



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



$$\begin{aligned} L^{-1} \left[\frac{s^2}{(s+1)(s^2+4)} \right] &= L^{-1} \left[\frac{1/5}{s+1} + \frac{4/5 s - 4/5}{s^2+4} \right] \\ &= \frac{1}{5} L^{-1} \left(\frac{1}{s+1} \right) + \frac{4}{5} L^{-1} \left(\frac{s}{s^2+4} \right) - \frac{4}{5} L^{-1} \left(\frac{1}{s^2+4} \right) \\ &= \frac{1}{5} e^{-t} + \frac{4}{5} \cos 2t - \frac{4}{5} \frac{\sin 2t}{2} \end{aligned}$$

3) Find $L^{-1} \left(\frac{s^2+2s+1}{(s+3)(s-3)(s+1)} \right)$

Sol $\therefore \frac{1}{3} e^{-3t} + \frac{2}{3} e^{3t}$

4) Find $L^{-1} \left(\frac{s}{(s-3)(s^2+4)} \right)$

Sol $\therefore \frac{3}{13} e^{3t} - \frac{3}{13} \cos 2t + \frac{2}{3} \sin t$

Convolution

Definition

If $f(t)$ and $g(t)$ are two functions defined for $t \geq 0$ then the Convolution of $f(t)$ and $g(t)$ is defined as

$$f(t) * g(t) = (f * g)(t) = \int_0^t f(u) g(t-u) du$$

Note $\therefore f(t) * g(t) = g(t) * f(t)$.

Convolution theorem

If $f(t)$ and $g(t)$ are two Laplace transformable functions defined for $t \geq 0$ then $L[f(t) * g(t)]$ is given by



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$$L^{-1}[F(s) \cdot G(s)] = L^{-1}[F(s)] * L^{-1}[G(s)]$$

Problems:-

i) Using convolution theorem, find the inverse transform of

i) $\frac{s^2}{(s^2+a^2)^2}$

ii) $\frac{s}{(s^2+a^2)(s^2+b^2)}$

iii) $\frac{1}{(s+a)(s+b)}$

Sol:- i) $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right] = L^{-1}\left[\frac{s}{s^2+a^2} \cdot \frac{s}{s^2+a^2}\right]$

$$= L^{-1}\left[\frac{s}{s^2+a^2}\right] * L^{-1}\left[\frac{s}{s^2+a^2}\right]$$

$$= \cos at * \cos at$$

$$= \int_0^t \cos au \cos a(t-u) du$$

$$= \int_0^t \left[\frac{\cos(au+at-au) + \cos(au-at+au)}{2} \right] du$$

$$= \frac{1}{2} \int_0^t [\cos at + \cos(2au-at)] du$$

$$= \frac{1}{2} \left[\cos at \cdot u + \frac{\sin(2au-at)}{2a} \right]_0^t$$

$$= \frac{1}{2} \left[\cos at \cdot t + \frac{\sin(2at-at)}{2a} - \frac{\sin(0-at)}{2a} \right]$$

$$= \frac{1}{2} \left[t \cos at + \frac{\sin at}{2a} + \frac{\sin at}{2a} \right]$$

$$= \frac{1}{2} \left[t \cos at + \frac{\sin at}{a} \right]$$

$$= \frac{1}{2a} (at \cos at + \sin at)$$



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$$L^{-1} \left[\frac{S}{(s^2+a^2)(s^2+b^2)} \right]$$

$$L^{-1} \left[\frac{S}{(s^2+a^2)(s^2+b^2)} \right] = L^{-1} \left[\frac{S}{s^2+a^2} \cdot \frac{1}{s^2+b^2} \right]$$

$$= L^{-1} \left[\frac{S}{s^2+a^2} \right] * L^{-1} \left[\frac{1}{s^2+b^2} \right]$$

$$= \cos at * \frac{1}{b} L^{-1} \left[\frac{b}{s^2+b^2} \right]$$

$$= \frac{1}{b} \int_0^t \cos au \sin b(t-u) du$$

$$= \frac{1}{b} \int_0^t \frac{\sin(bt-bu+au) + \sin(bt-bu-au)}{2} du$$

$$= \frac{1}{2b} \int_0^t \sin[bt+(a-b)u] + \sin(bt-(a+b)u) du$$

$$= \frac{1}{2b} \left\{ \frac{-\cos(bt+(a-b)u)}{a-b} - \frac{\cos(bt-(a+b)u)}{-(a+b)} \right\}_0^t$$

$$= \frac{1}{2b} \left\{ \frac{-\cos(bt+at-bt)}{a-b} + \frac{\cos bt}{a-b} + \frac{\cos(bt-at-bt)}{a+b} - \frac{\cos bt}{a+b} \right\}$$

$$= \frac{1}{2b} \left\{ \frac{-\cos at}{a-b} + \frac{\cos bt}{a-b} + \frac{\cos at}{a+b} - \frac{\cos bt}{a+b} \right\}$$

$$= \frac{1}{2b} \left\{ -\frac{\cos at}{a-b} + \frac{\cos bt}{a-b} + \frac{\cos at}{a+b} - \frac{\cos bt}{a+b} \right\}$$

$$= \frac{1}{2b} \left\{ -\frac{1}{a-b} [\cos at - \cos bt] + \frac{1}{a+b} (\cos at - \cos bt) \right\}$$

$$= \frac{1}{2b} (\cos at - \cos bt) \left\{ -\frac{1}{a-b} + \frac{1}{a+b} \right\}$$



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$$= \frac{1}{2b} \{ \cos at - \cos bt \} \left(- \frac{(a+b) + (a-b)}{a^2 - b^2} \right)$$

$$= \frac{1}{2b} (\cos at - \cos bt) \frac{(-2b)}{a^2 - b^2}$$

$$= \frac{\cos bt - \cos at}{a^2 - b^2}$$

$$\text{iii) } L^{-1} \left[\frac{1}{(s+a)(s+b)} \right] = L^{-1} \left(\frac{1}{s+a} \cdot \frac{1}{s+b} \right)$$

$$= L^{-1} \left(\frac{1}{s+a} \right) * L^{-1} \left(\frac{1}{s+b} \right)$$

$$= e^{-at} * e^{-bt}$$

$$= \int_0^t e^{-au} \cdot e^{-b(t-u)} du$$

$$= \int_0^t e^{-au-bt+bu} du$$

$$= \left[\frac{e^{-au-bt+bu}}{-a+b} \right]_0^t$$

$$= \left[\frac{e^{-at-bt+bt}}{-a+b} - \frac{e^{-bt}}{-a+b} \right]$$

$$= \frac{e^{-at} - e^{-bt}}{b-a}$$