SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 3 – FIR FILTER DESIGN

TOPIC – Linear Phase FIR Filter

FIR FILTERS

- **Finite Impulse Response (FIR) Systems:** Unit sample response (or) Impulse response h(n) has finite no. of terms
- **Finite Impulse Response (FIR) Filters:** The filters designed by considering all the finite samples of impulse response
- The specification of a digital filter will be desired frequency response $H_d(e^{j\omega})$. The desired impulse response $h_d(n)$ of the digital filter can be obtained by taking inverse Fourier transform $H_d(e^{j\omega})$. The $h_d(n)$ will be an infinite duration discrete time signal defined for all values of n in the range $-\infty$ to $+\infty$

FIR FILTERS

- The transfer function H(z) of the digital filter is obtained by taking Z transform of impulse response. Since $h_d(n)$ is an infinite duration signal, the transfer function obtained from $h_d(n)$ will have infinite terms, which cannot be realized or implemented in a digital system
- Therefore. Finite number of samples $h_d(n)$ are selected to form the impulse response, h(n) of the filter.
- The transfer function H(z) is obtained by taking Z transform of finite sample impulse response h(n). The filters designed by using finite samples of impulse response are called Finite Impulse Response Filters.

ADVANTAGES & DISADVANTAGES OF FIR FILTERS

- **Advantages:** FIR filters with exactly linear phase can be easily designed
- Efficient realizations of FIR filter exist as both recursive and nonrecursive structures
- FIR filters realized nonrecursively, i.e., by direct convolution are always stable
- Roundoff noise, which is inherent in realizations with finite precision arithmetic can easily be made small for nonrecursive realization of FIR filters
- **Disadvantages:** The duration of the impulse response should be large to adequately approximate sharp cutoff filter. Hence a large amount of processing is required to realize such filters when realized via slow convolution
- The delay of linear phase FIR filters need not always be an integer no. of samples. This non-integral delay can lead to problems in signal processing applications

STEPS IN DESIGNING FIR FILTER

- Choose an ideal (desired) frequency response, $H_d(e^{j\omega})$
- Take inverse Fourier transform of $H_d(e^{j\omega})$ to get $h_d(n)$ or sample $H_d(e^{j\omega})$ at finite number of points (N – Point) to get H(k)
- If $h_d(n)$ is determined then convert the infinite duration $h_d(n)$ to a finite duration h(n) or if H(k) is determined then take N-Point inverse DFT to get h(n).
- Take Z transform of h(n) to get H(z), Where H(z)-transfer function of the digital filter
- Choose a suitable structure and realize the filter
- Verify the design, In order to verify the design, determine the actual frequency response $H(e^{j\omega})$ of the filter, by letting $z = e^{j\omega}$ in H(z) and sketch the magnitude response $|H(e^{j\omega})|$

LTI SYSTEM AS FREQUENCY SELECTIVE FILTERS

- The frequency response $H(e^{j\omega})$ is a complex quantity,

$$H(e^{j\omega}) = |H(e^{j\omega})| \angle H(e^{j\omega}) = C e^{-j\alpha\omega}$$

$$\text{where, } |H(e^{j\omega})| = C$$

Magnitude

$$\angle H(e^{j\omega}) = -\alpha\omega$$

Phase

- Magnitude of frequency response is constant and its phase is a linear function of frequency. If the phase function of frequency response of a filter is linear function of frequency, then the filter is called Linear phase filter

- In order to examine the linear and nonlinear phase characteristics, two delay functions are defined and they are **Phase delay and Group delay**

Let, $\angle H(e^{j\omega}) = \theta(\omega)$

$$\theta(\omega) = -\alpha\omega$$

$$\text{Phase delay, } \tau_p = -\frac{\theta(\omega)}{\omega}$$

$$\therefore \tau_p = -\frac{\theta(\omega)}{\omega} = -\frac{-\alpha\omega}{\omega} = \alpha$$

$$\text{Group delay, } \tau_g = -\frac{d}{d\omega}\theta(\omega)$$

$$\tau_g = -\frac{d}{d\omega}\theta(\omega) = -\frac{d}{d\omega}(-\alpha\omega) = \alpha$$

IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS

- The filters are classified according to their frequency response characteristics. The ideal (desired) frequency response $H_d(e^{j\omega})$ of four major types of filters. They are Low pass, High pass, Band pass and Band stop filters
- The $H_d(e^{j\omega})$ is periodic, with periodicity of **0 to 2π** (or **-π** to **π**). Also any analog frequency Ω will map (or can be converted) to frequency of digital system ω within the range **0 to 2π** (or **-π** to **π**)
- Hence the frequency response of digital filters are defined in the interval **0 to 2π** (or **-π** to **π**)

IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS

Ideal Frequency Response of Low pass Filter $H_d(e^{j\omega})$

$$
\begin{aligned}
H_d(e^{j\omega}) &= 0 \quad ; \quad \text{for } \omega = -\pi \text{ to } -\omega_c \\
&= C\,e^{-j\alpha\omega} \quad ; \quad \text{for } \omega = -\omega_c \text{ to } +\omega_c \\
&= 0 \quad ; \quad \text{for } \omega = +\omega_c \text{ to } +\pi
\end{aligned}
$$

Ideal Frequency Response of High pass Filter $H_d(e^{j\omega})$

$$
\begin{aligned}
H_d(e^{j\omega}) &= C\,e^{-j\alpha\omega} \quad ; \quad \text{for } \omega = -\pi \text{ to } -\omega_c \\
&= 0 \quad ; \quad \text{for } \omega = -\omega_c \text{ to } +\omega_c \\
&= C\,e^{-j\alpha\omega} \quad ; \quad \text{for } \omega = +\omega_c \text{ to } +\pi
\end{aligned}
$$

IDEAL FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS

Ideal Frequency Response of Band pass Filter $H_d(e^{j\omega})$

$$
\begin{aligned}
H_d(e^{j\omega}) &= 0 \quad &; \quad \text{for } \omega = -\pi \text{ to } -\omega_{c2} \\
&= C\,e^{-j\alpha\omega} \quad &; \quad \text{for } \omega = -\omega_{c2} \text{ to } -\omega_{c1} \\
&= 0 \quad &; \quad \text{for } \omega = -\omega_{c1} \text{ to } +\omega_{c1} \\
&= C\,e^{-j\alpha\omega} \quad &; \quad \text{for } \omega = +\omega_{c1} \text{ to } +\omega_{c2} \\
&= 0 \quad &; \quad \text{for } \omega = +\omega_{c2} \text{ to } +\pi
\end{aligned}
$$

