



## SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Meromorphic function:

A function  $f(z)$  which is analytic everywhere in the finite plane except at finite number of Poles is called a meromorphic function.

$$\text{Eg: } f(z) = \frac{z+3}{(z-1)(z-2)^2}$$

Laurent's Series:

Let  $C_1$  and  $C_2$  be two concentric circles then

$$|z-a| = R_1 \text{ and } |z-a| = R_2 \text{ where } R_2 < R_1$$

Let  $f(z)$  be analytic inside and on the annular region  $R$  between  $C_1$  and  $C_2$ . Then for any  $z \in R$ ,

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$$

$$a_n = \frac{1}{2\pi i} \int_{C_1} \frac{f(z)}{(z-a)^{n+1}} dz$$

$$b_n = \frac{1}{2\pi i} \int_{C_2} \frac{f(z)}{(z-a)^{1-n}} dz$$

Problems:

1) Expand  $f(z) = \frac{z^2-1}{(z+2)(z+3)}$  in a Laurent's series if

(i)  $|z| < 2$  (ii)  $|z| > 3$  and (iii)  $2 < |z| < 3$ .

Sol: Using Partial fractions,

$$f(z) = \frac{z^2-1}{(z+2)(z+3)} = A + \frac{B}{z+2} + \frac{C}{z+3} \rightarrow \textcircled{1}$$



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$$\frac{z^2 - 1}{(z+2)(z+3)} = \frac{A(z+2)(z+3) + B(z+3) + C(z+2)}{(z+2)(z+3)}$$

$$z^2 - 1 = A(z+2)(z+3) + B(z+3) + C(z+2)$$

$$z = -2 \Rightarrow (-2)^2 - 1 = A(0) + B(-2+3) + 0$$

$$4 - 1 = B$$

$$B = 3$$

$$z = -3 \Rightarrow (-3)^2 - 1 = 0 + 0 + C(-3+2)$$

$$9 - 1 = -C$$

$$8 = -C$$

$$C = -8$$

$$z = 0 \Rightarrow 0 - 1 = A(2)(3) + B(3) + C(2)$$

$$-1 = 6A + 3B + 2C$$

$$-1 = 6A + 9 - 16$$

$$-1 = 6A - 7$$

$$6A = 7 - 1$$

$$6A = 6$$

$$A = 1$$

$$(i) \Rightarrow f(z) = 1 + \frac{3}{z+2} - \frac{8}{z+3}$$

$$(i) |z| < 2 \Rightarrow \frac{|z|^2}{3} < 1$$



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$$i) |z| < 2 \Rightarrow \frac{|z|}{2} < 1$$

$$(2) \Rightarrow f(z) = 1 + \frac{3}{2\left(1+\frac{z}{2}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)}$$

$$= 1 + \frac{3}{2} \left(1+\frac{z}{2}\right)^{-1} - \frac{8}{3} \left(1+\frac{z}{3}\right)^{-1}$$

$$= 1 + \frac{3}{2} \left[ 1 - \frac{z}{2} + \left(\frac{z}{2}\right)^2 - \dots \right]$$

$$- \frac{8}{3} \left[ 1 - \frac{z}{3} + \left(\frac{z}{3}\right)^2 - \dots \right]$$

$$(1+z)^{-1} = 1 - z + z^2 - z^3 + \dots$$

$$ii) |z| > 3$$

$$3 < |z|$$

$$\frac{3}{|z|} < 1$$

$$(2) \Rightarrow f(z) = 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{z\left(1+\frac{3}{z}\right)}$$

$$= 1 + \frac{3}{z} \left(1+\frac{2}{z}\right)^{-1} - \frac{8}{z} \left(1+\frac{3}{z}\right)^{-1}$$

$$= 1 + \frac{3}{z} \left[ 1 - \frac{2}{z} + \left(\frac{2}{z}\right)^2 - \dots \right] - \frac{8}{z} \left[ 1 - \frac{3}{z} + \left(\frac{3}{z}\right)^2 - \dots \right]$$

$$iii) 2 < |z| < 3$$

$$2 < |z| \text{ and } |z| < 3$$

$$\frac{2}{|z|} < 1 \text{ and } \frac{|z|}{3} < 1$$



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$$② \Rightarrow 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)}$$

$$= 1 + \frac{3}{z} \left(1 + \frac{2}{z}\right)^{-1} - \frac{8}{3} \left(1 + \frac{z}{3}\right)^{-1}$$

$$= 1 + \frac{3}{z} \left(1 - \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \dots\right) - \frac{8}{3} \left(1 - \frac{z}{3} + \left(\frac{z}{3}\right)^2 + \dots\right)$$

