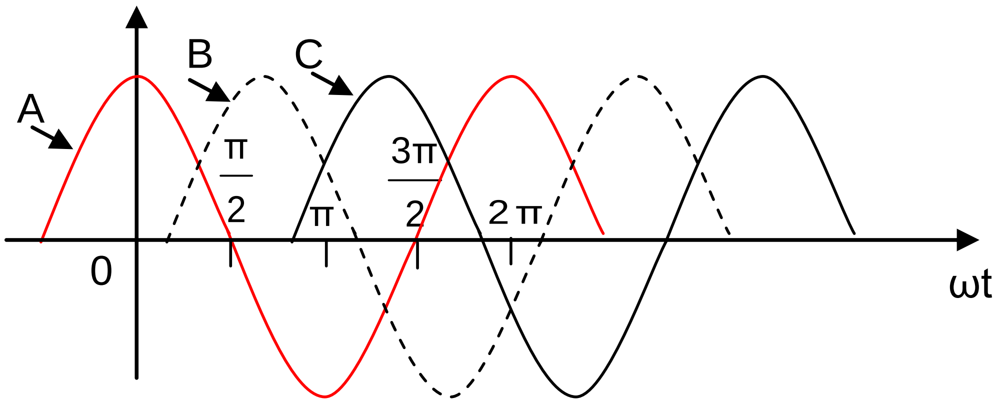
UNIT V– THREE-PHASE CIRCUITS, GRAPH THEORY AND TUNED CIRCUITS

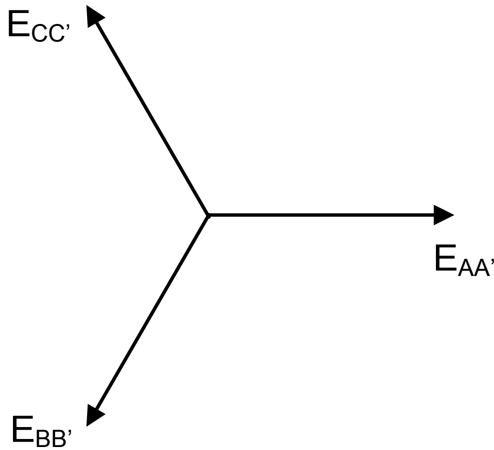
6.2 BALANCED THREE-PHASE VOLTAGES

Three-phase balanced voltages are generated by three-phase generators, also called as alternators. An alternator has three-phase winding with terminals A-A', B-B' and C-C'. The induced voltages in the three windings are equal in magnitude but out of phase by 120^0 as shown in equation 6.1.

$$\left.\begin{aligned} e_{AA'} &= E_m \cos \omega t \\ e_{BB'} &= E_m \cos (\omega t - 120^0) \\ e_{CC'} &= E_m \cos (\omega t - 240^0) \end{aligned}\right\} \quad (6.1)$$



Because of the symmetry, these voltages are known as balanced three-phase voltages. The phasor descriptions of these three voltages are shown in Fig. 6.1 in which voltage $E_{AA'}$ is taken as reference.



$$E_{AA'} = E\angle 0^0$$

$$E_{BB'} = E\angle -120^0$$

$$E_{CC'} = E\angle -240^0$$

$$|E_{AA'}| = |E_{BB'}| = |E_{CC'}| = E = \frac{E_m}{\sqrt{2}}$$

Fig. 6.1 Phasor representation of three-phase balanced voltages.

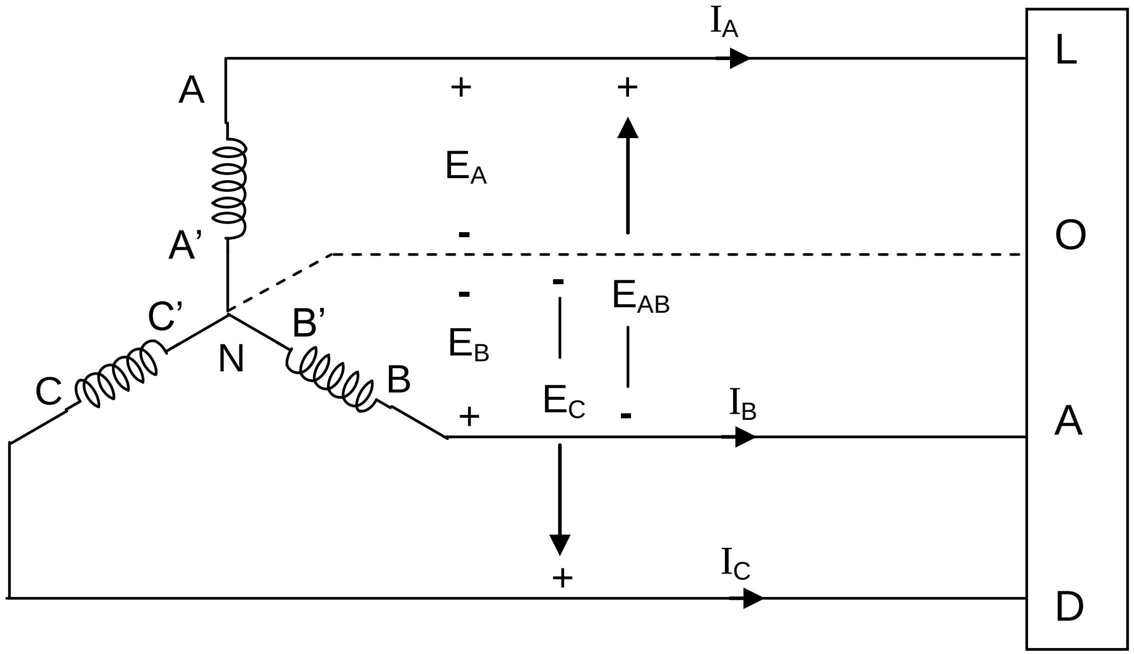
Generally, voltage $E_{AA'}$ is written as E_A and similar notation is true for other phasors also. Thus

$$\left.\begin{aligned} E_A &= E\angle 0^0 \\ E_B &= E\angle -120^0 \\ E_C &= E\angle -240^0 \end{aligned}\right\} \qquad (6.2)$$

It is to be noted that in case of balanced voltages, $E_A + E_B + E_C = 0$ (6.3)

6.3.1 THREE-PHASE GENERATOR WITH STAR-CONNECTED WINDING

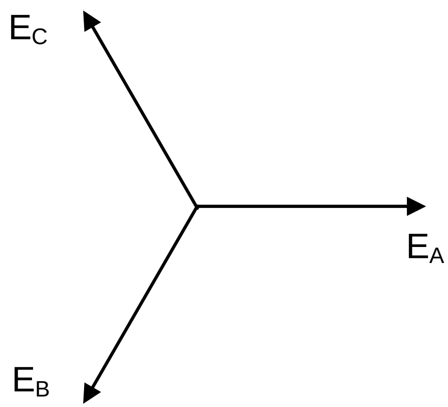
Fig. below shows the windings of three-phase star-connected generator, supplying power to a three-phase load.



Let I_ℓ and I_{ph} be the magnitude of line and phase currents and E_ℓ and E_{ph} be the magnitude of line and phase voltages. In the case of star-connected three-phase system, as seen from Fig. 6.2, line current is equal to the phase current, i.e.

$$I_\ell = I_{ph} \quad (6.4)$$

For the three-phase star-connected generator, taking voltage E_A as reference, the three phase voltages will be as shown in Fig. below.



$E_A = E_{ph} \angle 0^0$

$E_B = E_{ph} \angle -120^0$

$E_C = E_{ph} \angle -240^0$

For the three-phase star-connected generator, the relationship between the line voltage and the phase voltage can be obtained as follows. Referring to Fig. 6.2, we have

$E_A - E_B - E_{AB} = 0$ i.e. $E_{AB} = E_A - E_B$

Thus $E_{AB} = E_{ph} \angle 0^0 - E_{ph} \angle -120^0 = E_{ph} - E_{ph}(-0.5 - j\,0.866) = E_{ph}\,(1.5 + j\,0.866)$

$$= \sqrt{3}\, E_{ph} \angle 30^0 \qquad (6.5)$$

As seen from Eq. (6.5), $|E_{AB}| = \sqrt{3}\, E_{ph}$ i.e. $E_\ell = \sqrt{3}\, E_{ph}$ (6.6)

From Eqs. (6.4) and (6.6), we can state that, in the case of star-connected system,

$$\left.\begin{aligned} &\text{Line current, } I_\ell = \text{Phase current, } I_{ph} \\ &\text{Line voltage, } E_\ell = \sqrt{3} \times \text{Phase voltage} = \sqrt{3}\, E_{ph} \end{aligned}\right\} \qquad (6.7)$$

Real power supplied by the three-phase generator = 3 x power per phase

$$= 3\, E_{ph}\, I_{ph} \cos\theta \qquad (6.8)$$

Using the relations in Eq. (6.7),in the above

Real power supplied by the three-phase generator $= \sqrt{3}\,\sqrt{3}\; E_{ph}\, I_{ph} \cos\theta$

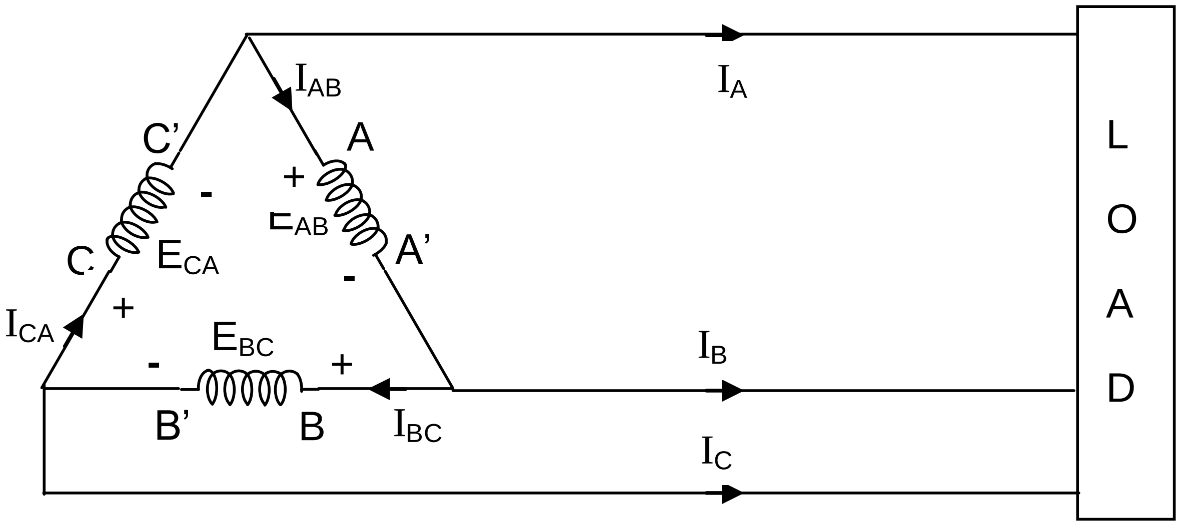
$$= \sqrt{3}\, E_{\ell}\, I_{\ell} \cos\theta \qquad (6.9)$$

Thus, the power supplied by the star-connected three-phase generator can be calculated by using the phase quantities as in Eq. (6.8) or by using the line quantities as in Eq. (6.9).

The generators, transformers and transmission lines in a three-phase system will have their own voltage ratings. The specified voltages associated with them are line voltages unless it is mentioned otherwise.

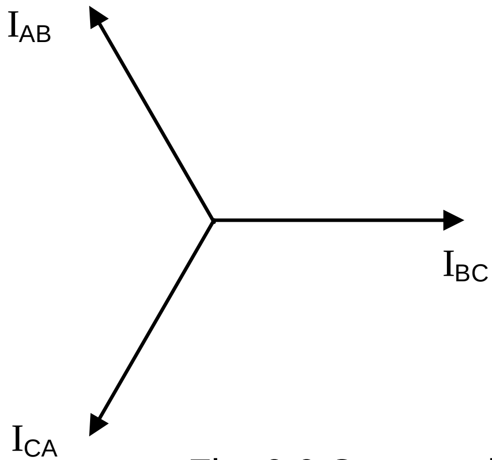
6.3.2 THREE-PHASE GENERATOR WITH DELTA-CONNECTED WINDING

Fig. below shows the windings of three-phase delta-connected generator, supplying power to a three-phase load.



Line voltage, E_ℓ = Phase voltage, E_{ph} (6.10)

Taking the phase current I_{BC} as reference, the phasor form of three phase currents I_{AB}, I_{BC} and I_{CA} are shown in Fig. 6.6.



$I_{BC} = I_{ph} \angle 0^0$

$I_{CA} = I_{ph} \angle -120^0$

$I_{AB} = I_{ph} \angle -240^0$

Fig. 6.6 Current phasors of phase currents in a delta system.

Referring to Fig. 6.5, KCL at junction A gives

$$I_A = I_{CA} - I_{AB} = I_{ph}(-0.5 - j\,0.866) - I_{ph}(-0.5 + j\,0.866) = -j\sqrt{3}\,I_{ph} \qquad (6.11)$$

i.e. $|I_A| = \sqrt{3}\,I_{ph}$ i.e. Line current, $I_\ell = \sqrt{3}\,I_{ph}$ (6.12)

From Eqs. (6.10) and (6.12), we can state that, in the case of delta-connected system.

Line voltage, E_ℓ = Phase current, E_{ph}

Line current, $I_\ell = \sqrt{3}$ x Phase current $= \sqrt{3}\,I_{ph}$

(6.13)

Real power supplied by the three-phase generator = 3 x power per phase

$$= 3\, E_{ph}\, I_{ph} \cos\theta \qquad (6.14)$$

Using the relations in Eq. (6.13), in the above

Real power supplied by the three-phase generator $= \sqrt{3}\, E_{ph}\, \sqrt{3}\, I_{ph} \cos\theta$

$$= \sqrt{3}\, E_{\ell}\, I_{\ell} \cos\theta \qquad (6.15)$$

It is to be noted that Eqs. (6.14) and (6.15) are the same as Eqs. (6.8) and (6.9) Thus, the real power supplied by the three-phase generator, whether it is connected in star or in delta, can be calculated by using the phase quantities as in Eq. (6.14) or by using the line quantities as in Eq. (6.15).

In a three-phase system, whenever the value of supply voltage is given, it refers to line voltage unless it is specifically mentioned as phase voltage.

Example 6.1 Each phase of a three-phase alternator, generates a voltage of 3810.5 V and can carry a maximum current of 30 A. Find the line current, line voltage and total kVA capacity, if the alternator is connected in (a) star (b) delta.

Solution: Given data: $E_{ph} = 3810.5$ V; $I_{ph} = 30$ A

Star

Line current $I_\ell = I_{ph} = 30$ A

Line voltage $E_\ell = \sqrt{3}\ E_{ph} = 6600$ V

Total kVA $= \sqrt{3}\, E_\ell\, I_\ell$

$= \sqrt{3} \times 6600 \times 30 \times 10^{-3}$

$= 342.95$

Delta

Line current $I_\ell = \sqrt{3}\ I_{ph} = 51.9615$ A

Line voltage $E_\ell = 3810.5$ V

Total kVA $= \sqrt{3}\, E_\ell\, I_\ell$

$= \sqrt{3} \times 3810.5 \times 51.9615 \times 10^{-3}$

$= 342.95$

6.4 BALANCED THREE-PHASE LOAD

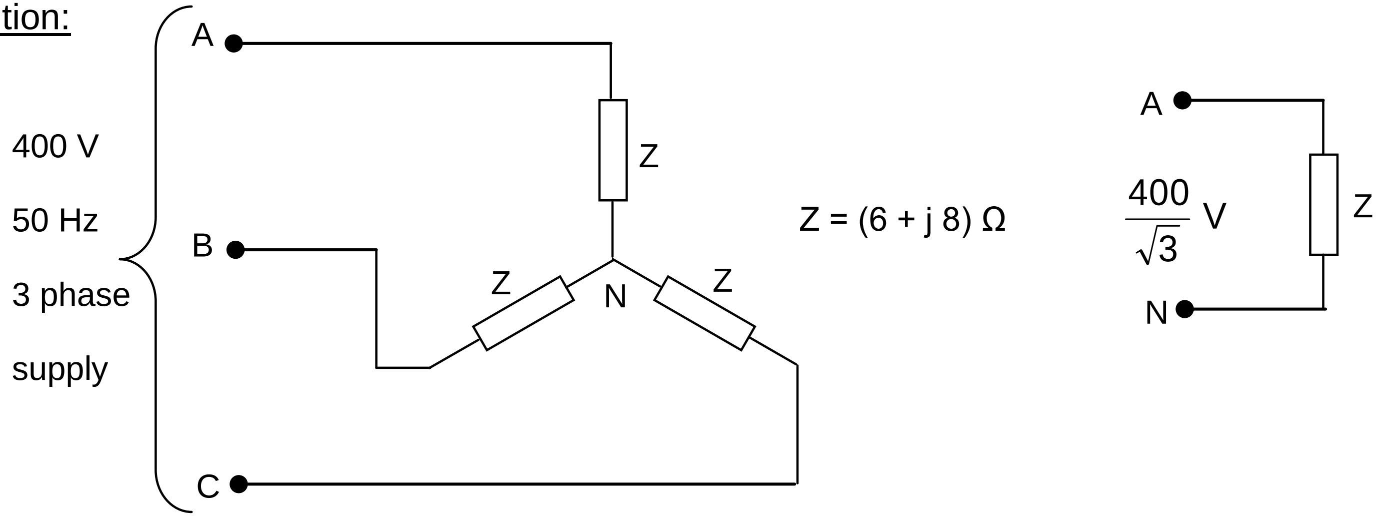
A three-phase load has three separate load impedances which may be connected in star or delta. When these three impedances are identical, then we say that the load is balanced; otherwise unbalanced.

