



DEPARTMENT OF MATHEMATICS

Conformal mapping :

① Find the image of the following region under the translation $w = 1/z$

(i) half plane $x > c$ when $c > 0$

(ii) the infinite strip $\frac{1}{4} < y < \frac{1}{2}$

(iii) the infinite strip $0 < y < \frac{1}{2}$

Soln: $w = \frac{1}{z}$

$$z = \frac{1}{w}$$

$$x + iy = \frac{1}{u + iv} = \frac{1}{u + iv} \cdot \frac{u - iv}{u - iv}$$

$$x + iy = \frac{u - iv}{u^2 + v^2} = \frac{u}{u^2 + v^2} - i \frac{v}{u^2 + v^2}$$

$$x = \frac{u}{u^2 + v^2}, \quad y = \frac{-v}{u^2 + v^2}$$

(i) Half plane $x > c$ when $c > 0$

$$x = c$$

$$\frac{u}{u^2 + v^2} = c$$

$$u = c(u^2 + v^2)$$

$$\frac{u}{c} = u^2 + v^2$$

$$u^2 - \frac{u}{c} + v^2 = 0$$

$$\left(u^2 - \frac{u}{c} + \left(\frac{1}{2c}\right)^2\right) + v^2 - \left(\frac{1}{2c}\right)^2 = 0$$

$$\left(u - \frac{1}{2c}\right)^2 + v^2 = \left(\frac{1}{2c}\right)^2$$

which is a circle with centre $\left(\frac{1}{2c}, 0\right)$ & radius $\frac{1}{2c}$.

$$u^2 - \frac{u}{c}$$

$$a = u$$

$$2ab = \frac{u}{c}$$

$$b = \frac{u}{2ac}$$

$$b = \frac{u}{2ac}$$

$$b = \frac{1}{2c}$$



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(ii) the infinite strip $\frac{i}{4} < y < \frac{i}{2}$

$$y = \frac{i}{4} \quad \text{and} \quad y = \frac{i}{2} \quad (1)$$

$$\frac{-v}{u^2+v^2} = \frac{1}{4} \quad \text{and} \quad \frac{-v}{u^2+v^2} = \frac{1}{2} \quad (i)$$

$$-v = \frac{1}{4}(u^2+v^2) \quad \text{and} \quad -2v = u^2+v^2 \quad (ii)$$

$$-4v = u^2+v^2 \quad \text{and} \quad u^2+v^2+2v=0 \quad (iii)$$

$$u^2+v^2+4v=0 \quad \text{and} \quad u^2+(v+1)^2-1=0$$

$$u^2+(v+2)^2-4=0 \quad \text{and} \quad u^2+(v+1)^2=1$$

$$u^2+(v+2)^2=4 \quad \text{and} \quad \text{centre: } (0, -1)$$

$$\text{which is a eqn of circle with centre } (0, -2)$$

$$\& r = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

(iii) $0 < y < 1/2$

$$y = 0 \quad \text{and} \quad y = \frac{1}{2}$$

$$\frac{-v}{u^2+v^2} = 0 \quad \text{and} \quad \frac{-v}{u^2+v^2} = \frac{1}{2} \quad (i)$$

$$\boxed{v = 0} \quad \text{and} \quad -2v = u^2+v^2$$

$$\text{which is a straight line} \quad u^2+v^2+2v=0$$

$$\text{in w-plane.} \quad u^2+(v+1)^2-1=0$$

$$u^2+(v+1)^2=1 \quad \text{centre: } (0, -1)$$

$$\text{radius} = 1$$

$$\left(0, -\frac{1}{2}\right) \quad \text{and} \quad \left(0, \frac{1}{2}\right)$$

$$\left(0, -\frac{1}{2}\right) \quad \text{and} \quad \left(0, \frac{1}{2}\right)$$