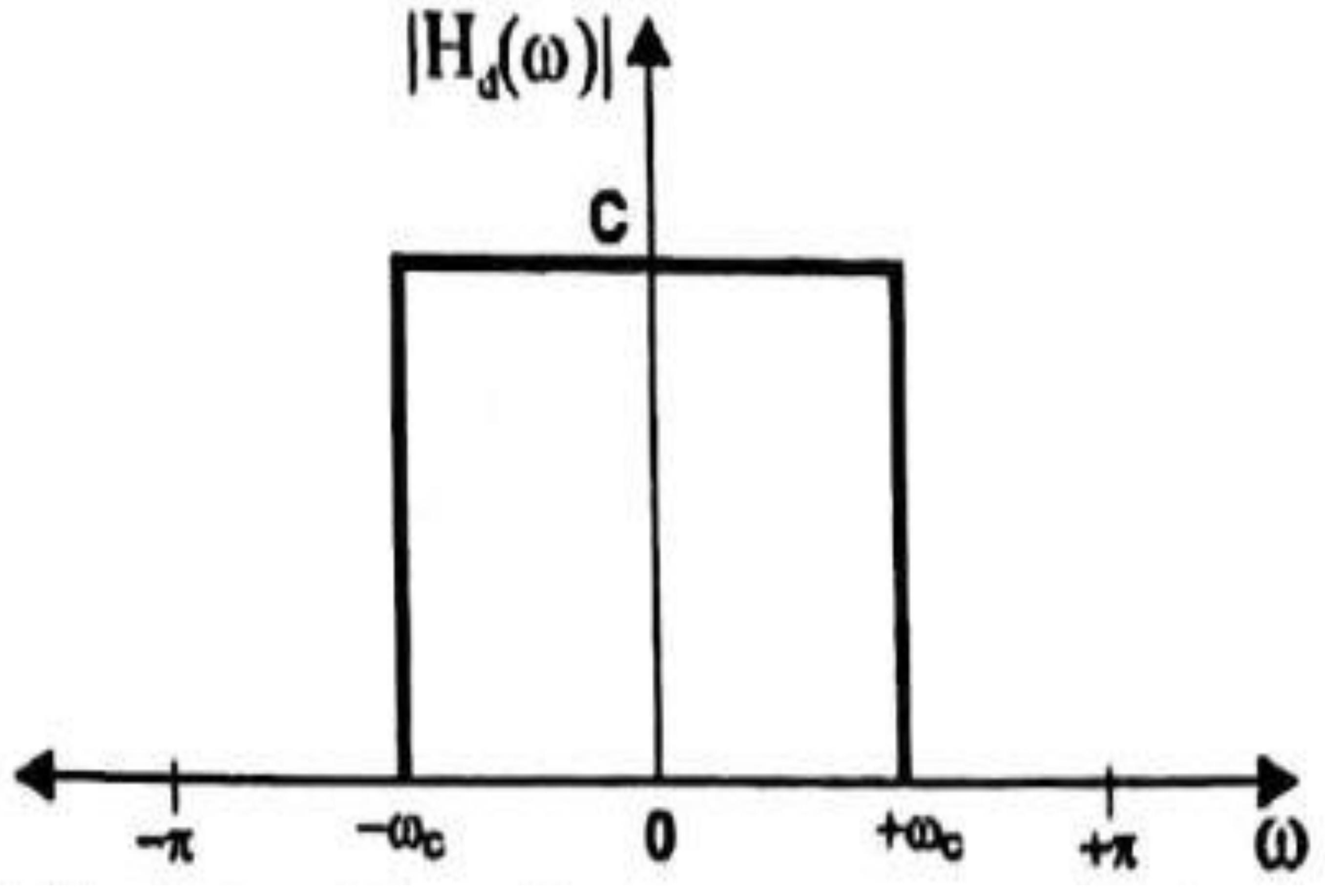
Ideal Frequency Response of Band stop Filter $H_d(e^{j\omega})$