Delta-connected load is more common than star-connected load. This is due to the ease with which load may be added or removed from each phase of a delta-connected load. Changing the load is difficult with star-connected load because of non-availability of the neutral.

When a three-phase balanced load is connected to a balanced three-phase supply, the analysis is simple. Because of perfect symmetry, there will not be current in the neutral wire. In this case, the three-phase circuit can be treated on single-phase basis. We first draw the single-phase equivalent and then work out the problem on per phase basis. Then, if necessary, three-phase quantities can be calculated.

Example 6.2 A balanced three-phase load connected in star consists of (6 + j 8) Ω impedance in each phase. It is connected to a three-phase supply of 400 V, 50 Hz. Find (a) magnitude of phase current and line current (b) per phase power and (c) total power.

Solution:



(a) (b)

Given three-phase circuit and its single-phase equivalent are shown above. Referring to Fig. (b), Voltage $E_{ph} = 400 / \sqrt{3} = 230.94$ V; $Z = (6 + j\,8)\ \Omega = 10\angle 53.13^0\ \Omega$

(a) Phase current, $I_{ph} = 230.94 / 10 = 23.094$ A

Line current, $I_\ell = I_{ph} = 23.094$ A

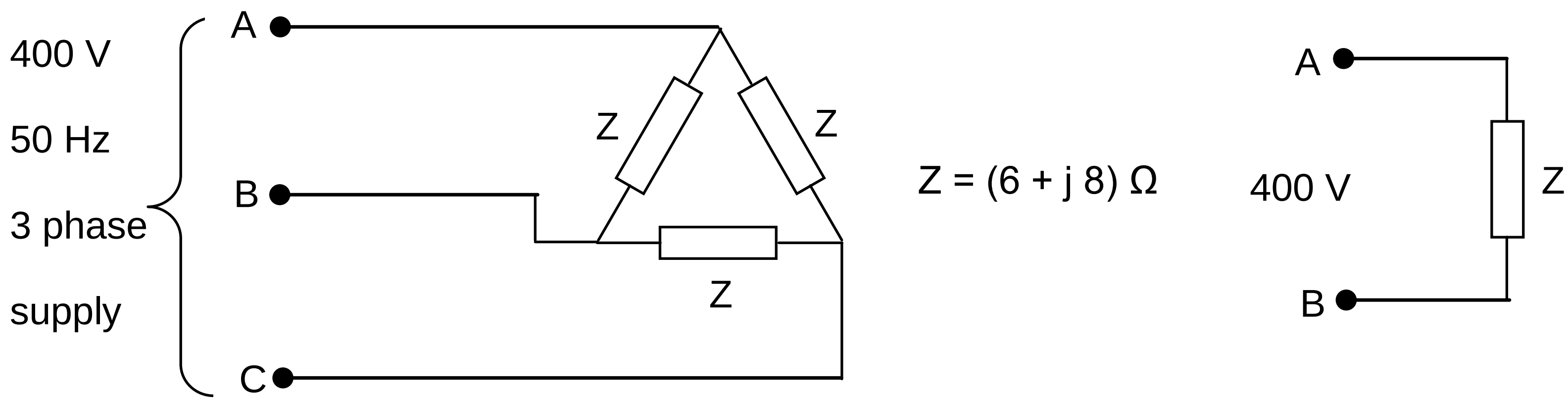
(b) Per phase power, $P = E_{ph}\, I_{ph} \cos\theta = 230.94 \times 23.094 \times \cos 53.13^0 = 3200$ W

(c) Total power, $P_T = \sqrt{3}\ E_\ell\, I_\ell \cos\theta = \sqrt{3} \times 400 \times 23.094 \times \cos 53.13^0 = 9600$ W

Alternatively, total power, $P_T = 3\,P = 9600$ W

Example 6.3 Repeat the previous example with the load impedance connected in delta.

Solution: Balanced three-phase circuit and its single-phase equivalent are shown.



(a) (b)

Referring to Fig. 6.9 (b), Voltage E_{ph} = 400 V $Z = (6 + j\,8)\ \Omega = 10\angle 53.13^0\ \Omega$

(a) Phase current, I_{ph} = 400 / 10 = 40 A

Line current, $I_\ell = \sqrt{3}\ I_{ph}$ = 69.282 A

(b) Per phase power, $P = E_{ph}\, I_{ph} \cos\theta = 400 \times 40 \times \cos 53.13^0 = 9600$ W

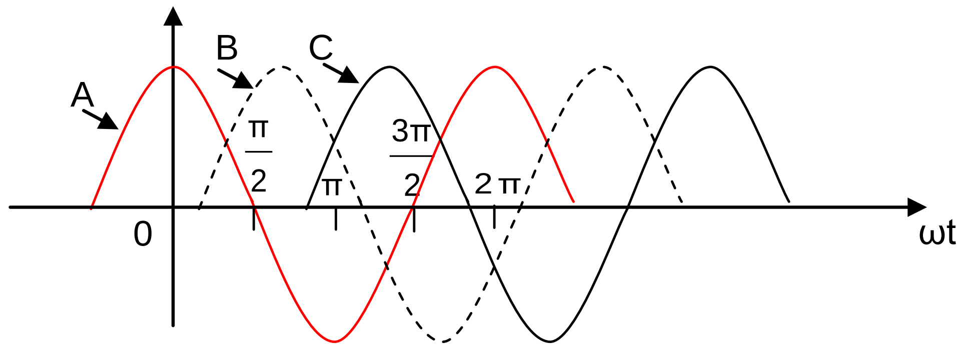
(c) Total power, $P_T = \sqrt{3}\ E_\ell\, I_\ell \cos\theta = \sqrt{3} \times 400 \times 69.282 \times \cos 53.13^0 = 28800$ W

Alternatively, total power, P_T = 3 P = 28800 W

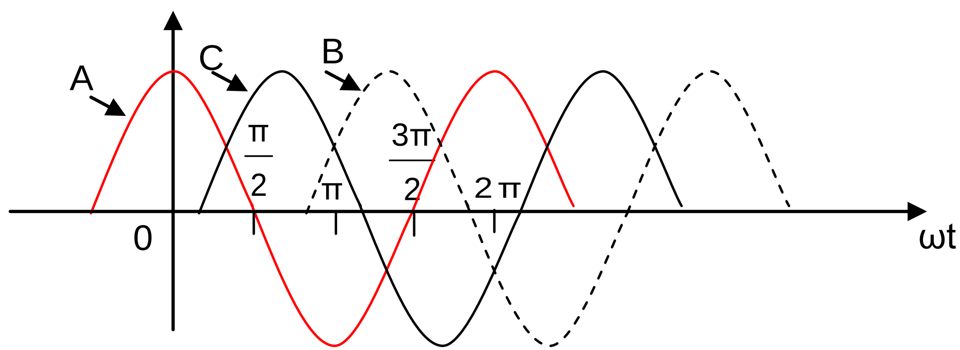
6.5 PHASE SEQUENCE

Phase sequence is the order in which different phase voltages reach maximum values. In a three-phase system, two phase sequences, namely ABC and ACB, are possible. To indicate the phase sequence as ABC, any three consecutive letters in ABCAB may be used. Similarly, to specify the phase sequence as ACB, any three consecutive letters in ACBAC may be used.

Phase sequence ABC



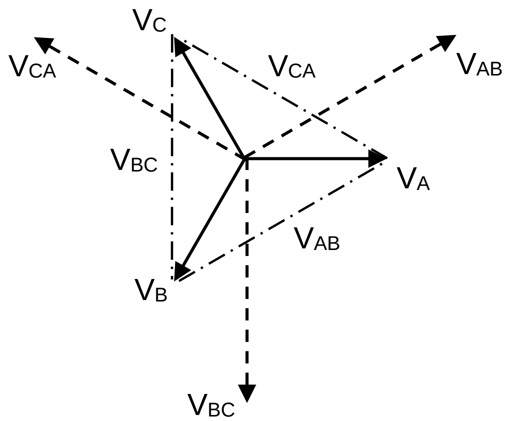
Phase sequence ACB



Phasor diagrams showing the phase and line voltage relationships for the two phase sequences ABC and ACB are shown in Figs. 6.10 and 6.11 respectively. Line to neutral voltages can be represented either by V_{AN}, V_{BN} and V_{CN} or simply by V_A, V_B and V_C.

Phase sequence is ABC:

V_A is taken as reference



Line to line Voltage V_{AB} leads phase voltage V_A by 30^0

V_{BC} is taken as reference

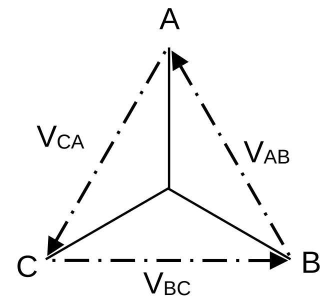
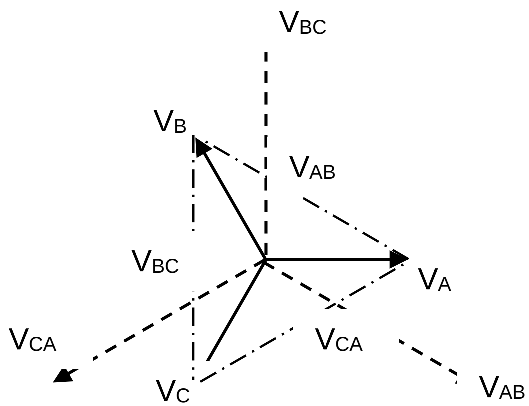


Fig. 6.10 Phasor diagrams - Phase sequence ABC.

Phase sequence is ACB:

V_A is taken as reference



Line to line Voltage V_{AB} lags phase voltage V_A by 30^0

V_{BC} is taken as reference

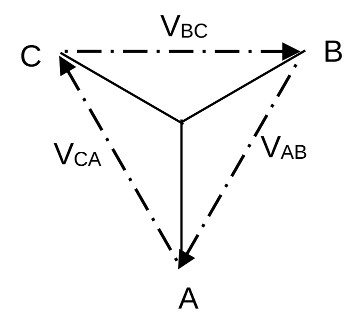


Fig. 6.11 Phasor diagrams - Phase sequence ACB.

If the phase sequence is not specified, it is understood that the phase sequence is ABC. Phase sequence ABC is also referred as phase sequence RYB.

It is to be noted that if the phase sequence is ABC, then, V_B lags V_A by 120^0 and V_C lags V_B by 120^0. Further, V_{BC} lags by V_{AB} by 120^0 and V_{CA} lags V_{BC} by 120^0.

On the other hand if the phase sequence is ACB, then, V_C lags V_A by 120^0 and V_B lags V_C by 120^0. Also the line voltage V_{CA} lags V_{AB} by 120^0 and V_{BC} lags V_{CA} by 120^0.

Phase sequence can be easily remembered by considering the phasors as rotating vectors rotating in anti-clockwise direction. If the phase sequence is ABC, at any stationary point phasors pass through in the order A-B-C or AB-BC-CA. On the other hand if the phase sequence is ACB, at any stationary point, phasors pass through in the order A-C-B or AB-CA-BC

Example 6.4

Determine the phase sequence of the set of voltages given by

v_{AN} = 400 cos (ωt - 10^0) V; v_{BN} = 400 cos (ωt - 250^0) V and v_{CN} = 400 cos (ωt - 130^0) V.

Solution:

Given voltage phasors are:

$V_{AN} = 282.8\angle -10^0$; $V_{BN} = 282.8\angle -250^0$ and $V_{CN} = 282.8\angle -130^0$

Phasors are shown in Fig. 6.12.

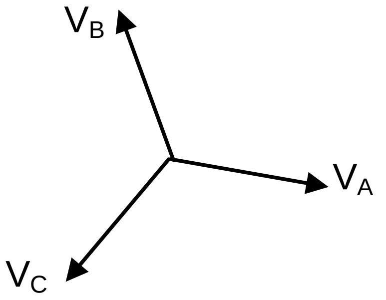


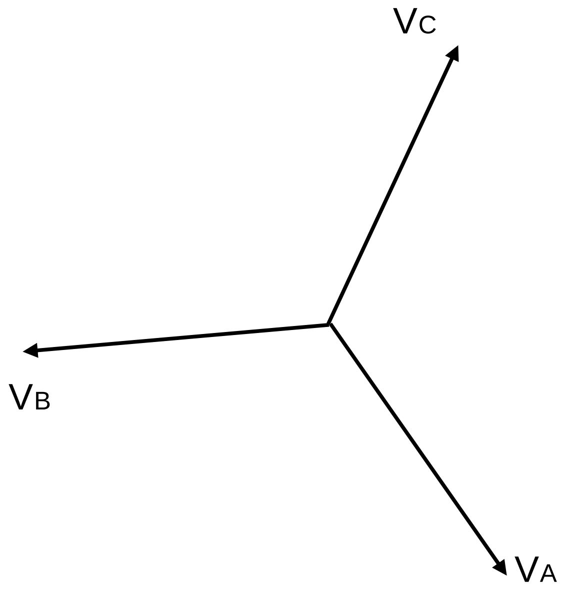
Fig. 6.12 Phasor diagrams - Example 6.4.

From the phasor diagram it is noted V_C is lagging V_A. Hence phase sequence is ACB

Example 6.5

In a three-phase balanced supply, voltage $V_C = 110\angle 65^0$ V. Taking the phase sequence as ABC, find the phase and line voltages.

Solution:



$V_A = V_C \times 1\angle -120^0 = 110\angle -55^0$ V

$V_B = V_C \times 1\angle 120^0 = 110\angle 185^0$ V

$V_C = 110\angle 65^0$ V

$V_{AB} = V_A - V_B = 190.5\angle -25^0$ V

$V_{BC} = V_B - V_C = 190.5\angle -145^0$ V

$V_{CA} = V_C - V_A = 190.5\angle 95^0$ V

Case 1: Unbalanced delta-connected load supplied by 3-wire system.

Case 2: Unbalanced star-connected load supplied by 4-wire system.

Case 3: Unbalanced star-connected load supplied by 3-wire system.

Case 1: Unbalanced delta-connected load supplied by 3-wire system.

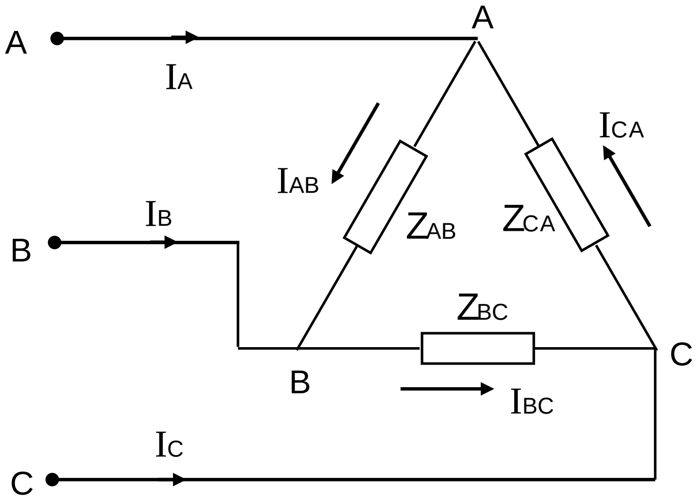
Example 6.6 A three-phase 3-wire 240 V ABC system has a delta-connected load with impedances $Z_{AB} = 10\angle 0^0\ \Omega$; $Z_{BC} = 10\angle 30^0\ \Omega$ and $Z_{CA} = 15\angle -30^0\ \Omega$. as shown. Taking V_{BC} as reference, determine the line voltages, phase currents, line currents and draw the phasor diagram. Also calculate the real power consumed by the load.

