSTANT (K_v)



- The steady-state error of the system for a unit-ramp input is

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \frac{s}{1 + G(s)} \frac{1}{s^2} \\ &= \lim_{s \rightarrow 0} \frac{1}{sG(s)} \end{aligned}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$



STATIC VELOCITY ERROR CONSTANT (K_v)



- For a **Type 0** system

$$K_v = \lim_{s \rightarrow 0} \frac{sK(T_a s + 1)(T_b s + 1) \cdots}{(T_1 s + 1)(T_2 s + 1) \cdots} = 0$$

- For **Type 1** systems

$$K_v = \lim_{s \rightarrow 0} \frac{sK(T_a s + 1)(T_b s + 1) \cdots}{s(T_1 s + 1)(T_2 s + 1) \cdots} = K$$

- For type 2 or higher order systems

$$K_v = \lim_{s \rightarrow 0} \frac{sK(T_a s + 1)(T_b s + 1) \cdots}{s^N (T_1 s + 1)(T_2 s + 1) \cdots} = \infty, \quad \text{for } N \geq 2$$



STATIC VELOCITY ERROR CONSTANT (K_v)



- For a ramp input the steady state error e_{ss} is

$$e_{ss} = \frac{1}{K_v} = \infty, \quad \text{for type 0 systems}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}, \quad \text{for type 1 systems}$$

$$e_{ss} = \frac{1}{K_v} = 0, \quad \text{for type 2 or higher systems}$$



STATIC ACCELERATION ERROR CONSTANT (K_A)



- The steady-state error of the system for parabolic input is

$$\begin{aligned}e_{ss} &= \lim_{s \rightarrow 0} \frac{s}{1 + G(s)} \frac{1}{s^3} \\ &= \frac{1}{\lim_{s \rightarrow 0} s^2 G(s)}\end{aligned}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$



STATIC ACCELERATION ERROR CONSTANT (K_A)



- For a **Type 0** system

$$K_a = \lim_{s \rightarrow 0} \frac{s^2 K (T_a s + 1)(T_b s + 1) \dots}{(T_1 s + 1)(T_2 s + 1) \dots} = 0$$

- For **Type 1** systems

$$K_a = \lim_{s \rightarrow 0} \frac{s^2 K (T_a s + 1)(T_b s + 1) \dots}{s (T_1 s + 1)(T_2 s + 1) \dots} = 0$$

- For **type 2** systems

$$K_a = \lim_{s \rightarrow 0} \frac{s^2 K (T_a s + 1)(T_b s + 1) \dots}{s^2 (T_1 s + 1)(T_2 s + 1) \dots} = K$$

- For **type 3** or higher order systems

$$K_a = \lim_{s \rightarrow 0} \frac{s^2 K (T_a s + 1)(T_b s + 1) \dots}{s^N (T_1 s + 1)(T_2 s + 1) \dots} = \infty, \quad \text{for } N \geq 3$$



STATIC ACCELERATION ERROR CONSTANT (K_A)



- For a parabolic input the steady state error e_{ss} is

$$e_{ss} = \infty, \quad \text{for type 0 and type 1 systems}$$

$$e_{ss} = \frac{1}{K}, \quad \text{for type 2 systems}$$

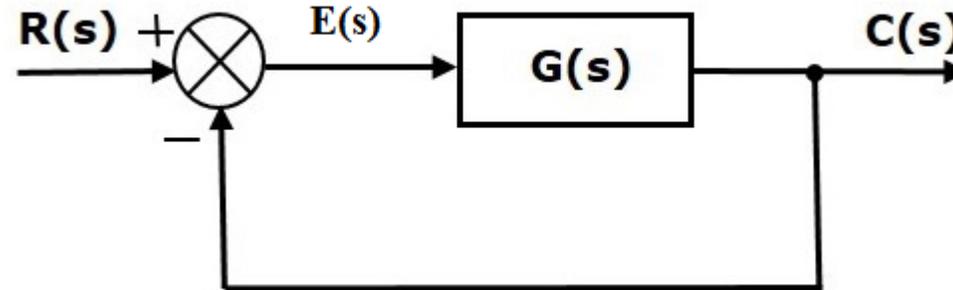
$$e_{ss} = 0, \quad \text{for type 3 or higher systems}$$



