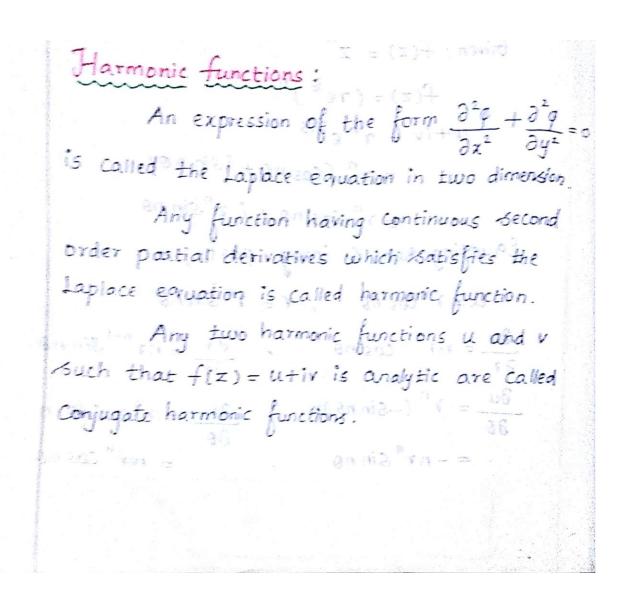




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#### **DEPATMENT OF MATHEMATICS**

# Note:

Both real and imaginary parts of an analytic function are harmonic. But the converse need not be true.

(1) Give an example such that u and v are harmonic but utiv is not analytic.

Let 
$$w = \overline{\chi}$$
:
$$u + iv = \chi - iy$$

$$\Rightarrow u = \chi$$

$$\frac{\partial u}{\partial \chi} = \chi$$

$$\frac{\partial u}{\partial y} = \chi$$

$$\frac{\partial u}{\partial y} = \chi$$

$$\frac{\partial v}{\partial z} = \chi$$

$$\frac{\partial^2 v}{\partial z^2} = \chi$$

$$\frac{\partial^2 v}{\partial z} = \chi$$

But  $u_x \neq v_y$  and  $u_y = -v_x$  f(z) = u + iv is not analytic. (2) Prove that  $u = e^x \cos y$  is a harmonic function.

Soln:

$$\frac{\partial u}{\partial x} = e^{x} \cos y ; \quad \frac{\partial u}{\partial y} = -e^{x} \sin y$$

$$\frac{\partial^{2}u}{\partial x^{2}} = e^{x} \cos y ; \quad \frac{\partial^{2}u}{\partial y^{2}} = -e^{x} \cos y$$

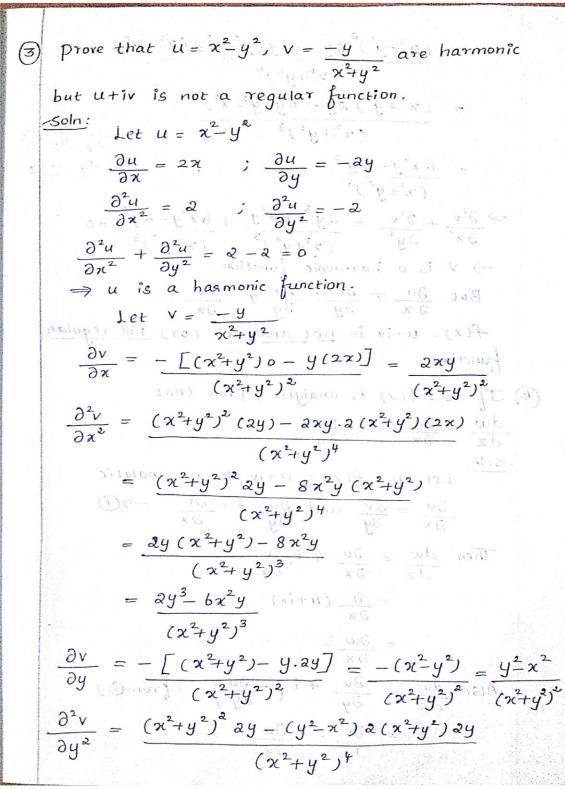
$$\Rightarrow \frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}} = 0$$

$$\therefore u \text{ is a harmonic function } .$$





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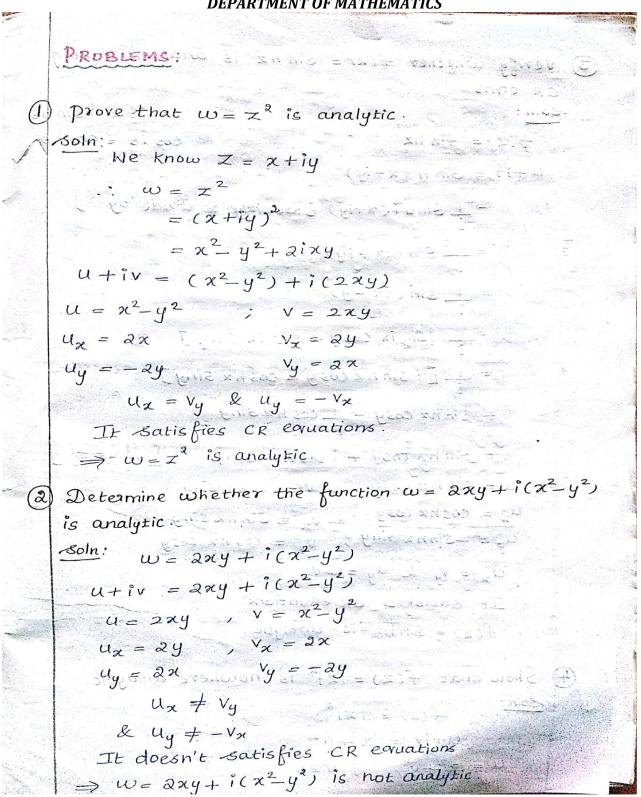






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$$(x^{2}+y^{2})^{2} 2y - 4y(y^{2}-x^{2})(x^{2}+y^{2})$$

$$(x^{2}+y^{2})^{4}$$

$$= (x^{2}+y^{2}) 2y - 4y(y^{2}-x^{2})$$

$$(x^{2}+y^{2})^{3}$$

$$= bx^{2}y - 2y^{3}$$

$$(x^{2}+y^{2})^{3}$$

$$\Rightarrow \frac{\partial^{2}v}{\partial x^{2}} + \frac{\partial^{2}v}{\partial y^{2}} = \frac{2y^{3}-bx^{2}y}{(x^{2}+y^{2})^{3}} + \frac{bx^{2}y-2y^{3}}{(x^{2}+y^{2})^{3}} = 0$$

$$\Rightarrow v \text{ is a harmonic function.}$$

$$\text{But } \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}; \frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$$

$$f(z) = u + iv \text{ is not analytice (or) not regular function.}$$