Solution: Given line and phase voltages are:

$$V_{BC} = 240\angle 0^0\ V$$

$$V_{CA} = 240\angle -120^0\ V$$

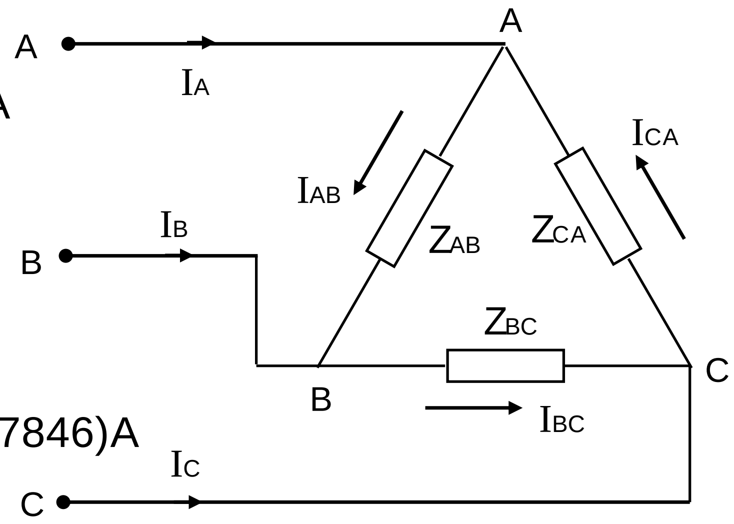
$$V_{AB} = 240\angle -240^0\ V$$



$V_{BC} = 240\angle 0^0$ V; $V_{CA} = 240\angle -120^0$ V; $V_{AB} = 240\angle -240^0$ V;

$Z_{AB} = 10\angle 0^0\ \Omega$; $Z_{BC} = 10\angle 30^0\ \Omega$ and $Z_{CA} = 15\angle -30^0\ \Omega$

Phase currents are:



$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{240\angle 0^0}{10\angle 30^0} = 24\angle -30^0 \text{ A} = (20.7846 - j12)\text{A}$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{240\angle -120^0}{15\angle -30^0} = 16\angle -90^0 \text{ A} = (0 - j16)\text{A}$$

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{240\angle -240^0}{10\angle 0^0} = 24\angle -240^0 \text{ A} = (\text{-}12 + j20.7846)\text{A}$$

Line currents are obtained by applying KCL at junction A, B and C. Thus

$I_A = I_{AB} - I_{CA} = (-12 + j\,20.7846) - (-j\,16) = (-12 + j\,36.7846) = 38.6925\angle 108.07^0$ A

$I_B = I_{BC} - I_{AB} = (32.7846 - j\,32.7846) = 46.3644\angle -45^0$ A

$I_C = I_{CA} - I_{BC} = (-20.7846 - j\,4) = 21.166\angle -169.11^0$ A

Phase voltages, phase currents and the line currents are shown in Fig. 6.14

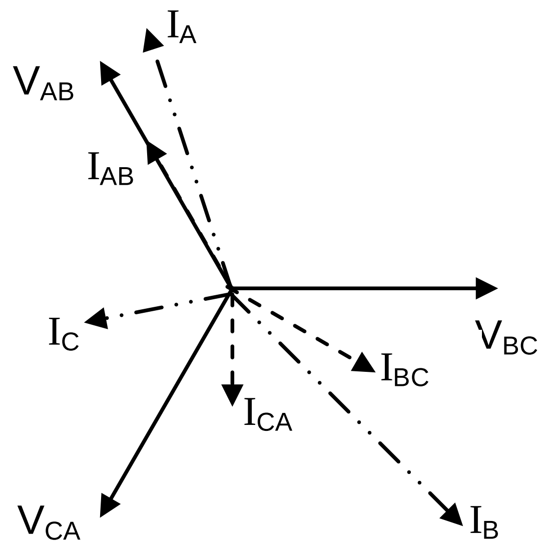


Fig. 6.14 Phasor diagram - Example 6.6.

Real power consumed by the load can be calculated from the real power taken by the load in each phase. Noting that

R_{AB} = 10 Ω; R_{BC} = 10 cos 30^0 = 8.6603 Ω and R_{CA} = 15 cos 30^0 = 12.9904 Ω

Real power consumed by the load = (24^2 x 10) + (24^2 x 8.6603) + (16^2 x 12.9904)

= 14073.8 W = 14.0738 kW

Case 2: Unbalanced star-connected load supplied by 4-wire system.

When an unbalanced star-connected load whose neutral point is designated by O, is connected to a 4-wire balanced load with the neutral wire designated as N, there will be current in the neutral wire. Impedance of the neutral wire will be of negligible value as compared to the load. Hence, N and O will be at the same potential.

Example 6.7

A three-phase 4-wire, 208 V system has a star connected load with impedances $Z_A = 6\angle 0^0\ \Omega$; $Z_B = 6\angle 30^0\ \Omega$ and $Z_C = 5\angle 45^0\ \Omega$. Taking the voltage V_{AN} as reference, determine line and neutral currents. Draw the phasor diagram. Also calculate the real power consumed by the load.

Solution: $Z_A = 6\angle 0^0\ \Omega$; $Z_B = 6\angle 30^0\ \Omega$ and $Z_C = 5\angle 45^0\ \Omega$

Taking the phase sequence as ABC,

$$V_{AN} = \frac{208}{\sqrt{3}}\angle 0^0\ \text{V} = 120.0889\angle 0^0\ \text{V}$$

$$V_{BN} = 120.0889\angle -120^0\ \text{V}$$

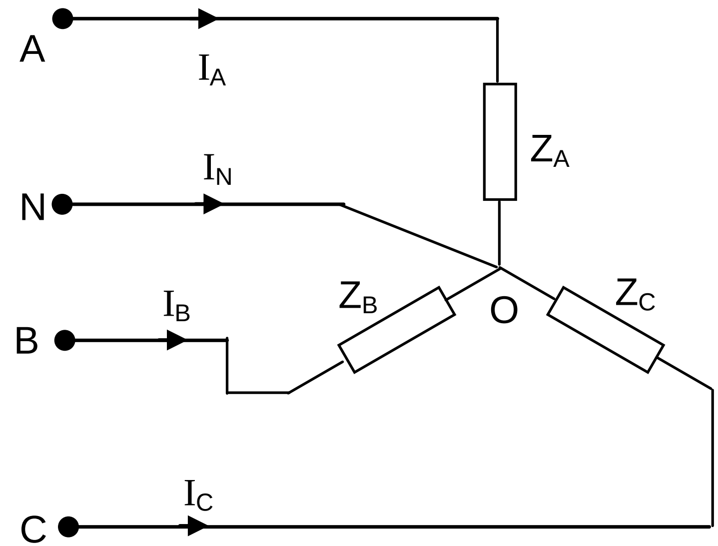
$$V_{CN} = 120.0889\angle -240^0\ \text{V}$$

Currents are calculated as follows:

$$I_A = \frac{V_{AN}}{Z_A} = \frac{120.0889}{6\angle 0^0} = 20.0148\angle 0^0\ \text{A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{120.0889\angle -120^0}{6\angle 30^0} = 20.0148\angle -150^0\ \text{A}$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{120.0889\angle -240^0}{5\angle 45^0} = 24.0178\angle 75^0\ \text{A}$$



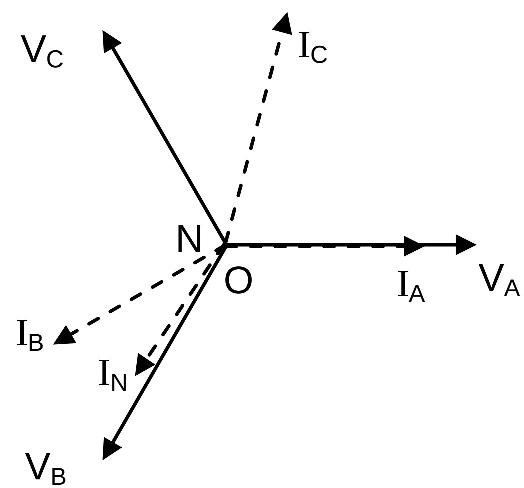
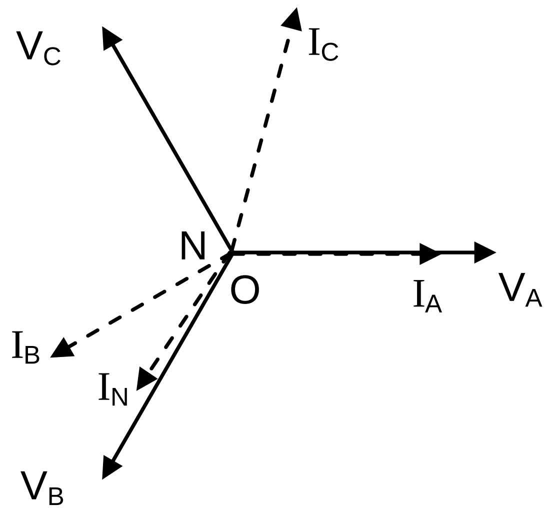


Fig. 6.16 Phasor diagram - Example 6.7.



Use of KCL at O gives

$I_N = -(I_A + I_B + I_C) = (-8.8978 - j\,13.192)\text{ A} = 15.9123\angle -124^0\text{ A}$

In this case, N and O are electrically same point. Resistances in different phases are:

$R_A = 6\ \Omega$; $R_B = 6\cos 30^0 = 5.1962\ \Omega$ and $R_C = 3.5355\ \Omega$.

Real power consumed by the load

$= (20.0148^2 \times 6) + (20.0148^2 \times 5.1962) + (24.0178^2 \times 3.5355)$

$= 6524.58\text{ W} = 6.5246\text{ kW}$

Case 3: Unbalanced star-connected load supplied by 3-wire system.

When an unbalanced star-connected load is connected to a 3-wire system, the connection diagram will be as shown in Fig. below.

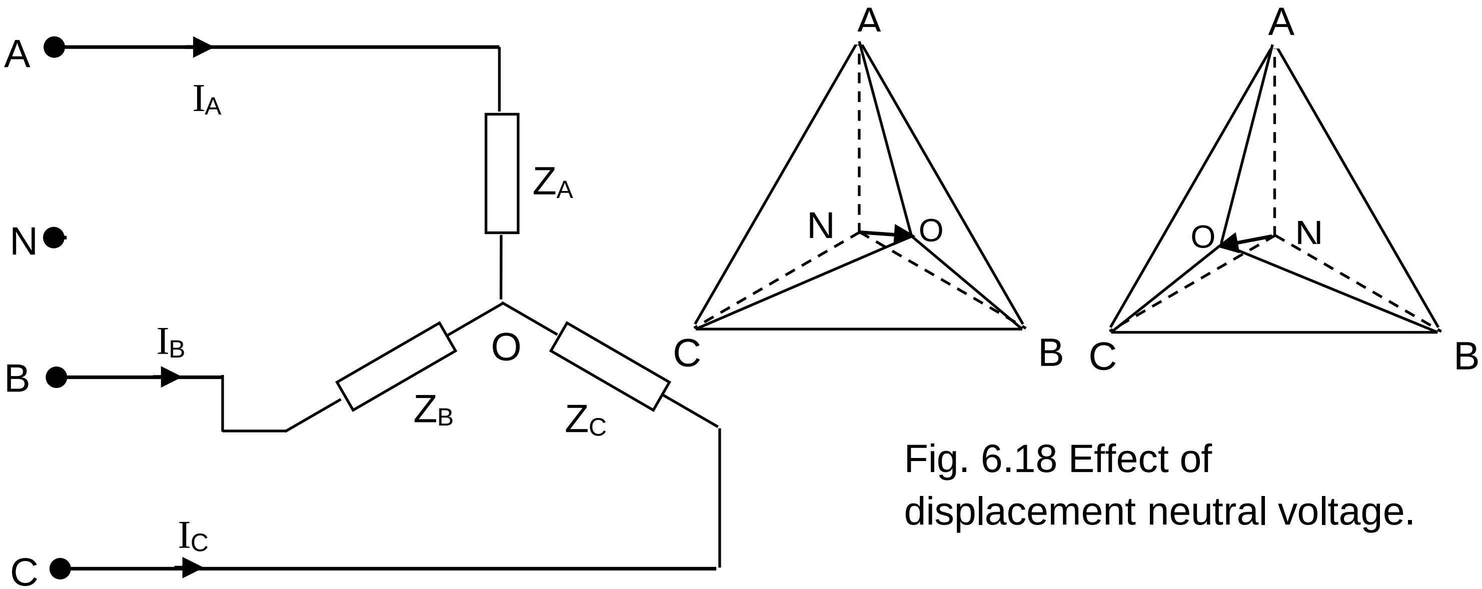


Fig. 6.18 Effect of displacement neutral voltage.

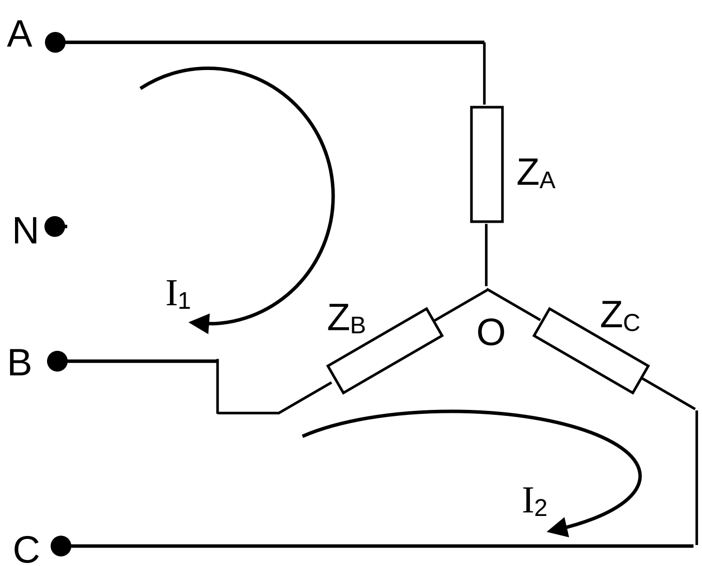
In this case, the system neutral N and the common point of the load O will not be at the same potential. The voltages across the three impedances V_{AO}, V_{BO} and V_{CO} will vary considerably from the line to neutral voltages, depending upon the voltage of O with respect to N, as seen from Fig. 6. 18. V_{ON}, the voltage of load neutral O with respect to system neutral N, is known as displacement neutral voltage.

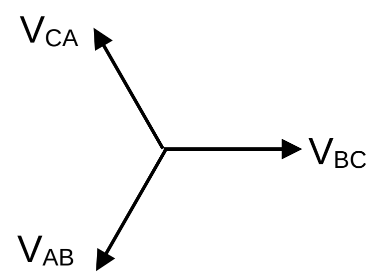
Example 6.8 A three-phase, 3-wire, 208 V, ACB system has star connected load with impedances $Z_A = 6\angle 0^0\ \Omega$; $Z_B = 6\angle 30^0\ \Omega$ and $Z_C = 5\angle 45^0\ \Omega$. Determine the line currents and the voltage across load impedances taking V_{BC} as reference.

Construct the voltage triangle and determine the displacement neutral voltage.