STEADY STATE ERROR

- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss}

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} E(s)$$





STEADY STATE ERROR

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1 + G(s)}$$

$$E(s) = R(s) - C(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)}$$



STEADY STATE ERROR

- The following table shows the steady state errors and the error constants for standard input signals like unit step, unit ramp & unit parabolic signals.

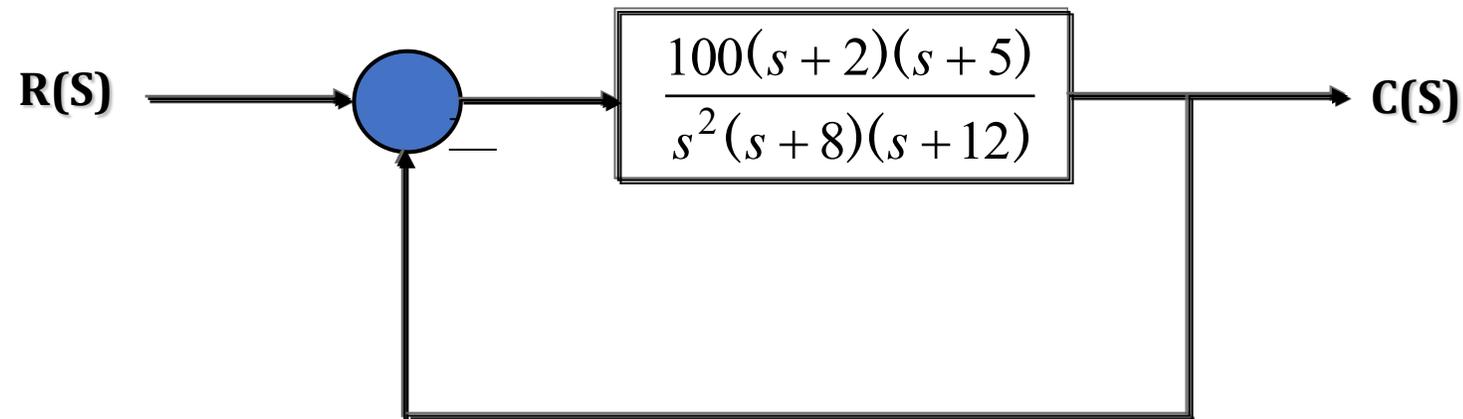
Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p = \lim_{s \rightarrow 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v = \lim_{s \rightarrow 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a = \lim_{s \rightarrow 0} s^2 G(s)$

- Where K_p , K_v , K_a are the position error constant, velocity error constant and acceleration error constant respectively.



EXAMPLE

- For the system shown in figure below evaluate the static error constants and find the expected steady state errors for the standard step, ramp and parabolic inputs.





EXAMPLE



$$G(s) = \frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)$$

$$K_p = \lim_{s \rightarrow 0} \left(\frac{100(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$$K_p = \infty$$

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

$$K_v = \lim_{s \rightarrow 0} \left(\frac{100s(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$K_v = \infty$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

$$K_a = \lim_{s \rightarrow 0} \left(\frac{100s^2(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$$K_a = \left(\frac{100(0+2)(0+5)}{(0+8)(0+12)} \right) = 10.4$$



EXAMPLE



$$K_p = \infty$$

$$K_v = \infty$$

$$K_a = 10.4$$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

$$e_{ss} = \frac{1}{K_v} = 0$$

$$e_{ss} = \frac{1}{K_a} = 0.09$$



SUMMARY



	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1 + K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$



SUMMARY