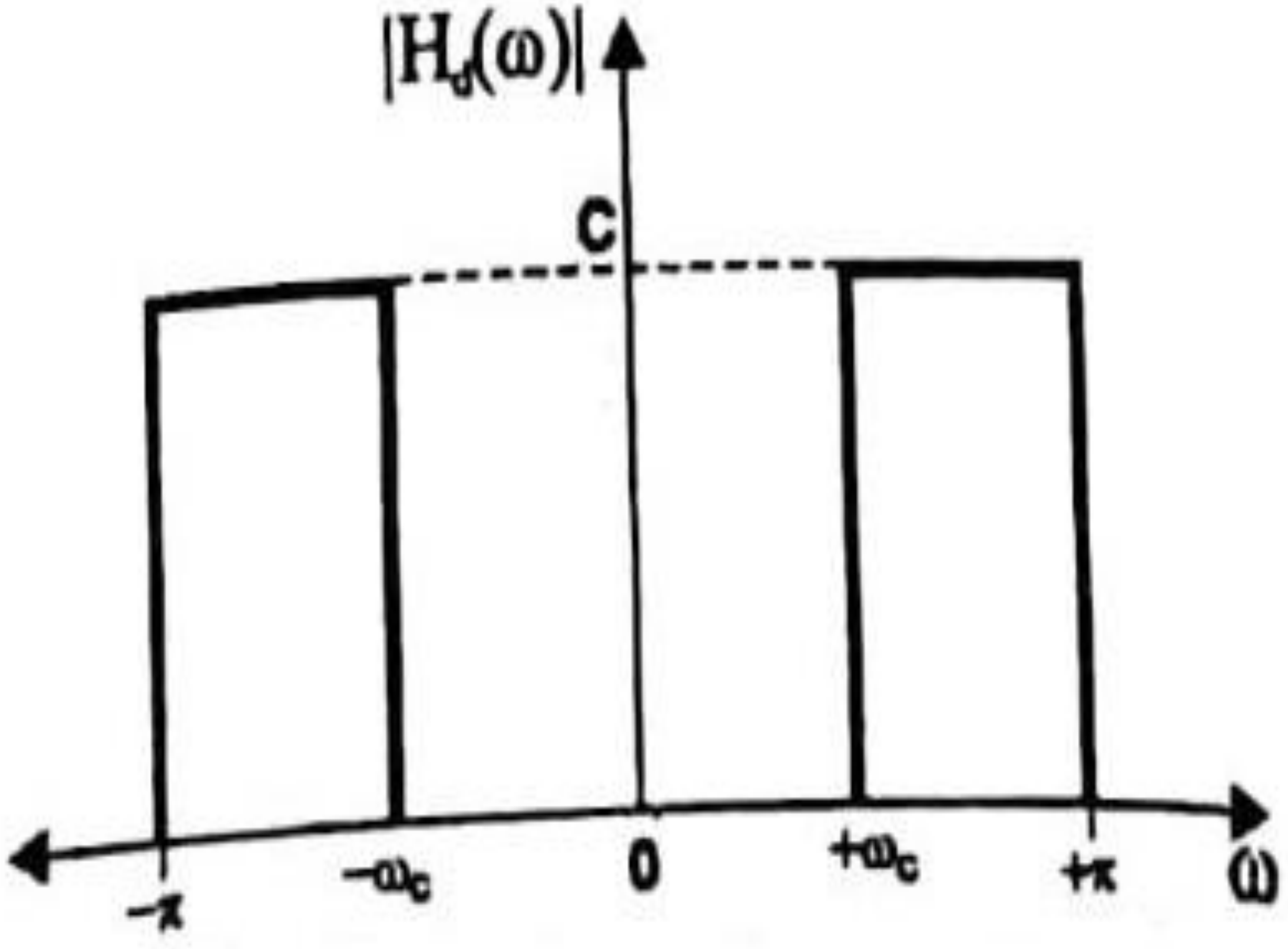
$$
\begin{aligned}
H_d(e^{j\omega}) &= C\,e^{-j\alpha\omega} \quad &; \quad \text{for } \omega = -\pi \text{ to } -\omega_{c2} \\
&= 0 \quad &; \quad \text{for } \omega = -\omega_{c2} \text{ to } -\omega_{c1} \\
&= C\,e^{-j\alpha\omega} \quad &; \quad \text{for } \omega = -\omega_{c1} \text{ to } +\omega_{c1} \\
&= 0 \quad &; \quad \text{for } \omega = +\omega_{c1} \text{ to } +\omega_{c2} \\
&= C\,e^{-j\alpha\omega} \quad &; \quad \text{for } \omega = +\omega_{c2} \text{ to } +\pi
\end{aligned}
$$