Solution: $V_{BC} = 208\angle 0^0$ V; $V_{CA} = 208\angle 120^0$ V; $V_{AB} = 208\angle -120^0$ V

Selecting two mesh currents I_1 and I_2 as shown, mesh current equations are:

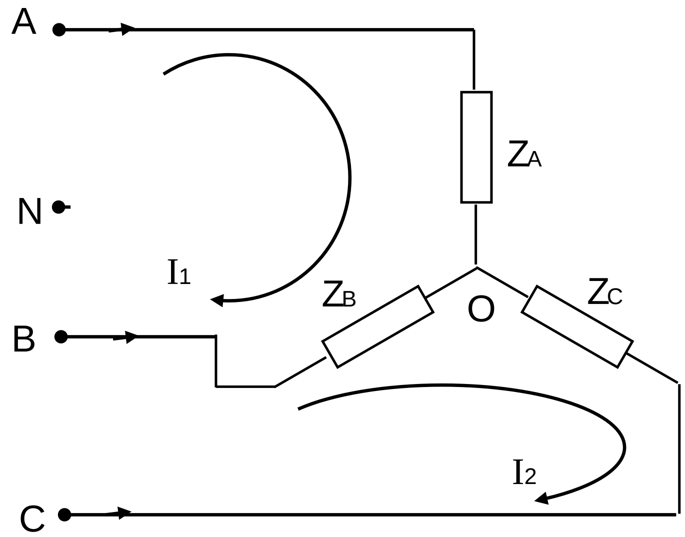




$Z_A = 6\ \Omega$

$Z_B = (5.1962 + j\ 3)\ \Omega$

$Z_C = (3.5355 + j\ 3.5355)\ \Omega$



$Z_A = 6\ \Omega$

$Z_B = (5.1962 + j\ 3)\ \Omega$

$Z_C = (3.5355 + j\ 3.5355)\ \Omega$

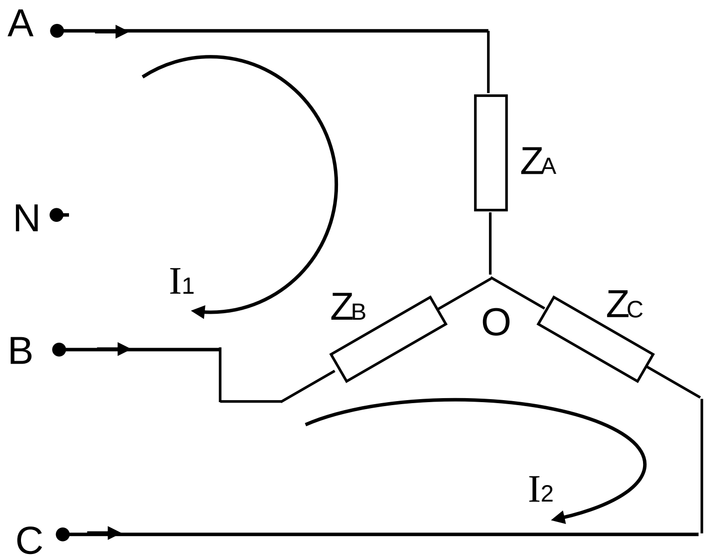
$$\begin{bmatrix} Z_A + Z_B & -Z_B \\ -Z_B & Z_B + Z_C \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{AB} \\ V_{BC} \end{bmatrix}$$

$$\begin{bmatrix} 11.1962 + j\,3 & -5.1962 - j\,3 \\ -5.1962 - j\,3 & 8.7317 + j\,6.5355 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 208\angle -120^0 \\ 208\angle 0^0 \end{bmatrix}$$

Determinant of the coefficient matrix = $90.9323 \angle 48.58^0$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{90.9323\angle 48.58^0} \begin{bmatrix} 8.7317 + j6.5355 & 5.1962 + j3 \\ 5.1962 + j3 & 11.1962 + j3 \end{bmatrix} \begin{bmatrix} 208\angle -120^0 \\ 208\angle 0^0 \end{bmatrix}$$

$$= \begin{bmatrix} -3.6097 - j22.9799 \\ 11.7974 - j23.7446 \end{bmatrix} = \begin{bmatrix} 23.2617\angle -98.93^0 \\ 26.5139\angle -63.58^0 \end{bmatrix}$$



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -3.6097 - j22.9799 \\ 11.7974 - j23.7446 \end{bmatrix}$$

From the mesh currents, line currents can be calculated:

$I_A = I_1 = (-3.6109 - j\,22.9797)\ A = 23.2617 \angle -98.93^0\ A$

$I_B = I_2 - I_1 = (15.4071 - j\,0.7647)\ A = 15.426 \angle -2.84^0\ A$

$I_C = -I_2 = (-11.7974 + j\,23.7446)\ A = 26.5139 \angle 116.42^0\ A$

Voltages across the loads can be calculated as:

$V_{AO} = Z_A I_A = (-21.6654 - j\,137.8782)\ A = 139.5702 \angle -98.93^0\ V$

$V_{BO} = Z_B I_B = (82.3503 + j\,42.2497)\ A = 92.556 \angle 27.16^0\ V$

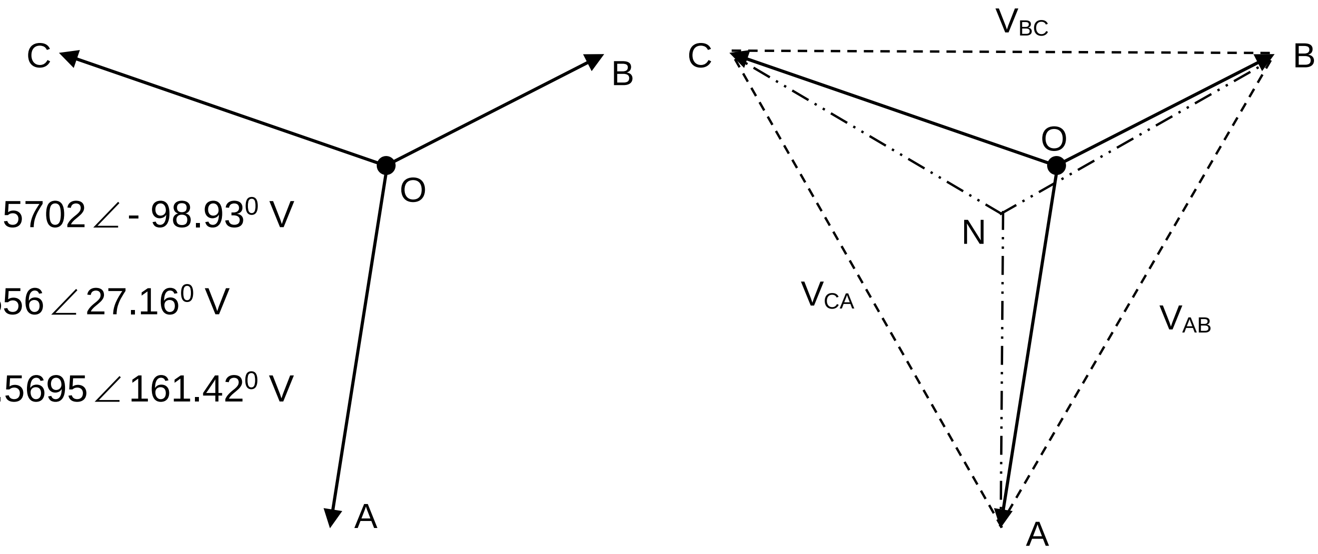
$V_{CO} = Z_C I_C = (-125.6599 + j\,42.2404)\ A = 132.5695 \angle 161.42^0\ V$

These voltage phasors are shown in Fig. 6.20 (a).

$V_{AO} = 139.5702 \angle -98.93^0$ V

$V_{BO} = 92.556 \angle 27.16^0$ V

$V_{CO} = 132.5695 \angle 161.42^0$ V



(a). (b)

Fig. 6.20 Displacement neutral voltage - Illustration.

Also, V_{AB}, V_{BC}, V_{CA} and hence the system neutral N are located as shown in Fig. 6.20 (b). Considering the voltage triangle ANO

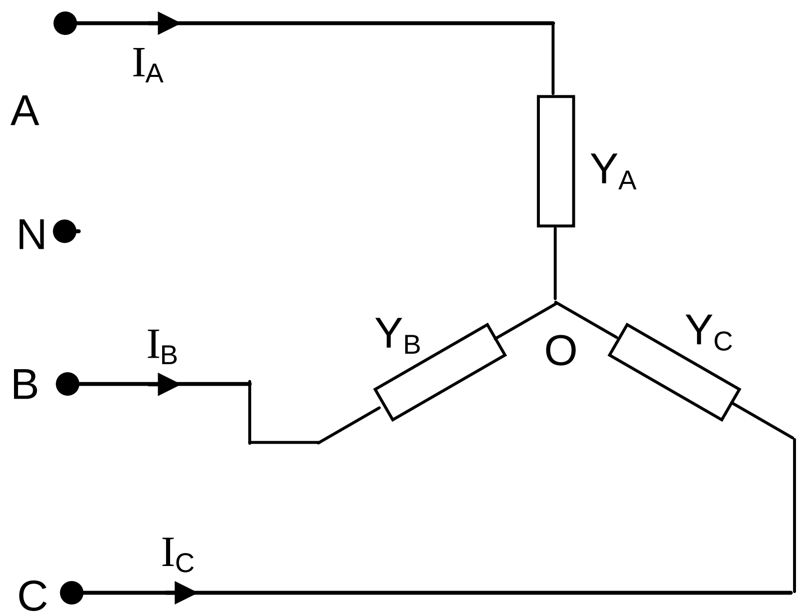
$V_{ON} + V_{AO} + V_{NA} = 0$ Thus displacement neutral voltage V_{ON} is given by

$$V_{ON} = -V_{AO} - V_{NA} = -(-21.6654 - j\,137.8782) - j\frac{208}{\sqrt{3}}$$

$$= (21.6654 + j\,17.7893)\text{ V} = 28.033 \angle 39.39^0 \text{ V}$$

6.6.1 EXPRESSION FOR DISPLACEMENT NEUTRAL VOLTAGE

Consider a star-connected load connected to a balanced three-wire supply as shown.



Applying KCL at point O we get $I_A + I_B + I_C = 0$ i.e. (6.16)

$$Y_A V_{AO} + Y_B V_{BO} + Y_C V_{CO} = 0 \quad (6.17)$$

$$Y_A V_{AO} + Y_B V_{BO} + Y_C V_{CO} = 0 \tag{6.17}$$

Recalling that $V_{ab} = V_{ac} - V_{bc}$, voltages V_{AO}, V_{BO} and V_{CO} can be written as

$$V_{AO} = V_{AN} - V_{ON}; \qquad V_{BO} = V_{BN} - V_{ON}; \qquad V_{CO} = V_{CN} - V_{ON} \tag{6.18}$$

Substitution of the above in Eq. (6.17) results in

$$Y_A (V_{AN} - V_{ON}) + Y_B (V_{BN} - V_{ON}) + Y_C (V_{CN} - V_{ON}) = 0 \text{ i.e.}$$

$$Y_A V_{AN} + Y_B V_{BN} + Y_C V_{CN} = (Y_A + Y_B + Y_C) V_{ON}.$$

Thus Displacement neutral voltage $V_{ON} = \dfrac{Y_A V_{AN} + Y_B V_{BN} + Y_C V_{CN}}{Y_A + Y_B + Y_C}$ (6.19)

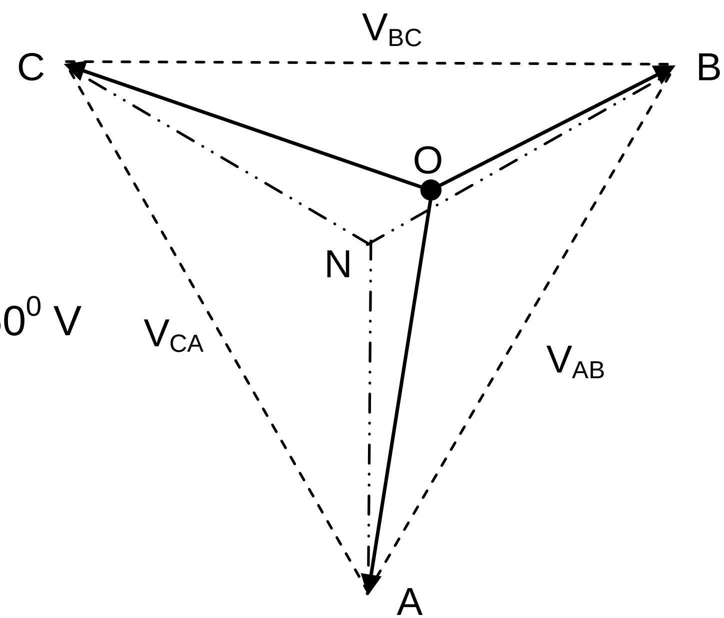
V_{AN}, V_{BN} and V_{CN} are known from the supply system. Once we calculate the voltage V_{ON} from Eq. (6.19), load voltages V_{AO}, V_{BO} and V_{CO} can be calculated from Eq. (6.18). Then, using the load admittances, the currents I_A, I_B and I_C can be calculated.

Example 6.9

For the system described in the previous example, first determine the displacement neutral voltage and hence compute the line currents.

Solution:

Referring to the phasors in Fig. 6.20 (b)



$$V_{AN} = \frac{208}{\sqrt{3}} \angle -90^0 = 120.0889 \angle -90^0 \text{ V}$$

$V_{BN} = 120.0889 \angle 30^0$ V and $V_{CN} = 120.0889 \angle 150^0$ V

Load admittances are:

$Y_A = 0.1667 \angle 0^0 = (0.1667 + j\,0)$ ℧

$Y_B = 0.1667 \angle -30^0 = (0.1443 - j\,0.0833)$ ℧

$Y_C = 0.2 \angle -45^0 = (0.1414 - j\,0.1414)$ ℧

Thus, $Y_A + Y_B + Y_C = (0.4524 - j\,0.2447)$ ℧

Also,

$$Y_A\ V_{AN} = 20.0188 \angle -90^0 = -j\ 20.0188$$

$$Y_B\ V_{BN} = 20.0188 \angle 0^0 = 20.0188$$

$$Y_C\ V_{CN} = 24.0178 \angle 105^0 = -6.2163 + j\ 23.1994$$

Displacement neutral voltage

$$V_{ON} = \frac{Y_A\ V_{AN} + Y_B\ V_{BN} + Y_C\ V_{CN}}{Y_A + Y_B + Y_C} = \frac{13.8025 + j\ 3.1806}{0.4524 - j0.2247}$$

$$= 28.0424 \angle 39.39^0\ \text{V} = (21.6724 + j\ 17.7956)\ \text{V}$$

Now, the load voltages are calculated as

$V_{AO} = V_{AN} - V_{ON} = - j120.0889 - (21.6724 + j\,17.7956)$

$= (- 21.6724 - j\,137.7956)\ V = 139.5773 \angle - 98.93^0\ V$

$V_{BO} = V_{BN} - V_{ON} = (104.0 + j60.0445 - (21.6724 + j\,17.7956)$

$= (82.3276 + j\,42.2489)\ V = 92.5354 \angle 27.17^0\ V$

$V_{CO} = V_{CN} - V_{ON} = (- 104.0 + j60.0445 - (21.6724 + j\,17.7956)$

$= (- 125.6724 + j\,42.2489)\ V = 132.5840 \angle 161.42^0\ V$

Multiplying these load voltages and the corresponding load admittances, line currents are computed as

$I_A = 139.5773 \angle - 98.93^0 \times 0.1667 \angle 0^0 = 23.2675 \angle - 98.93^0\ A$

$I_B = 92.5354 \angle 27.17^0 \times 0.1667 \angle - 30^0 = 15.4257 \angle - 2.83^0\ A$

$I_C = 132.5840 \angle 161.42^0 \times 0.2 \angle - 45^0 = 26.5168 \angle 116.42^0\ A$

These results agree with the results obtained in the previous example.

6.8 MEASUREMENT OF POWER IN THREE-PHASE SYSTEM

For measuring the power consumed by a three phase load, three wattmeters can be connected as shown in Fig. 6.27. Here, each wattmeter measures the real power of the respective phase.

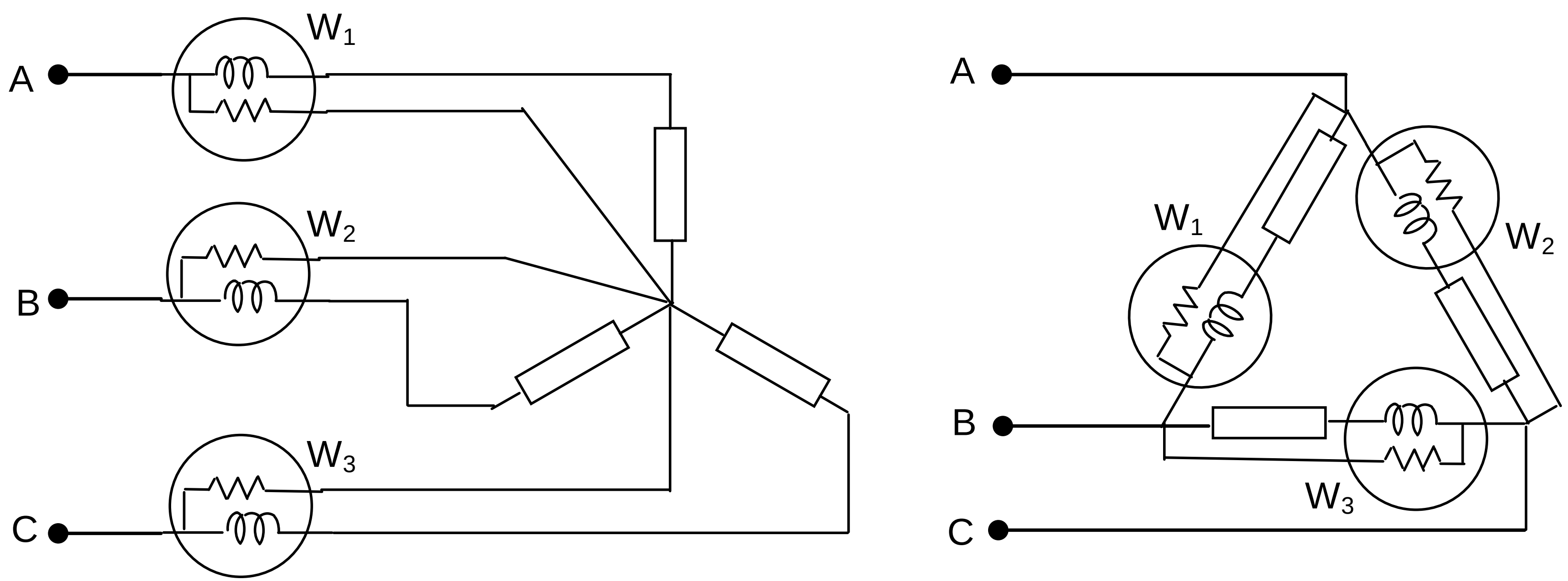


Fig. 6.27 Power measurement - Three wattmeters.