2) Expand  $f(z) = \frac{7z-2}{z(z-2)(z+1)}$  in Laurent's Series if

i)  $|z| < 2$  ii)  $|z| > 3$  iii)  $2 < |z| < 3$  iv)  $1 < |z+1| < 3$

Sol: Given  $f(z) = \frac{7z-2}{z(z-2)(z+1)}$

$$\frac{7z-2}{z(z-2)(z+1)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+1}$$

$$7z-2 = A(z-2)(z+1) + Bz(z+1) + Cz(z-2)$$

When  $z=0$ ,  $A = 1$

$z=-1$ ,  $C = -3$

$z=2$ ,  $B = 2$

$$f(z) = \frac{1}{z} + \frac{2}{z-2} - \frac{3}{z+1}$$

i)  $|z| < 2 \Rightarrow \left|\frac{z}{2}\right| < 1$

$$f(z) = \frac{1}{z} + \frac{2}{z-2} - \frac{3}{z+1}$$



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$$\begin{aligned}
 &= \frac{1}{z} + \frac{2}{z-2} - \frac{3}{z+1} \\
 &= \frac{1}{z} + \left(1 - \frac{z}{2}\right)^{-1} - 3(z+1)^{-1} \\
 &= \frac{1}{z} + \left[1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \dots\right] - 3[1 - z + z^2 - z^3 + \dots] \\
 &= \frac{1}{z} - \left[1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \dots\right] - 3[1 - z + z^2 - z^3 + \dots]
 \end{aligned}$$

ii)  $|z| > 3 \Rightarrow \frac{3}{|z|} < 1$

$$\begin{aligned}
 f(z) &= \frac{1}{z} + \frac{2}{z-2} - \frac{3}{z+1} \\
 &= \frac{1}{z} + \frac{2}{z\left(1 - \frac{2}{z}\right)} - \frac{3}{z\left(1 + \frac{1}{z}\right)} \\
 &= \frac{1}{z} + \frac{2}{z} \left(1 - \frac{2}{z}\right)^{-1} - \frac{3}{z} \left(1 + \frac{1}{z}\right)^{-1} \\
 &= \frac{1}{z} + \frac{2}{z} \left(1 - \frac{2}{z}\right)^{-1} - \frac{3}{z} \left(1 + \frac{1}{z}\right)^{-1} \\
 &= \frac{1}{z} + \frac{2}{z} \left[1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + \dots\right] - \frac{3}{z} \left[1 - \frac{1}{z} + \dots\right] \\
 &= \frac{1}{z} + \frac{2}{z} \left[1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + \dots\right] - \frac{3}{z} \left[1 - \frac{1}{z} + \dots\right]
 \end{aligned}$$

iii)  $2 < |z| < 3$

$|z| > 2, |z| < 3$

$\frac{2}{|z|} < 1, \frac{|z|}{3} < 1$

$$f(z) = \frac{1}{z} + \frac{2}{z-2} - \frac{3}{z+1}$$



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$$= \frac{1}{z} + \frac{2}{z(1+\frac{2}{z})} - \frac{3}{z(1+\frac{1}{z})}$$

$$= \frac{1}{z} + \frac{2}{z} \left( 1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \dots \right) - \frac{3}{z} \left( 1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \dots \right)$$

iv)  $|z+1| < 3$

Let  $u = z+1$

$z = u-1$

$|z+1| < 3$

$\frac{1}{|u|} < 1, \quad \left| \frac{u}{3} \right| < 1$

$f(z) = \frac{1}{u-1} + \frac{2}{u-3} - \frac{3}{u}$

$= \frac{1}{u(1-\frac{1}{u})} + \frac{2}{u(1-\frac{u}{3})} - \frac{3}{u}$

$= \frac{1}{u} \left( 1 + \frac{1}{u} + \left(\frac{1}{u}\right)^2 + \dots \right) + \frac{2}{u} \left( 1 + \frac{u}{3} + \left(\frac{u}{3}\right)^2 + \dots \right) - \frac{3}{u}$

$= \frac{1}{u} \left( 1 + \frac{1}{u} + \left(\frac{1}{u}\right)^2 + \dots \right) + \frac{2}{u} \left( 1 + \frac{u}{3} + \left(\frac{u}{3}\right)^2 + \dots \right) - \frac{3}{u}$

$f(z) = \frac{1}{z+1} \left( 1 + \left(\frac{1}{z+1}\right) + \left(\frac{1}{z+1}\right)^2 + \dots \right)$

$+ \frac{2}{3} \left( 1 + \frac{z+1}{3} + \left(\frac{z+1}{3}\right)^2 + \dots \right) - \frac{3}{z+1}$