SNS COLLEGE OF TECHNOLOGY
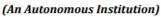
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

Unit-V

Laplace Transforms

Introduction:-

Laplace transformation, named after a great French Mathematician Pierre Simon De Laplace (1749-1827) who used such transformation in the "Theory of Probability".

Uses of Laplace Transformation:-

1. It is used to find the solution of linear differential equations ordinary as well as Partial.

2. It helps in solving the differential equation with boundary values without finding the general solution and then finding the values of the arbitrary constants.

Transformation:

A transformation is an operation which converts a mathematical expression to a different but equivalent form.

Laplace Transformation : Definition

Let $f(t)$ be a function of t defined for $t > 0$ then the laplace transform of $f(t)$, denoted by $L\{f(t)\}$ or $f(s)$ is defined by,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s)$$

Provided the integral exists.

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

Conditions for existence of Laplace transform :

i) $f(t)$ should be Continuous or Piecewise continuous in the given close interval $[a,b]$ where $a > 0$.

ii) $f(t)$ should be of exponential order.

Exponential order:

A function $f(t)$ is said to be of exponential order if

$$\underset{t \to \infty}{lt} \, e^{-st} f(t) = 0.$$

Example:

i) t^2 is of exponential order.

$$\underset{t \to \infty}{lt} \, e^{-st} f(t) = \underset{t \to \infty}{lt} \, e^{-st} t^2$$

$$= \underset{t \to \infty}{lt} \, \frac{t^2}{e^{st}} \qquad \left(\frac{\infty}{\infty} \text{ indeterminant form} \right)$$

$$= \underset{t \to \infty}{lt} \, \frac{2t}{s e^{st}} \qquad \text{Apply L'Hospital rule}$$

$$= \underset{t \to \infty}{lt} \, \frac{2}{s^2 e^{st}}$$

$$= \frac{2}{\infty}$$

$$= 0$$

ii) e^{t^2} is not of exponential order.

$$\underset{t \to \infty}{lt} \, e^{-st} f(t) = \underset{t \to \infty}{lt} \, e^{-st} e^{t^2}$$

$$= \underset{t \to \infty}{lt} \, e^{-st + t^2}$$

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

$$= e^{\infty}$$
$$= \infty$$

$\therefore \quad e^{t^2}$ is not of exponential order.

Transforms of elementary functions:-

1) $L(1) = \dfrac{1}{s}$ where $s > 0$.

Proof:-

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$L(1) = \int_0^{\infty} e^{-st} \cdot 1\,dt$$

$$= \left[\dfrac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= -\dfrac{1}{s}(0-1) = \dfrac{1}{s}$$

$$L(1) = \dfrac{1}{s}$$

2) $L(K) = \dfrac{K}{s}$.

3) $L(t) = \dfrac{1!}{s^2}$

$$L(t) = \int_0^{\infty} e^{-st} \cdot t\,dt$$

$$= \left[\dfrac{t\,e^{-st}}{-s} - \dfrac{e^{-st}}{s^2}\right]_0^{\infty}$$

$$L(t) = \dfrac{1!}{s^2}$$

Bernoulli's formula:-

$$I = uv_1 - u'v_2 + u'''v_3$$

$u = t \qquad \begin{cases} v = e^{-st} \\ v' = e^{\frac{-st}{-s}} \\ v_2 = \dfrac{e^{-st}}{s^2} \end{cases}$

$u' = 1$

$u'' = 0$