If W_1, W_2 and W_3 are the readings of the wattmeters, then total real power consumed by the three phase load is given by

$$P = W_1 + W_2 + W_3 \qquad (6.35)$$

This method of measuring three-phase power can be used whether the load is balanced or not. However, there are certain practical difficulties in this method of measuring three-phase power. They are,

1. Sometime the neutral point in the star-connected load may not be readily available.

2. Usually it is not possible to cut through the delta-connected load to introduce the current coil.

Because of these difficulties, the application of this method is limited.

6.9 MEASUREMENT OF THREE-PHASE POWER BY TWO-WATTMETER METHOD

Two-wattmeter method can be used to measure the real power consumed by three-phase load. The merits of this method are

1. The three-phase load can be balanced or unbalanced.
2. The three-phase load can be connected in either star or delta.
3. Neutral point of star-connected load is not required to connect the wattmeters.
4. There is no need to cut open the delta-connected load to introduce the current coil.
5. The power factor of the balanced load can be calculated from the two wattmeter readings.

6.9.1 MEASUREMENT OF POWER CONSUMED BY STAR-CONNECTED BALANCED LOAD

Consider star-connected balanced load connected to a three-phase supply. Fig. 6.28 (a) shows the physical connection of two wattmeters.

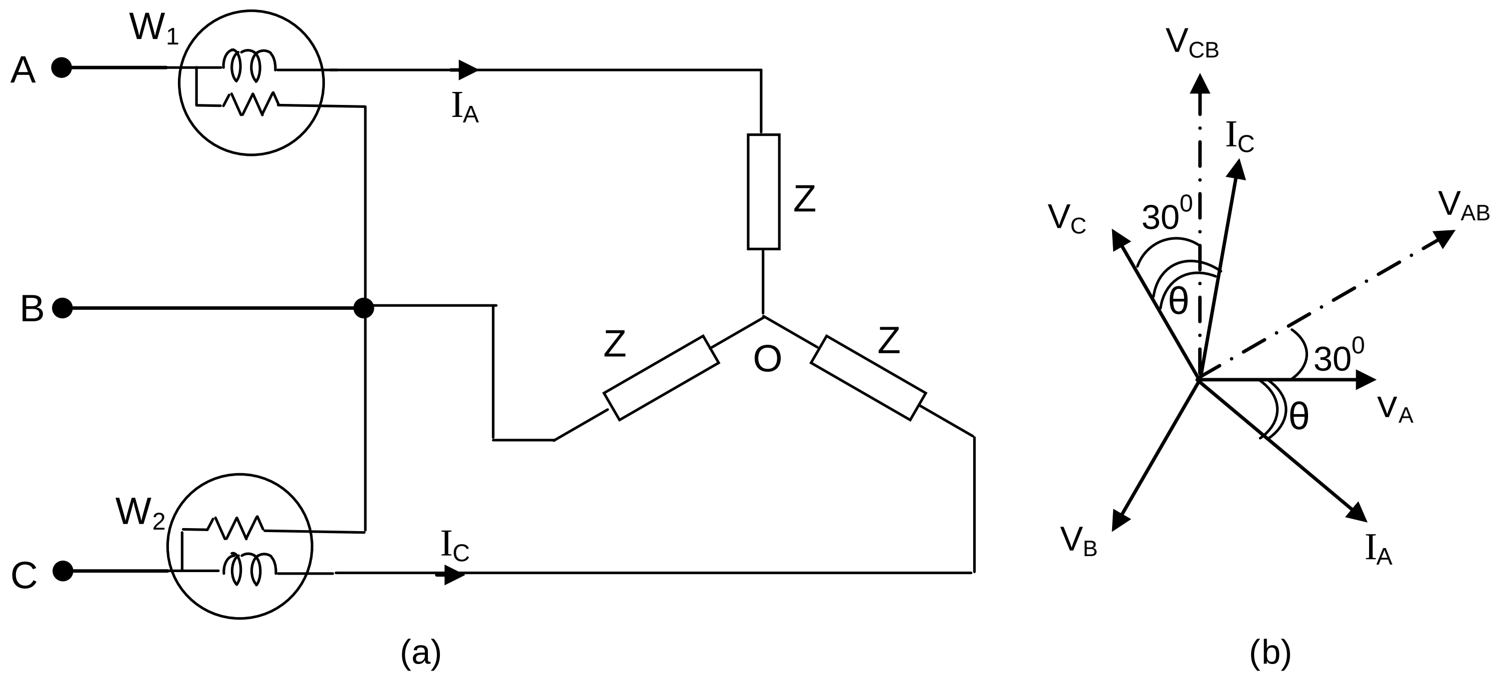
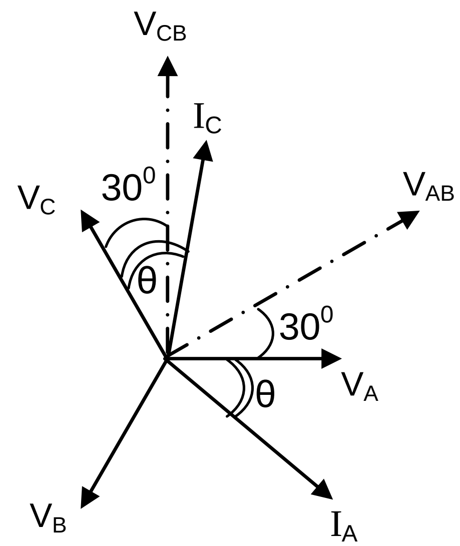


Fig. 6.28 Two wattmeter method - Star-connected balanced load.

Let us assume that the load is inductive having an impedance angle of θ. Let V_{ph} and I_{ph} be the magnitudes of phase voltage and phase current respectively.

In wattmeter 1, the voltage across the pressure coil is V_{AB} and the current in the current coil is I_A. In wattmeter 2, the voltage across the pressure coil is V_{CB} and the current in the current coil is I_C.



$$W_1 = |V_{AB}|\ |I_A|\ \cos(\text{angle between } V_{AB} \text{ and } I_A)$$

$$W_2 = |V_{CB}|\ |I_C|\ \cos(\text{angle between } V_{CB} \text{ and } I_C)$$

The phasor diagram, taking voltage V_A as reference is shown in Fig. 6.28 (b). The phasors V_{AB}, V_{CB}, I_A and I_C are indicated. Thus,

$$V_A = V_{ph}\angle 0^0, \qquad V_B = V_{ph}\angle -120^0 \qquad \text{and} \qquad V_C = V_{ph}\angle -240^0 \tag{6.36}$$

$$\text{Voltage } V_{AB} = V_A - V_B = \sqrt{3}\ V_{ph}\angle 30^0 \text{ V} \tag{6.37}$$

$$\text{Voltage } V_{CB} = V_C - V_B = \sqrt{3}\ V_{ph}\angle 90^0 \text{ V} \tag{6.38}$$

$V_{AB} = V_A - V_B = \sqrt{3}\ V_{ph} \angle 30^0$ V; $\quad$ $V_{CB} = V_C - V_B = \sqrt{3}\ V_{ph} \angle 90^0$ V

Knowing the impedance angle as θ,

Current $I_A = I_{ph} \angle -\theta^0$ and current $I_C = I_{ph} \angle 120 - \theta^0$ (6.39)

Wattmeter reading $W_1 = |V_{AB}|\ |I_A| \cos$ (angle between V_{AB} and I_A)

$$= \sqrt{3}\ V_{ph}\ I_{ph} \cos[30^0 - (-\theta)] = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta + 30^0) \quad (6.40)$$

Wattmeter reading $W_2 = |V_{CB}|\ |I_C| \cos$ (angle between V_{CB} and I_C)

$$= \sqrt{3}\ V_{ph}\ I_{ph} \cos[90 - (120^0 - \theta)] = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta - 30^0) \quad (6.41)$$

From the above two equations,

$$W_1 + W_2 = \sqrt{3}\ V_{ph}\ I_{ph} [\cos\theta\ \cos 30^0 - \sin\theta\ \sin 30^0 + \cos\theta\ \cos 30^0 + \sin\theta\ \sin 30^0]$$

$$= \sqrt{3}\ V_{ph}\ I_{ph}\ 2\cos\theta\ \cos 30^0 = 3\ V_{ph}\ I_{ph} \cos\theta$$

= Total three-phase real power consumed by the load (6.42)

6.9.2 MEASUREMENT OF POWER CONSUMED BY DELTA-CONNECTED BALANCED LOAD

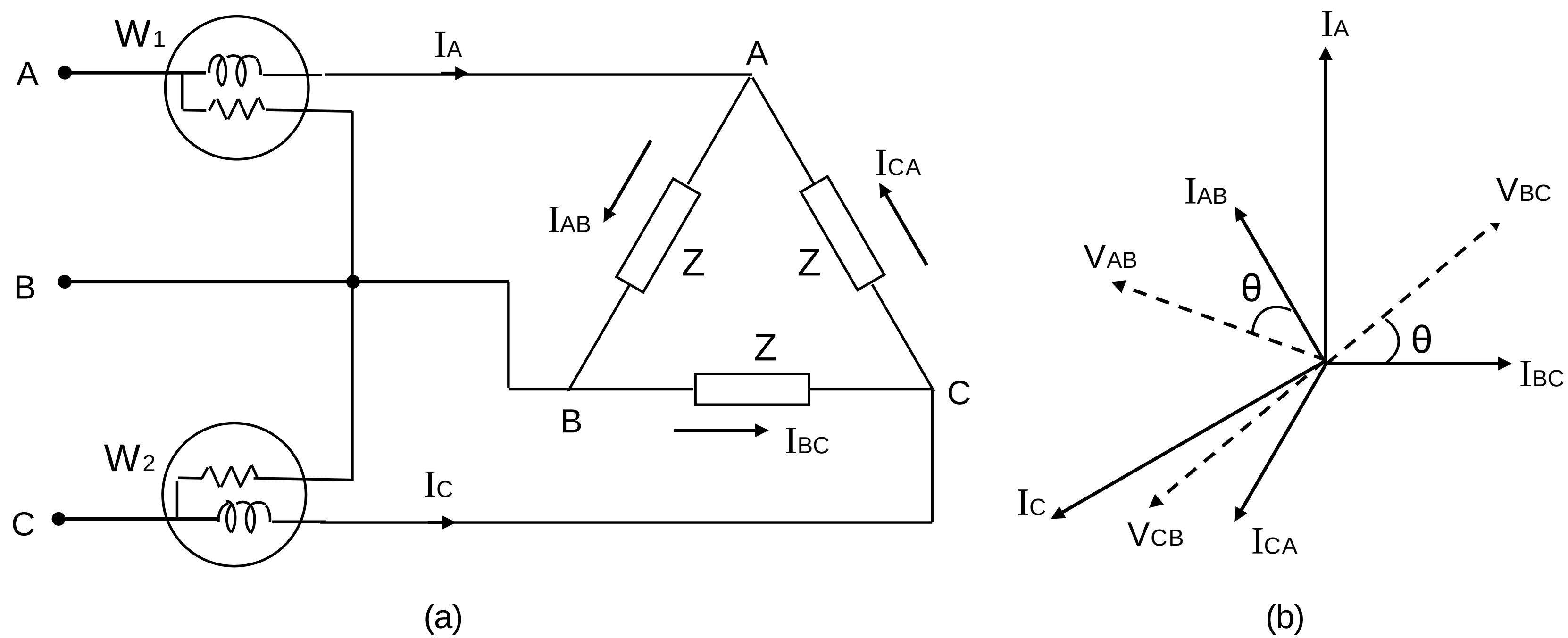


Fig. 6.29 Two wattmeter method - Delta-connected balanced load.

Fig. 6.29 (a) shows the physical connection of wattmeters corresponding to delta-connected balanced load. It is assumed that the load is inductive with an impedance angle of θ. Let V_{ph} and I_{ph} be the magnitudes of phase voltage and phase current respectively.

In wattmeter 1, the current through the current coil is I_A and the voltage across the pressure coil is V_{AB}. In wattmeter 2, the current through the current coil is I_C and the voltage across the pressure coil is V_{CB}.

The phasor diagram, taking current I_{BC} as reference is shown in Fig. 6.25 (b). The phasors V_{AB}, V_{CB}, I_A and I_C are indicated. Thus,

$$I_{BC} = I_{ph} \angle 0^0,\ I_{CA} = I_{ph} \angle -120^0 \text{ and } I_{AB} = I_{ph} \angle -240^0 \qquad (6.43)$$

Current $I_A = I_{AB} - I_{CA} = \sqrt{3}\ I_{ph} \angle 90^0$; Current $I_C = I_{CA} - I_{BC} = \sqrt{3}\ I_{ph} \angle 210^0$

$I_A = I_{AB} - I_{CA} = \sqrt{3}\ I_{ph} \angle 90^0$; $I_C = I_{CA} - I_{BC} = \sqrt{3}\ I_{ph} \angle 210^0$

Knowing the impedance angle as θ,

Voltage $V_{AB} = V_{ph} \angle \theta + 120^0$ and voltage $V_{CB} = - V_{BC} = - V_{ph} \angle \theta = V_{ph} \angle \theta + 180^0$

Wattmeter reading $W_1 = |V_{AB}|\ |I_A|$ cos (angle between V_{AB} and I_A)

$$= \sqrt{3}\ V_{ph}\ I_{ph} \cos\,[\theta + 120^0 - 90^0] = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta + 30^0) \qquad (6.44)$$

Wattmeter reading $W_2 = |V_{CB}|\ |I_C|$ cos (angle between V_{CB} and I_C)

$$= \sqrt{3}\ V_{ph}\ I_{ph} \cos\,[\theta + 180^0 - 210] = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta - 30^0) \qquad (6.45)$$

The above two equations are the same as those obtained in the case of star-connected balanced load. Therefore, on simplification, we get

$W_1 + W_2 = 3\ V_{ph}\ I_{ph} \cos\theta$

= Total three-phase real power consumed by the load (6.46)

Thus, irrespective of whether the load is connected in star or delta, sum of the readings of the two wattmeter readings will give the total real power consumed by the three-phase load.

6.9.3 POWER FACTOR OF BALANCED LOAD IN TERMS OF WATTMETER READINGS

Power factor of the balanced three-phase load can be calculated from the two wattmeter readings.

