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DEPARTMENT OF MATHEMATICS

Construction of Analytic function: =) U is harmo Milne-Thomson method: Let f(z) = u + iv is to be constructed. (i) Suppose the real part u is given, then $f(z) = \int_{-\infty}^{\infty} f(z) + \int_{-\infty}^{$ $f(z) = \int \left[\varphi_1(z, 0) - i \varphi_2(z, 0) \right] dz$ Where $\varphi_1(z,0) = \frac{\partial u}{\partial x}(z,0)$, $\varphi_2(z,0) = \frac{\partial u}{\partial y}(z,0)$. (ii) Suppose imaginary part v is given, then $-f(z) = \int [\varphi_{1}(z,0) + i\varphi_{2}(z,0)]dz$ where $\varphi_1(z,0) = \frac{\partial v}{\partial y}(z,0), \quad \varphi_2(z,0) = \frac{\partial v}{\partial x}(z,0)$





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DEPATMENT OF MATHEMATICS

() Show that the
$$f_{12}^{n}$$
 $u = x^{3} + x^{2} - 3xy^{2} + 2xy - y^{2}$
is harmonic and find the corresponding analytic
 $f_{12}^{n} - f(x) = u + iv$.
Soln:
 $u = x^{3} + x^{2} - 3xy^{2} + 2xy - y^{2}$
 $u_{x} = 3x^{2} + 2x - 3y^{2} + 2y$; $u_{y} = -bxy + 2x - 2y^{2}$
 $u_{xx} = bx + 2$; $u_{yy} = -bx - 2$
 $u_{xx} + u_{yy} = 0$
 $\Rightarrow u$ is hasmonic. $f(z) = \int \left[\frac{\partial u}{\partial x}(z, 0) - i\frac{\partial u}{\partial y}(z, 0)\right] dz$
where $\varphi_{1}(z, 0) = \frac{\partial u}{\partial x}(z, 0) = 3z^{2} + 2z$
 $f(z) = \int \int \left[(3z^{2} + 2z) - i\cdot 2z\right] dz$
 $f(z) = \int \int \left[(3z^{2} + 2z) - i\cdot 2z\right] dz$
 $= \frac{3z^{3}}{3} + \frac{2z^{2}}{4} - i \cdot \frac{2z^{2}}{2} + c$
 $f(z) = z^{3} + z^{2}(1 - i) + c$.
(2) Find an analytic f_{12} whose imaginaxy past is
 $V = e^{2x} (y \cos 2y + x \sin 2y)$
 $\frac{soln:}{V} = e^{2x} (y \cos 2y + x \sin 2y) + e^{2x} \sin 2y$
 $v_{y} = e^{2x} (\cos 2y + y (-2\sin 2y) + 2x\cos 2y)$
 $f(z) = \int \left[\frac{\partial v}{\partial y}(z, 0) + i\frac{\partial v}{\partial x}(z, 0)\right] dz$
 $= \int \left[\frac{\partial v}{\partial y}(z, 0) + i\frac{\partial v}{\partial x}(z, 0)\right] dz$



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DEPARTMENT OF MATHEMATICS

(3) Prove that
$$u = x^2 - y^2$$
, $v = -\frac{y}{x^2 + y^2}$ are harmonic
but u + iv is not a regular function.
Soln: Let $u = x^2 - y^2$
 $\frac{\partial u}{\partial x} = 2x$; $\frac{\partial u}{\partial y} = -2y$
 $\frac{\partial^2 u}{\partial x^2} = 2$; $\frac{\partial^2 u}{\partial y^2} = -2$
 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 - 2 = 0$
 $\Rightarrow u$ is a hasmonic function.
Let $v = -\frac{y}{x^2 + y^2}$
 $\frac{\partial v}{\partial x} = -\frac{[(x^2 + y^2) \circ - y(2x)]}{(x^2 + y^2)^2} = \frac{2xy}{(x^2 + y^2)^2}$
 $\frac{\partial^2 v}{\partial x^2} = \frac{(x^2 + y^2)^2 (2y) - 2xy \cdot 2(x^2 + y^2)}{(x^2 + y^2)^4}$
 $= \frac{2y(x^2 + y^2)^2 (2y) - 2xy \cdot 2(x^2 + y^2)}{(x^2 + y^2)^3}$
 $= \frac{2y(x^2 + y^2)^2 (2y) - 8x^2y}{(x^2 + y^2)^3}$
 $= \frac{2y^3 - 6x^2y}{(x^2 + y^2)^3}$
 $\frac{\partial^2 v}{\partial y} = -\frac{[(x^2 + y^2) - y \cdot 2y]}{(x^2 + y^2)^2} = -(x^2 - y^2) = \frac{y^2 - x^2}{(x^2 + y^2)^2}$
 $\frac{\partial^2 v}{\partial y^2} = \frac{(x^2 + y^2)^2 (2y - (y^2 - x^2)) 2((x^2 + y^2))^2}{(x^2 + y^2)^4}$





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DEPARTMENT OF MATHEMATICS

$$\begin{aligned} \varphi_{i}(z, o) &= \frac{\partial v}{\partial y}(z, o) = e^{2z}(1+o+az) \\ &= e^{3z} + az e^{2z} \\ \varphi_{a}(z, o) &= \frac{\partial v}{\partial x}(z, o) = ae^{2z}(o) + e^{2z}(o) = o \\ f(z) &= \int [e^{2z} + az e^{2z}] dz + c \\ &= \int e^{az} dz + a \int z e^{az} dz + c \\ &= \frac{e^{az}}{2} + a \left(\frac{ze^{2z}}{2} - \frac{e^{2z}}{4}\right) + c \\ &= \frac{e^{az}}{2} + ze^{az} - \frac{e^{2z}}{2} + c \\ &= ze^{az} + c \end{aligned}$$

$$(3) \text{ Griven that } u = \underline{Sin 2x} \\ \underline{cosh 2y - cos 2x} \text{, find the} \\ \underline{Sohn} \quad u = \underline{Sin 2x} \\ \underline{cosh 2y - cos 2x} \end{aligned}$$

$$u_{x} = (\underline{cosh 2y - cos 2x}) \underline{cos 2x \cdot a} - \underline{Sin 2x} (a \sin ay) \\ (\underline{cosh 2y - cos 2x})^{2} \\ u_{y} = (\underline{cosh 2y - cos 2x}) (\underline{cos 2x \cdot a} - \underline{Sin 2x} (a \sin ay) \\ \underline{cosh 2y - cos 2x})^{2} \\ f(z) &= \int [\varphi_{i}(z, o) + \varphi_{i}(z; o)] dz \\ \varphi_{i}(z, o) = \frac{\partial z}{\partial x} (z, o) \\ (\underline{cos - cos 2z})^{2} \underline{cos 2z} - \underline{Sin 2z} (a \sin ay) \\ \underline{cos - cos 2z} - \underline{cos 2z} - \underline{Sin 2z} (a \sin ay) \\ \underline{cos - cos 2z} - \underline{cos 2$$





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