MAGNITUDE RESPONSE

IDEAL LOWPASS FILTER

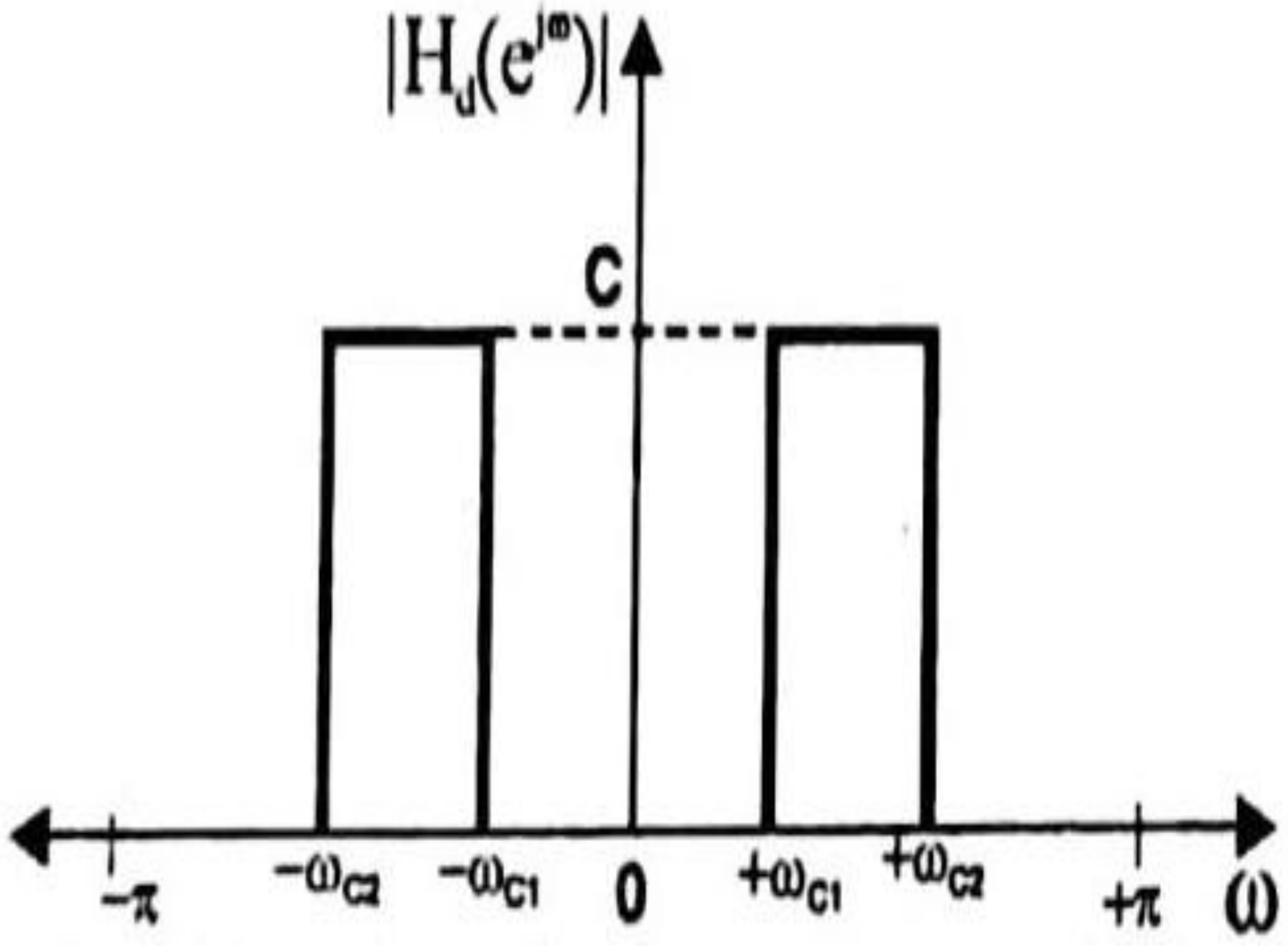
IDEAL HIGHPASS FILTER



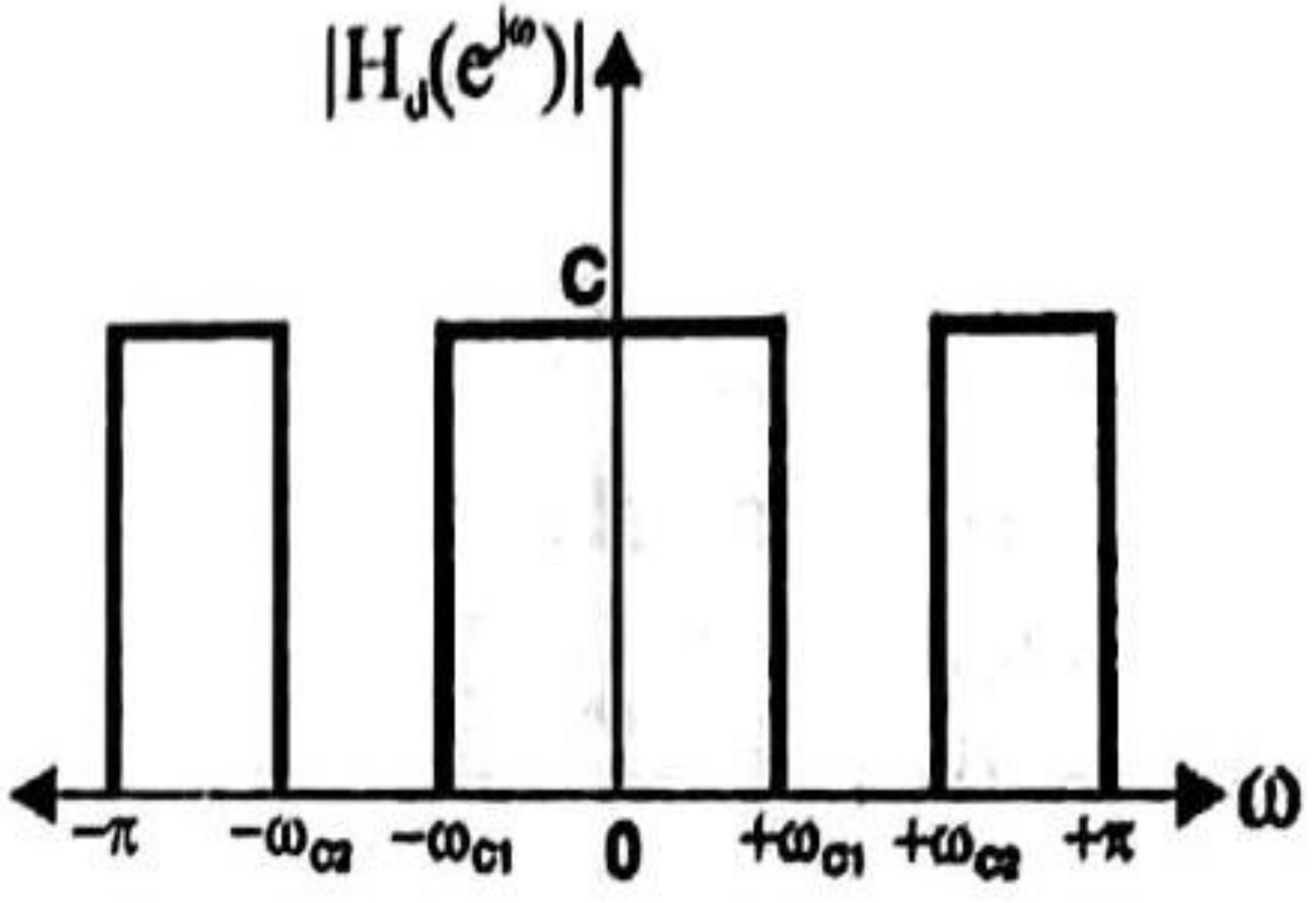


MAGNITUDE RESPONSE

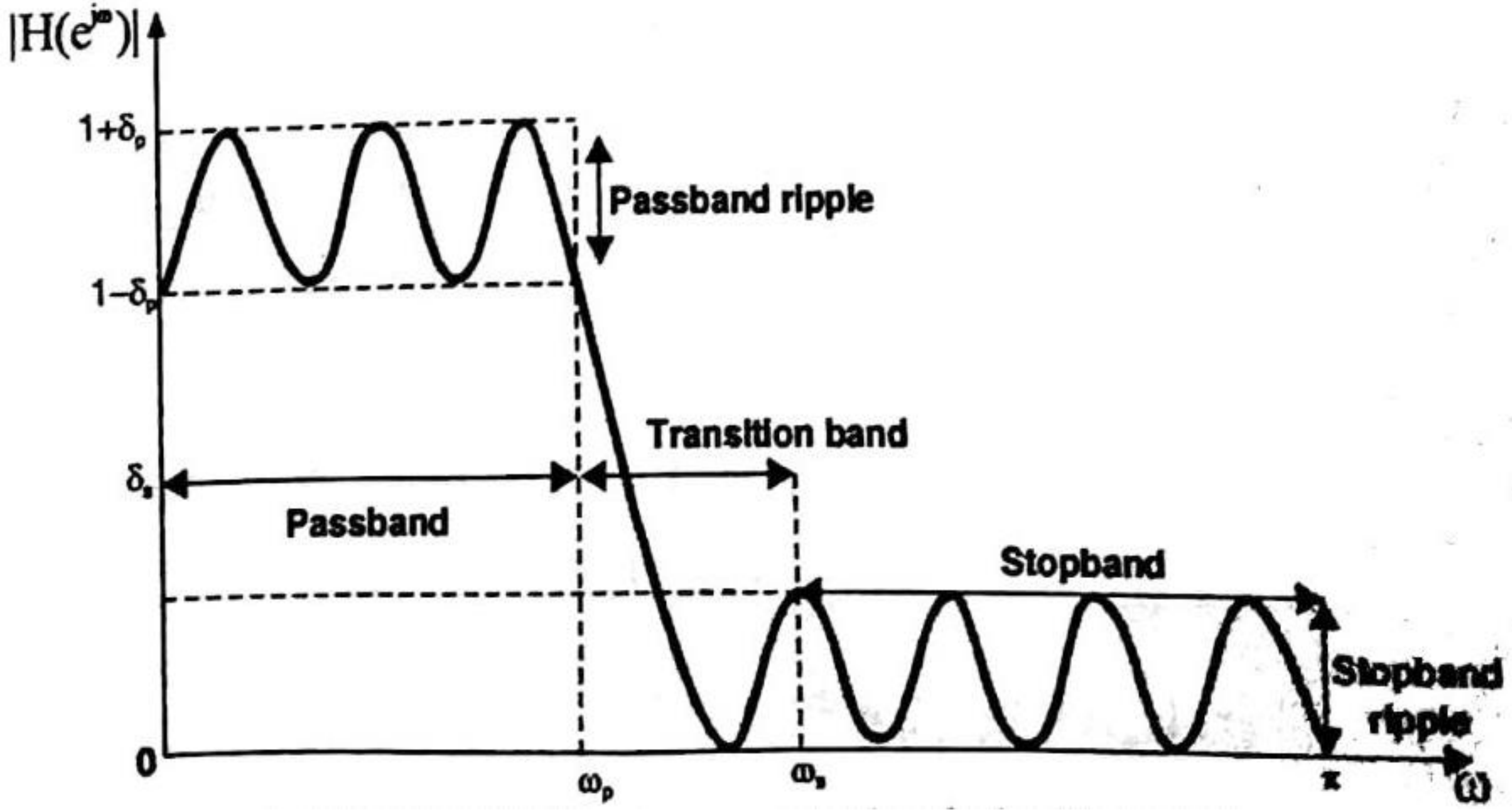
IDEAL BANDPASS FILTER



IDEAL BANDSTOP FILTER



MAGNITUDE RESPONSE OF A PRACTICAL LOWPASS FILTER



MAGNITUDE RESPONSE OF A PRACTICAL LOWPASS FILTER

- The transition of the frequency response from pass band to stop band defines the transition band or transition region of the filter
- The pass band edge frequency ω_p defines the edge of the pass band, while the stop band edge frequency ω_s denotes the beginning of the stop band
- $\boldsymbol{\delta_p}$ - Pass band ripple
- $\boldsymbol{\delta_s}$ - Stop band ripple
- $\boldsymbol{\omega_p}$ - Pass band edge frequency
- $\boldsymbol{\omega_s}$ - Stop band edge frequency