Wattmeter reading $W_1 = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta + 30^0)$ (6.40)

Wattmeter reading $W_2 = \sqrt{3}\ V_{ph}\ I_{ph} \cos(\theta - 30^0)$ (6.41)

Therefore,

$$W_2 - W_1 = \sqrt{3}\ V_{ph}\ I_{ph}\ [\cos\theta\ \cos 30^0 + \sin\theta\ \sin 30^0 - \cos\theta\ \cos 30^0 + \sin\theta\ \sin 30^0]$$

$$= \sqrt{3}\ V_{ph}\ I_{ph}\ 2 \sin\theta\ \sin 30^0 = \sqrt{3}\, V_{ph}\ I_{ph} \sin\theta \quad (6.47)$$

We know that, $W_1 + W_2 = 3\ V_{ph}\ I_{ph} \cos\theta$ (6.48)

Therefore, $\dfrac{W_2 - W_1}{W_1 + W_2} = \dfrac{1}{\sqrt{3}} \tan\theta$ Thus, $\tan\theta = \sqrt{3}\ \dfrac{W_2 - W_1}{W_1 + W_2}$ (6.49)

$$\text{Power factor, } \cos\theta = \cos\left[\tan^{-1}\left(\sqrt{3}\ \frac{W_2 - W_1}{W_1 + W_2}\right)\right] \quad (6.50)$$

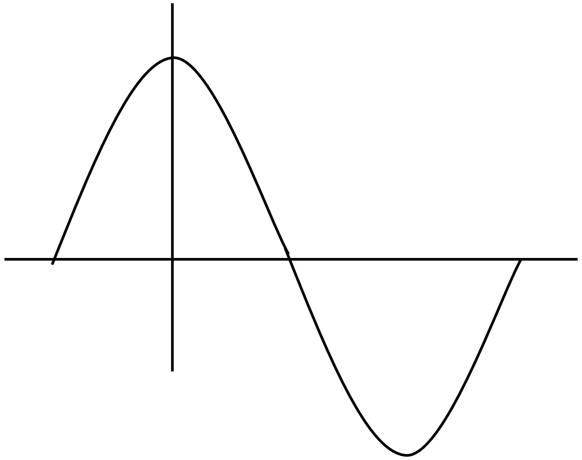
Reactive power Q

Reactive power, $Q = 3\, V_{ph}\, I_{ph}\, \sin\theta$ (6.51)

Knowing that $W_2 - W_1 = \sqrt{3}\, V_{ph}\, I_{ph}\, \sin\theta$ (6.47)

Reactive power, $Q = \sqrt{3}\, (W_2 - W_1)$ (6.53)

Sign of wattmeter readings



$W_1 = \sqrt{3}\, V_{ph}\, I_{ph}\, \cos(\theta + 30^0)$

$W_2 = \sqrt{3}\, V_{ph}\, I_{ph}\, \cos(\theta - 30^0)$

For inductive load, wattmeter reading

W_1 is positive for $0 \le \theta < 60^0$; W_1 is zero for $\theta = 60^0$; W_1 is negative for $\theta > 60^0$

W_2 is always positive; Always $W_2 \ge W_1$; $W_2 = W_1$ for $\theta = 0$

When the <u>load is capacitive</u>, wattmeter readings W_1 and W_2 get interchanged as compared to those of inductive load.

Example 6.14

The power input to a 2000 V, 50 Hz, 3-phase motor is measured by two wattmeters which indicate 100 kW and 300 kW respectively. Calculate (a) the input power (b) the power factor and (c) the line current.

Solution:

(a) $W_1 = 100$ kW and $W_2 = 300$ kW; Input power $P = W_1 + W_2 = 400$ kW

(b) $\tan\theta = \sqrt{3}\,\dfrac{W_2 - W_1}{W_1 + W_2} = \sqrt{3} \times \dfrac{200}{400} = 0.8660$

$\theta = 40.8934^0$; $\cos\theta = 0.7559$; Power factor = 0.7559 lagging

(c) Three-phase power, $P = \sqrt{3}\, V_\ell I_\ell \cos\theta$

Line current, $I_\ell = \dfrac{400 \times 10^3}{\sqrt{3} \times 2000 \times 0.7559} = 152.76$ A

Example 6.15

Two wattmeters are connected to measure the power in a 3-phase 3-wire balanced load. Determine the total power and power factor, if the two wattmeters read (a) 1000 W each, both positive and (b) 1000 W each of opposite sign.

Solution:

(a) W_1 = 1000 W; W_2 = 1000 W; Total power P = $W_1 + W_2$ = 2000 W

$$\tan\theta = \sqrt{3}\,\frac{W_2 - W_1}{W_1 + W_2} = 0;\ \text{Power factor angle}\ \theta = 0$$

Power factor = cos 0^0 = 1.0

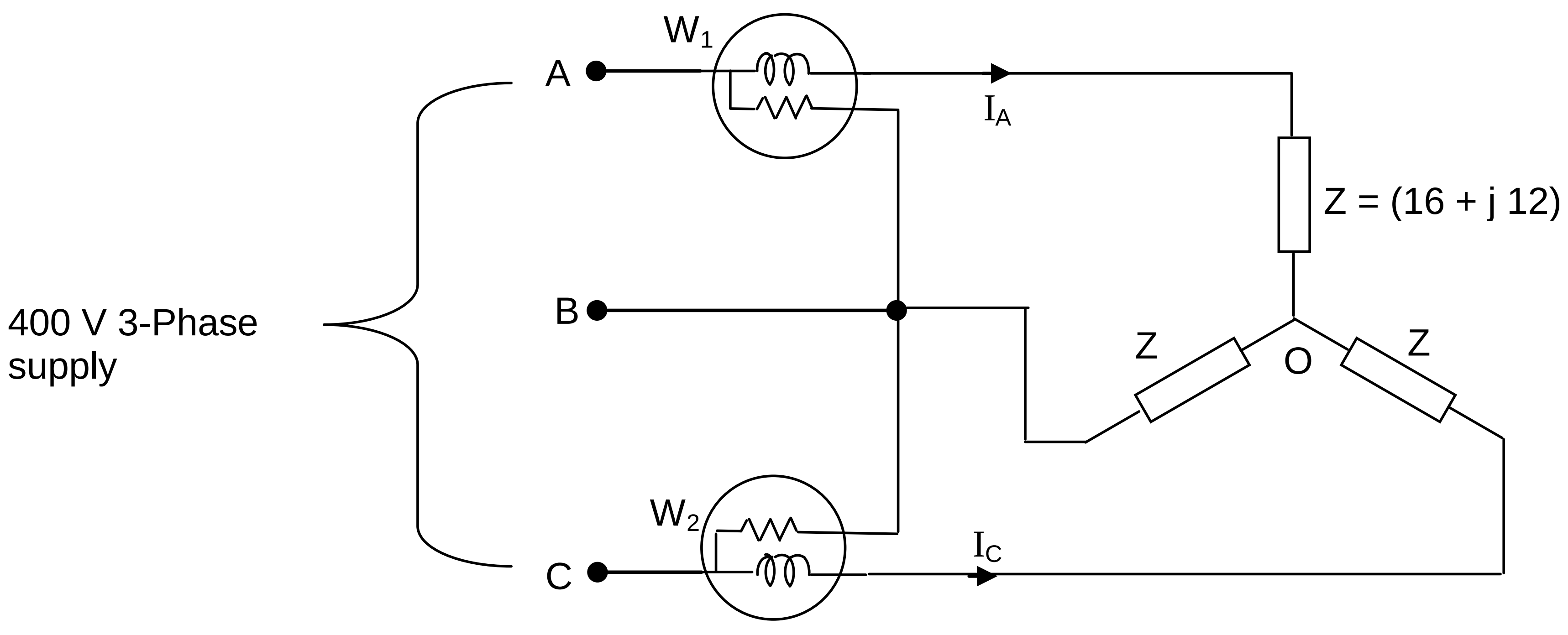
(b) W_1 = - 1000 W; W_2 = 1000 W; Total power P = $W_1 + W_2$ = 0

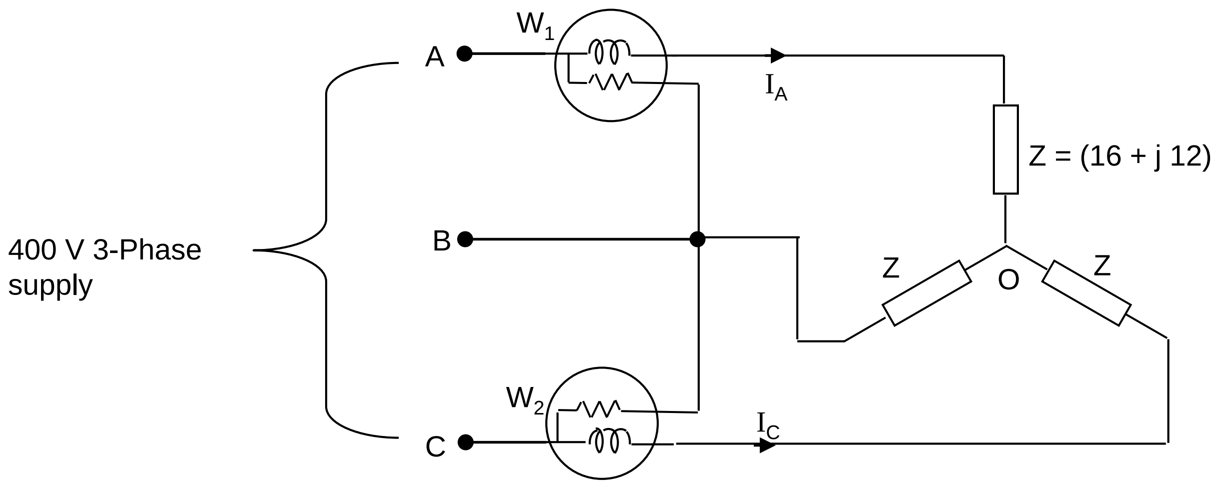
$$\tan\theta = \sqrt{3}\,\frac{W_2 - W_1}{W_1 + W_2} = \infty;\ \text{Power factor angle}\ \theta = 90^0;$$

Power factor = 0 lagging.

Example 6.16

Across 400 V, 3-phase supply mains, a star-connected balanced load of (16 + j 12) Ω impedance is connected. (a) Taking V_A as reference, determine the line currents and the power absorbed by the load. (b) If two wattmeters are used to measure the power, what will be the readings of the wattmeters?





(a) $V_A = \dfrac{400}{\sqrt{3}}\angle 0^0 = 230.9410\angle 0^0$ V ; Z = (16 + j 12) = 20 $\angle$ 36.87^0 Ω

Line currents are:

$I_A = \dfrac{V_A}{Z} = 11.547\angle -36.87^0$ A ; I_B = 11.547 $\angle$ - 156.87^0 A: I_C =11.547 $\angle$ 83.13^0 A

Total power P = $\sqrt{3}\ V_\ell\, I_\ell \cos\theta = \sqrt{3}$ x 400 x 11.547 cos 36.87^0 = 6400 W

(b) $W_1 = \sqrt{3}\ V_{ph}\, I_{ph} \cos(\theta + 30^0) = \sqrt{3}$ x 230.9401 x 11.547 cos 66.87^0 = 1814.35 W

$W_2 = \sqrt{3}\ V_{ph}\, I_{ph} \cos(\theta - 30^0) = \sqrt{3}$ x 230.9401 x 11.547 cos 6.87^0 = 4585.64 W

6.10 POWER MEASUREMENT IN UNBALANCED THREE-PHASE LOAD

Total power consumed by three-phase unbalanced load also can be measured using two wattmeter method.

6.10.1 STAR-CONNECTED UNBALANCED LOAD

Fig. 6.32 shows the physical connections of two wattmeters to measure the power consumed by a three-phase star-connected unbalanced load.

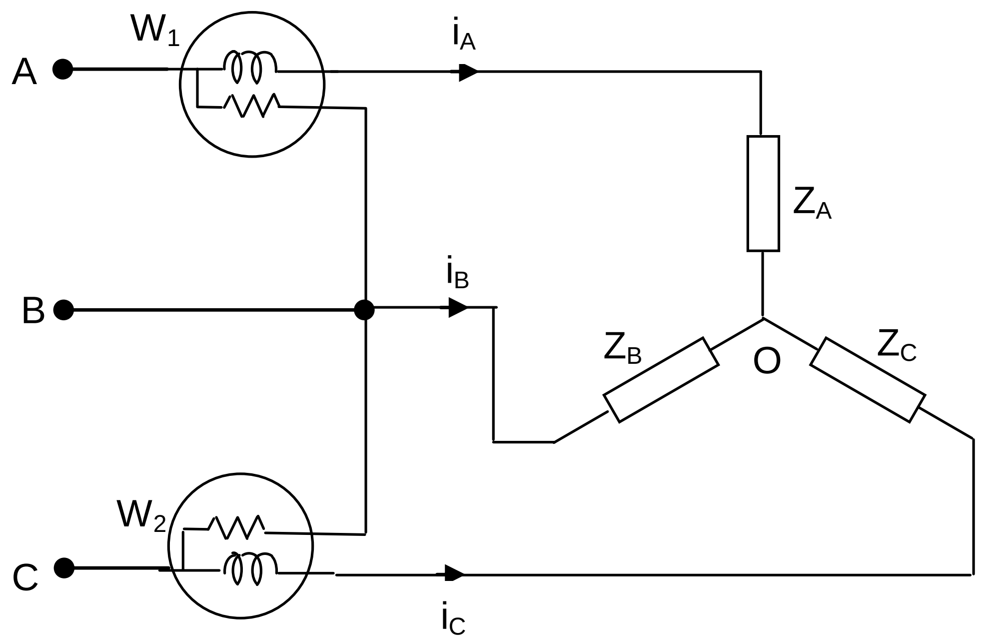


Fig. 6.32 Two wattmeter method - Star-connected unbalanced load.

At any instant of time, the total power supplied to the load is given by

$$p = v_{AO}\, i_A + v_{BO}\, i_B + v_{CO}\, i_C \qquad (6.59)$$

Note that $i_A + i_B + i_C = 0$ and current i_B is not used in wattmeter readings

$$i_B = - i_A - i_C. \qquad (6.60)$$

Therefore,

$$p = v_{AO}\, i_A + v_{BO}\, (- i_A - i_C) + v_{CO}\, i_C = (v_{AO} - v_{BO})\, i_A + (v_{CO} - v_{BO})\, i_C$$

$$= v_{AB}\, i_A + v_{CB}\, i_C \qquad (6.61)$$

From the above equation,

$$\text{Average power } P = |V_{AB}|\,|I_A| \cos(\text{angle between } V_{AB} \text{ and } I_A) +$$

$$|V_{CB}|\,|I_C| \cos(\text{angle between } V_{CB} \text{ and } I_C) \qquad (6.62)$$

It is clear from the diagram in Fig. 6.32 that in wattmeter 1, the voltage across the pressure coil is V_{AB} and the current through the current coil is I_A. Further, in wattmeter 2, the voltage across the pressure coil is V_{CB} and the current through the current coil is I_C. Therefore, wattmeter readings will be

It is clear from the diagram in Fig. 6.32 that in wattmeter 1, the voltage across the pressure coil is V_{AB} and the current through the current coil is I_A. Further, in wattmeter 2, the voltage across the pressure coil is V_{CB} and the current through the current coil is I_C. Therefore, wattmeter readings will be

$$W_1 = |V_{AB}|\,|I_A| \cos(\text{angle between } V_{AB} \text{ and } I_A) \tag{6.63}$$

$$W_2 = |V_{CB}|\,|I_C| \cos(\text{angle between } V_{CB} \text{ and } I_C) \tag{6.64}$$

Therefore, average power consumed by the load

$$P = W_1 + W_2 \tag{6.65}$$

Thus, the two wattmeters connected as shown will measure the total average power consumed by the three-phase unbalanced load.

6.10.2 DELTA-CONNECTED UNBALANCED LOAD

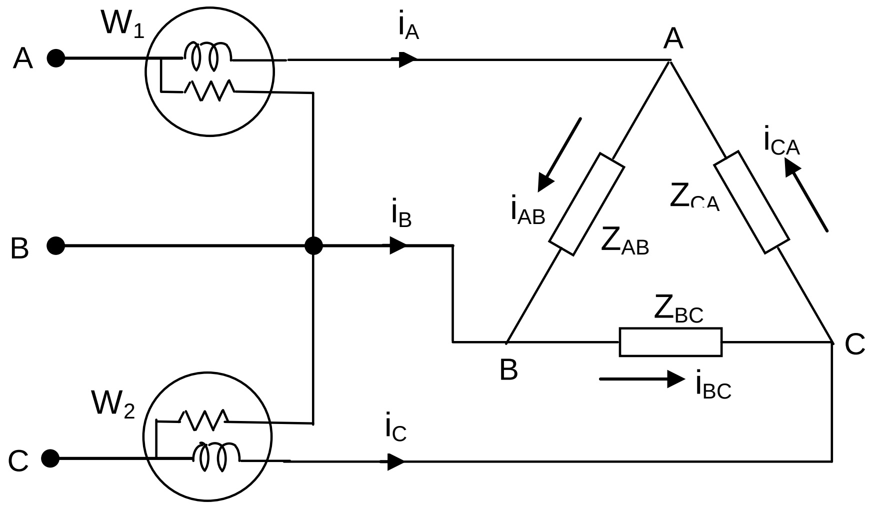


Fig. 6.33 Two wattmeter method - Delta-connected unbalanced load.

When the unbalanced load is connected in delta, the two wattmeters are connected as shown in Fig. 6.33. Now, at any instant of time $v_{AB} + v_{BC} + v_{CA} = 0$. Voltage v_{CA} is not used in wattmeter reading.

$$v_{CA} = -(v_{AB} + v_{BC}) \tag{6.66}$$

Instantaneous power is given by

$$p = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} + v_{CA}\, i_{CA} = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} - (v_{AB} + v_{BC})\, i_{CA}$$

$$= v_{AB}\,(i_{AB} - i_{CA}) + v_{BC}\,(i_{BC} - i_{CA}) = v_{AB}\,(i_{AB} - i_{CA}) + v_{CB}\,(i_{CA} - i_{BC})$$

$$= v_{AB}\, i_A + v_{CB}\, i_C \tag{6.67}$$

Instantaneous power is given by

$$p = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} + v_{CA}\, i_{CA} = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} - (v_{AB} + v_{BC})\, i_{CA}$$

$$= v_{AB}\,(i_{AB} - i_{CA}) + v_{BC}\,(i_{BC} - i_{CA}) = v_{AB}\,(i_{AB} - i_{CA}) + v_{CB}\,(i_{CA} - i_{BC})$$

$$= v_{AB}\, i_{A} + v_{CB}\, i_{C} \qquad (6.67)$$

The above equation is the same as Eq. (6.46) in the previous case. Thus as discussed earlier, average power consumed by the load can be obtained as

$$P = W_1 + W_2 \qquad (6.68)$$

Therefore, it can be concluded that two wattmeters properly connected, always measure the total power consumed by the three-phase load, irrespective of the load is balanced or unbalanced and the load is star-connected or delta-connected.

It is to be noted that the power factor is not defined for three-phase unbalanced load.

Example 6.22

In the two wattmeter method, wattmeter readings are noted as - 100 W and 300 W. Find the total real power, power factor and the total reactive power.

Solution:

P = - 100 + 300 = 200 W

$$\tan\theta = \sqrt{3}\,\frac{W_2 - W_1}{W_1 + W_2} = \sqrt{3}\,\frac{400}{200} = 3.4641;\ \ \theta = 73.8979^0$$

Power factor = cos θ = 0.2774 lagging

$Q = \sqrt{3}\,(W_2 - W_1) = \sqrt{3} \times 400 = 692.82$ VAR

Example 6.23

For a purely inductive load, wattmeter 2 in the two wattmeter method indicates 520 W.

What will be the reading of wattmeter 1?

Solution:

For purely inductive load, P = 0. i.e. $W_1 + W_2 = 0$ Thus $W_1 = - W_2 = - 520$ W

Example 6.24

Two wattmeters are used to measure power consumed by a three-phase balanced inductive load. The wattmeter readings are 663 W and 5097 W. Find (a) total power (b) power factor of the load (c) phase current if the phase voltage is 400 V.

Solution:

W_1 = 663 W; W_2 = 5097 W

(a) Total power = 663 + 5097 = 5760 W

(b) $\tan\theta = \sqrt{3}\,\dfrac{W_2 - W_1}{W_1 + W_2} = \sqrt{3}\,\dfrac{4434}{5760} = 1.3333;\ \ \theta = 53.13^0$

Power factor = 0.6 lagging

(c) 3 x 400 x I_{ph} x 0.6 = 5760

Current I_{ph} = 8 A

Example 6.25

The reading of two wattmeters to measure power in a three-phase capacitive load are 800 W and - 300 W. Calculate (a) input power (b) power factor of the load.

Solution:

$W_1 = 800$ W; $W_2 = -300$ W

Input power $P = W_1 + W_2 = 500$ W

$$\tan\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{1100}{500} = 3.8105; \;\; \theta = 75.3^0$$

Power factor = 0.2538 leading

9.1 INTRODUCTION TO NETWORK TOPOLOGY

A network consists of elements such as

i) Voltage source

ii) Current source

iii) Resistance

iv) Inductance

v) Capacitance and

vi) Transformer

Network topology concerns itself with the manner in which the various elements are grouped and interconnected.

In the network topology, each element in a network is represented merely by a line with small circles or dots at the two ends denoting the terminals as shown in Fig. 9.1.

Fig. 9.1 Representation of an element.

9.2 GRAPH

Graphical portrayal, showing the geometric interconnection of elements of the network is called the GRAPH of the given network. Fig. 9.2 shows a network and the corresponding graph.

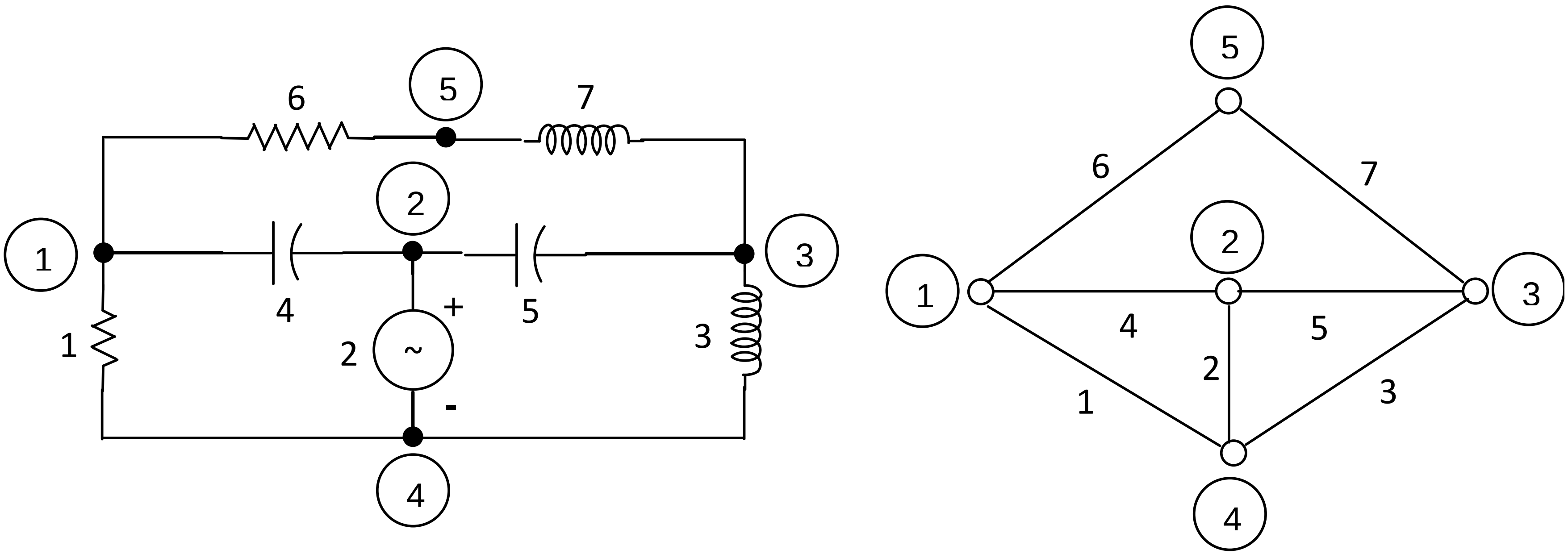


Fig. 9.2 A network and its graph.

A node is the meeting point of two or more elements.

A sequence of elements travelled from one node to another is called a PATH.

If there exists at least one path from each node in the graph to every other node of the graph, the graph is said to be CONNECTED or said to be in one PART; otherwise the graph is UNCONNECTED or in more than one part.

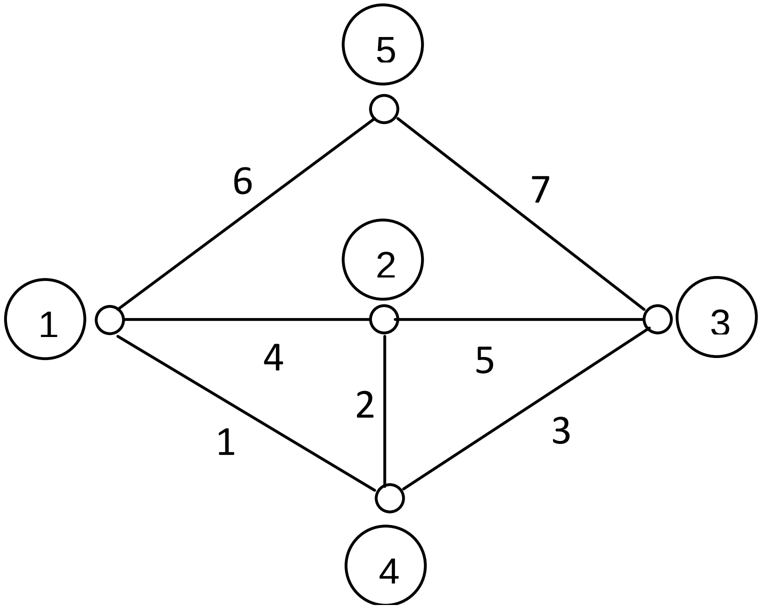
The graph shown in Fig. 9.2, has seven elements and five nodes. In this

4-2-3 is the path from node 1 to 3.

4-5 is a path from node 1 to 3.

2-3-7 is a path from node 2 to 5.

5-7 is a path from node 2 to 5.



If the elements in a graph are assigned orientations, the resulting graph is called ORIENTED GRAPH. Fig. 9.3 shows an oriented graph.

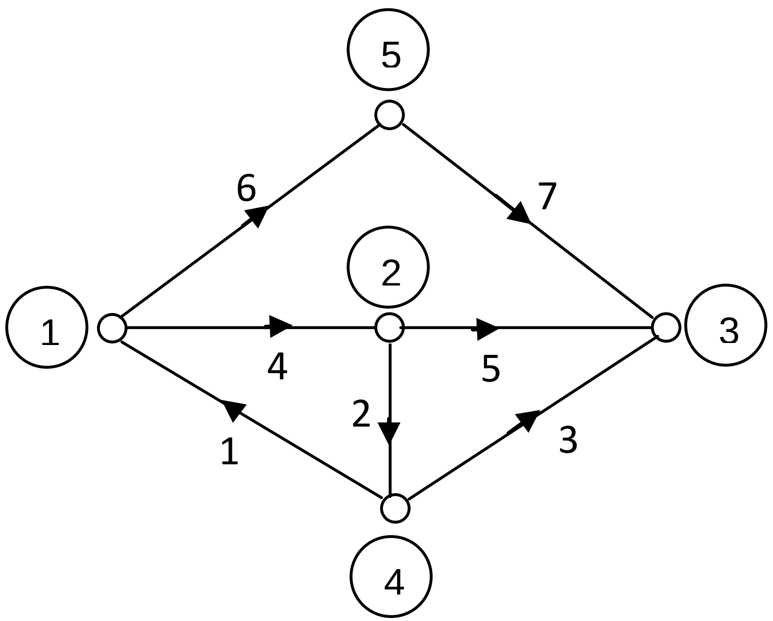


Fig. 9.3 Oriented graph.

In a network having N nodes, one node can be designated as reference node and it may be marked as node 0. If so, other nodes are numbered as 1, 2, …. , N-1.

9.2.1 SUB-GRAPH

For a graph G, a sub-graph G_1 is a collection of elements and nodes of G, such that every element and node of G_1 is contained in G. The number of elements in a sub-graph may be just as few as one or as many as all those in G. A sub-graph may be connected or unconnected.

9.3 TREE AND CO-TREE

TREE of a connected graph is defined as a sub-graph that contains a set of elements, which together, connects all the nodes of the graph, without forming any closed loop.

In general, a graph has more than one tree. The elements of a tree are called TREE BRANCHES.

The set of all the remaining elements of the graph, which are not in the tree, form the compliment of the tree and is known as CO-TREE. The elements of a co-tree are called LINKS. Links are also called CHORDS. Co-tree of a graph need not be a connected graph.

Consider the network graph shown in Fig. 9.4.

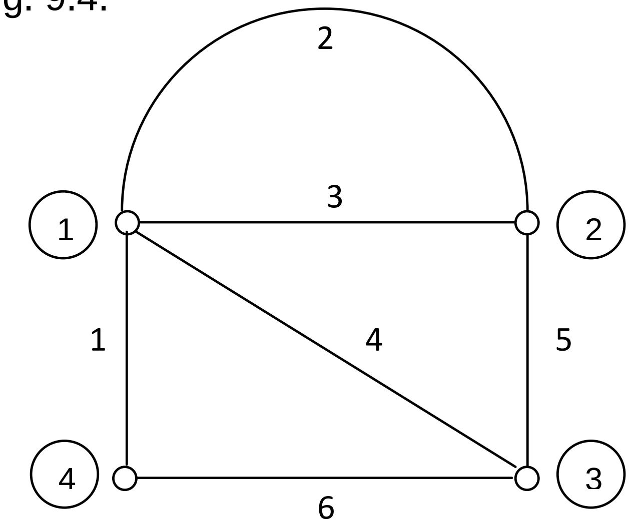


Fig. 9.4 A sample network

For this network graph, one of the trees and the corresponding co-tree are shown in Fig. 9.5.

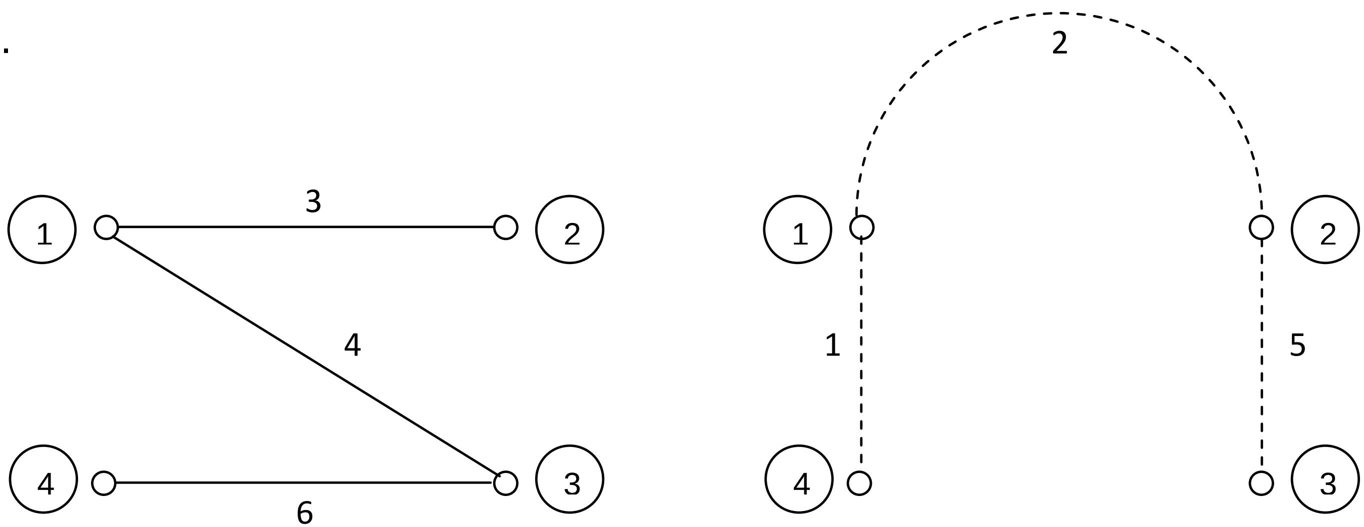
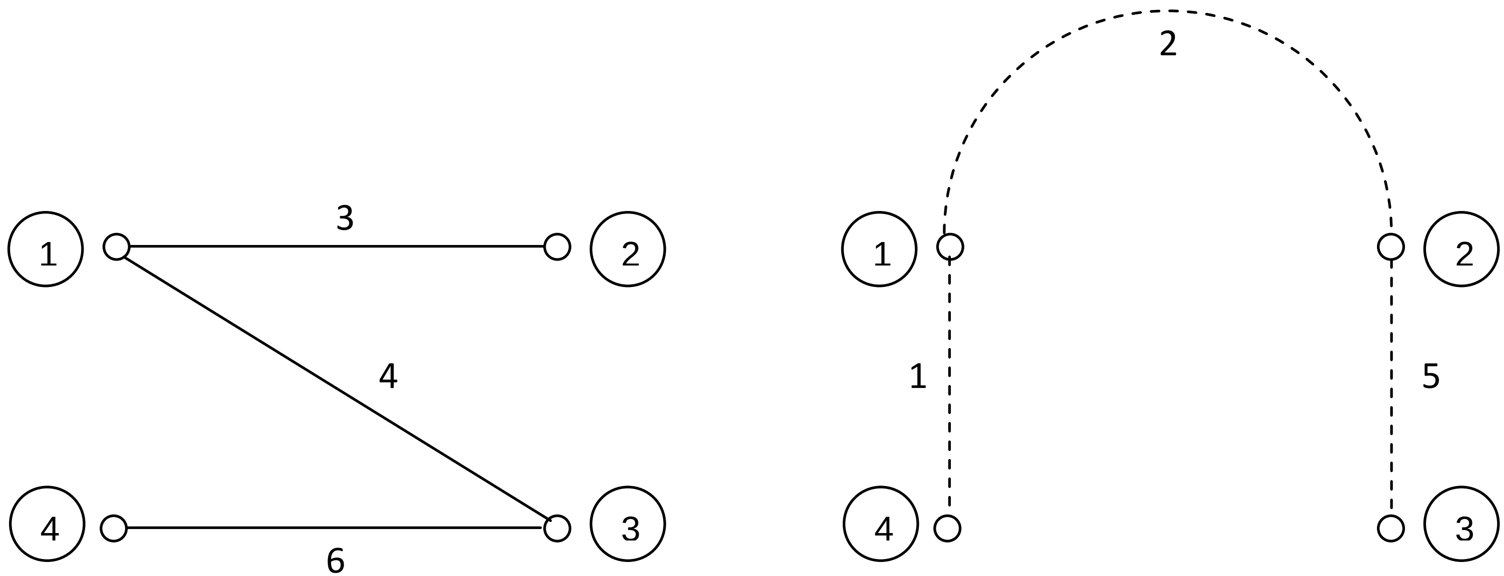


Fig. 9.5 A tree and the corresponding co-tree of a network graph.