CHARACTERISTICS OF FIR FILTERS WITH LINEAR PHASE

- Let h(n) be the causal finite duration sequence defined over the interval $0 \le n \le N-1$ and the samples of h(n) be real
- The Fourier transform of h(n) is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

Which is periodic in frequency with period **2π**

$$\therefore H(e^{j\omega}) = H(e^{j\omega + 2\pi m}); \quad \text{for} \quad m = 0, \pm 1, \pm 2, \ldots$$

CHARACTERISTICS OF FIR FILTERS WITH LINEAR PHASE

- Since $H(e^{j\omega})$ is complex it can be expressed as **Amplitude function, Magnitude function and Phase function**

$$H(e^{j\omega}) = \pm \left|H(e^{j\omega})\right| e^{j\angle H(e^{j\omega})} = A(\omega)\, e^{j\theta(\omega)}$$

$$\text{where,}\quad A(\omega) = \pm \left|H(e^{j\omega})\right| = \text{Amplitude function}$$

$$\theta(\omega) = \angle H(e^{j\omega}) = \text{Phase function}$$

$$\left|H(e^{j\omega})\right| = \text{Magnitude function}$$

- When h(n) is real, the magnitude function is a symmetric function and the phase function is an asymmetric function

$$\therefore\ |H(e^{j\omega})| = |H(-e^{j\omega})|$$

$$|\theta(\omega)| = -|\theta(-\omega)|$$

ASSESSMENT

1. Define FIR Systems.
2. Mention the advantages and disadvantages of FIR Filters.
3. Based on frequency response the filters are classified into four basic types. They are ------------, --------------, ------------------- and ----------------
4. What are the steps involved in designing FIR Filter?
5. In order to examine the linear and nonlinear phase characteristics, two delay functions are -------------- and ---------------
6. The Fourier transform of h(n) is ----------------------

THANK YOU