Elements 3, 4 and 6 are the tree branches. Elements 1, 2 and 5 are the links.

Each time we add a link to the tree, a closed loop will be formed.

The following are the characteristics of a tree.

1. Tree contains all the nodes in the graph.
2. It does not have any closed loop.
3. There exists a path from any one node to every other node.

To make our ideas more clear, we may consider another network graph shown.

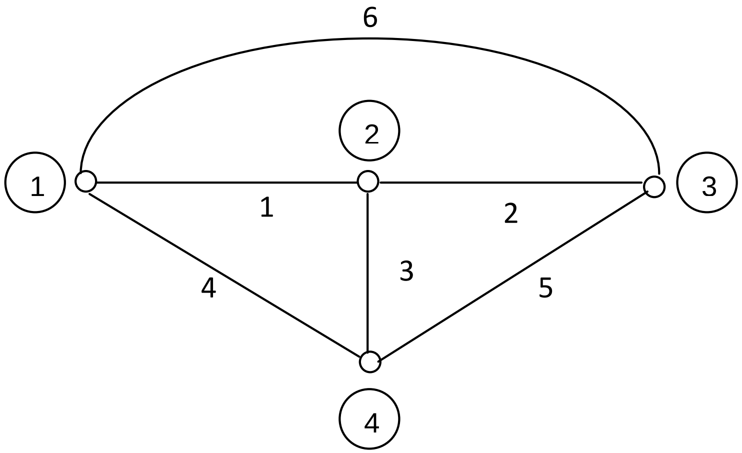
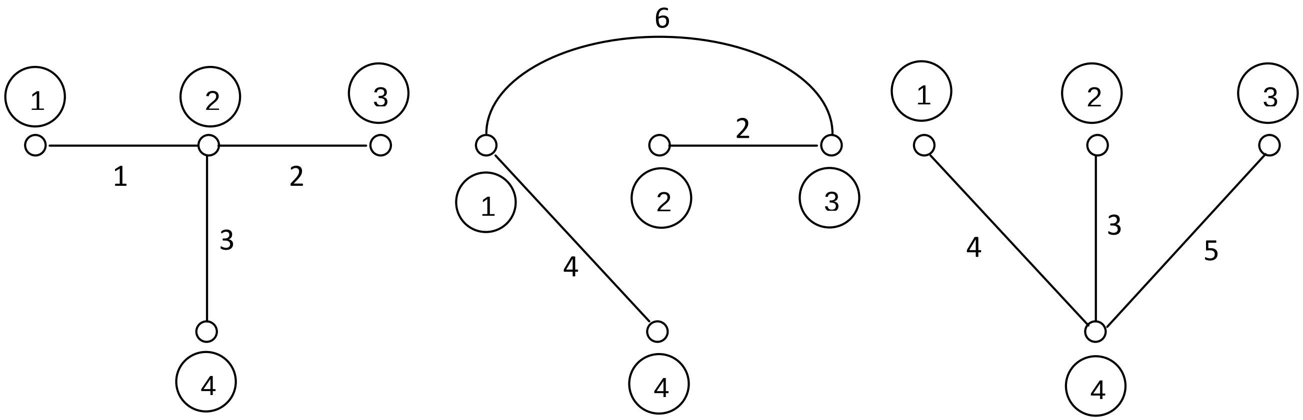


Fig. 9.6 A sample network graph.

Three possible trees of the graph are shown in Fig. 9.7 (some more trees are also possible).



(a) (b) (c)

Fig. 9.7 Three possible trees of network graph shown in Fig. 9.6.

Tree branches and the links of the three trees shown in Fig. 9.7, are given in Table 9.1.

TABLE 9.1 Tree branches and the corresponding links.

Fig.	Tree branches	Links
9.7 (a)	1, 2 and 3	4, 5 and 6
9.7 (b)	2, 4 and 6	1, 3 and 5
9.7 (c)	3, 4 and 5	1, 2 and 6

If there are n elements and N nodes in the network graph, then

Number of tree branches = N - 1.

Hence, number of links = n - (N - 1) = n - N + 1

Each time when a link is added to the tree, one loop is formed. Thus,

Number of independent loops = n - N + 1

A CUT-SET is a set of elements that, if removed, divides a connected graph into two connected graphs.

Consider the oriented connected graph shown in Fig. 9.8.

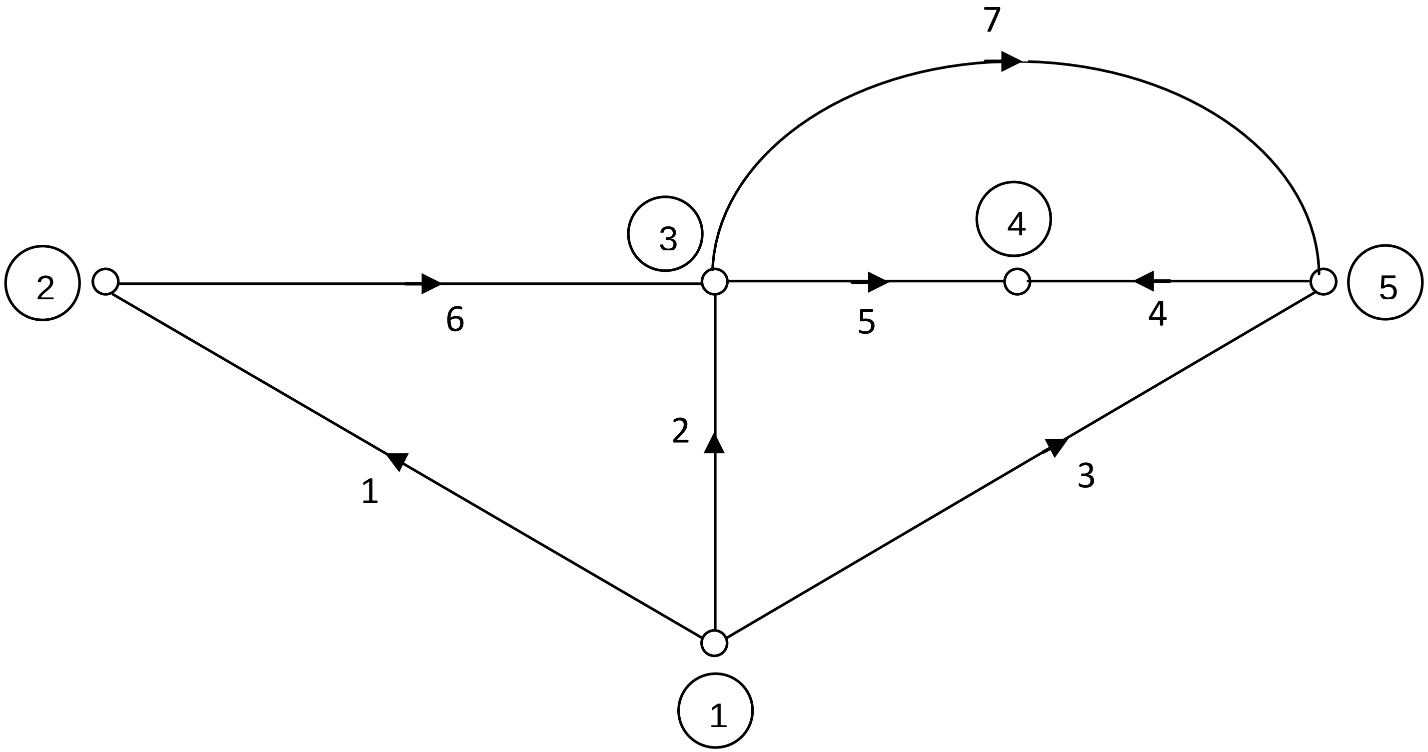


Fig. 9.8 Oriented connected graph.

Elements 1, 2, 5 and 7 constitute a cut-set as removal of these elements will result in two connected sub-graphs as shown in Fig. 9.9.

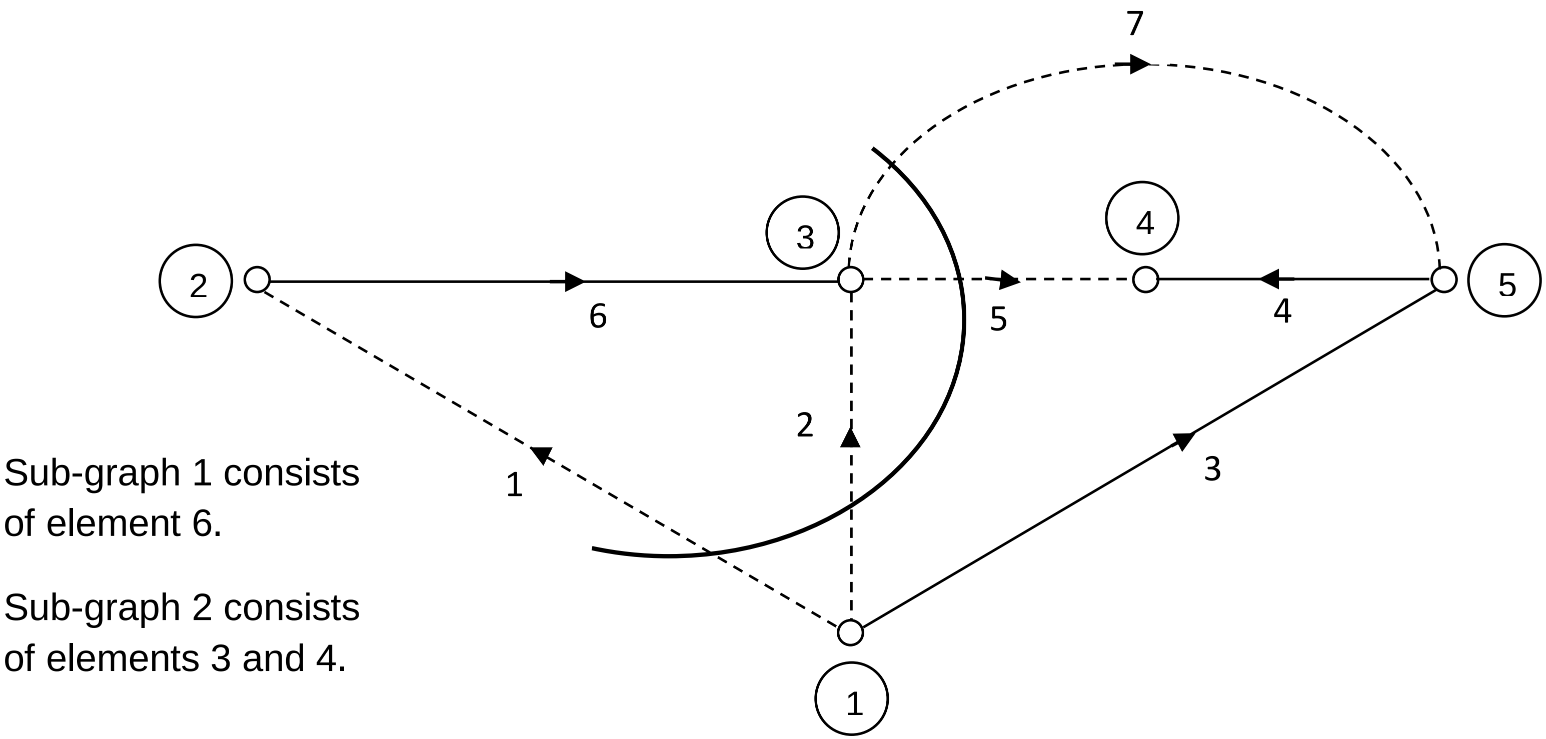
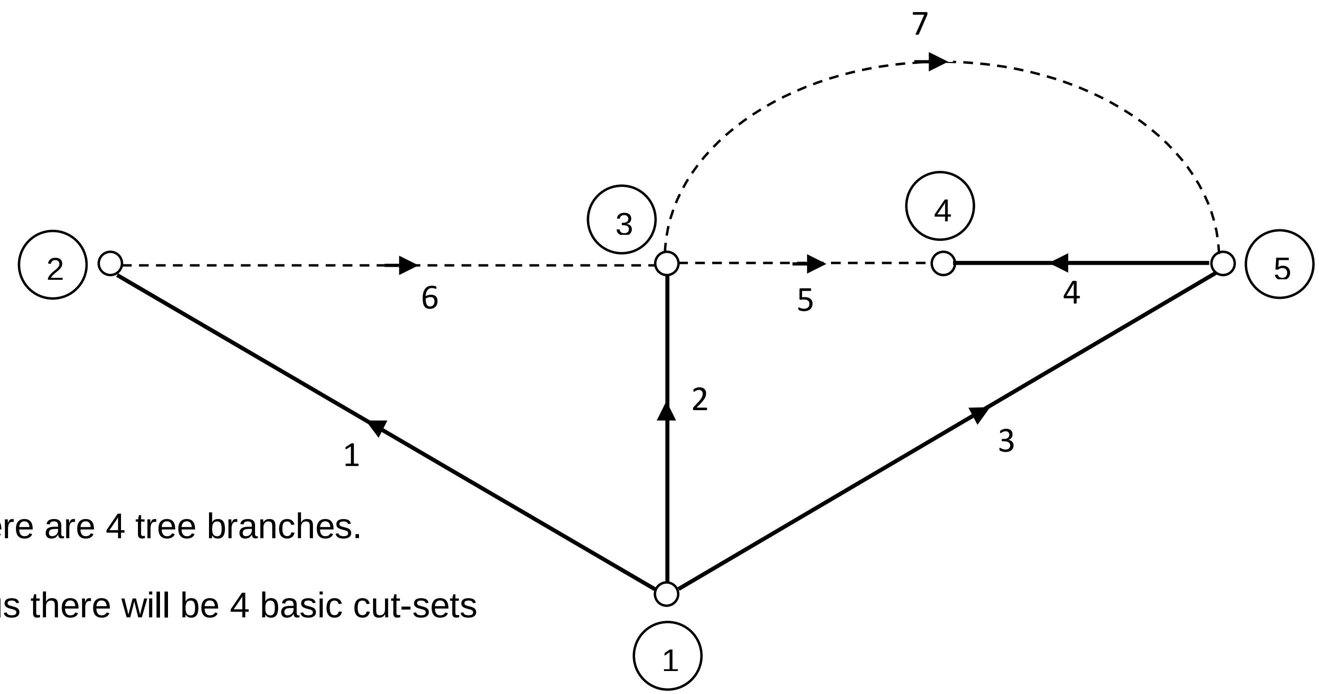


Fig. 9.9 Illustration of a cut-set.

Elements 1, 6 and elements 2, 3, 6 are examples of two other cut-sets. It is possible to identify more cut-sets.

A unique independent group of cut-sets may be chosen if each cut-set contains only one tree branch. Such independent cut-sets are called BASIC CUT-SETS. The number of basic cut-sets is equal to the number of tree branches. Orientation of a basic cut-set is chosen to be the same as that of the corresponding tree branch. Basic cut-sets will be different for different tree selected.

To explain the basic cut-sets, we need to consider a tree of the graph. Consider the tree consisting of elements 1, 2, 3 and 4 as shown in Fig. 9.10.



There are 4 tree branches.

Thus there will be 4 basic cut-sets

Fig. 9.10 Tree of a graph.

Corresponding to each and every tree branch, there will be one basic cut-set. Orientation of basic cut-set is the same as the orientation of the corresponding tree branch. Fig. 9.11 shows the four basic cut-sets A , B, C and D corresponding to the tree marked in Fig. 9.10.

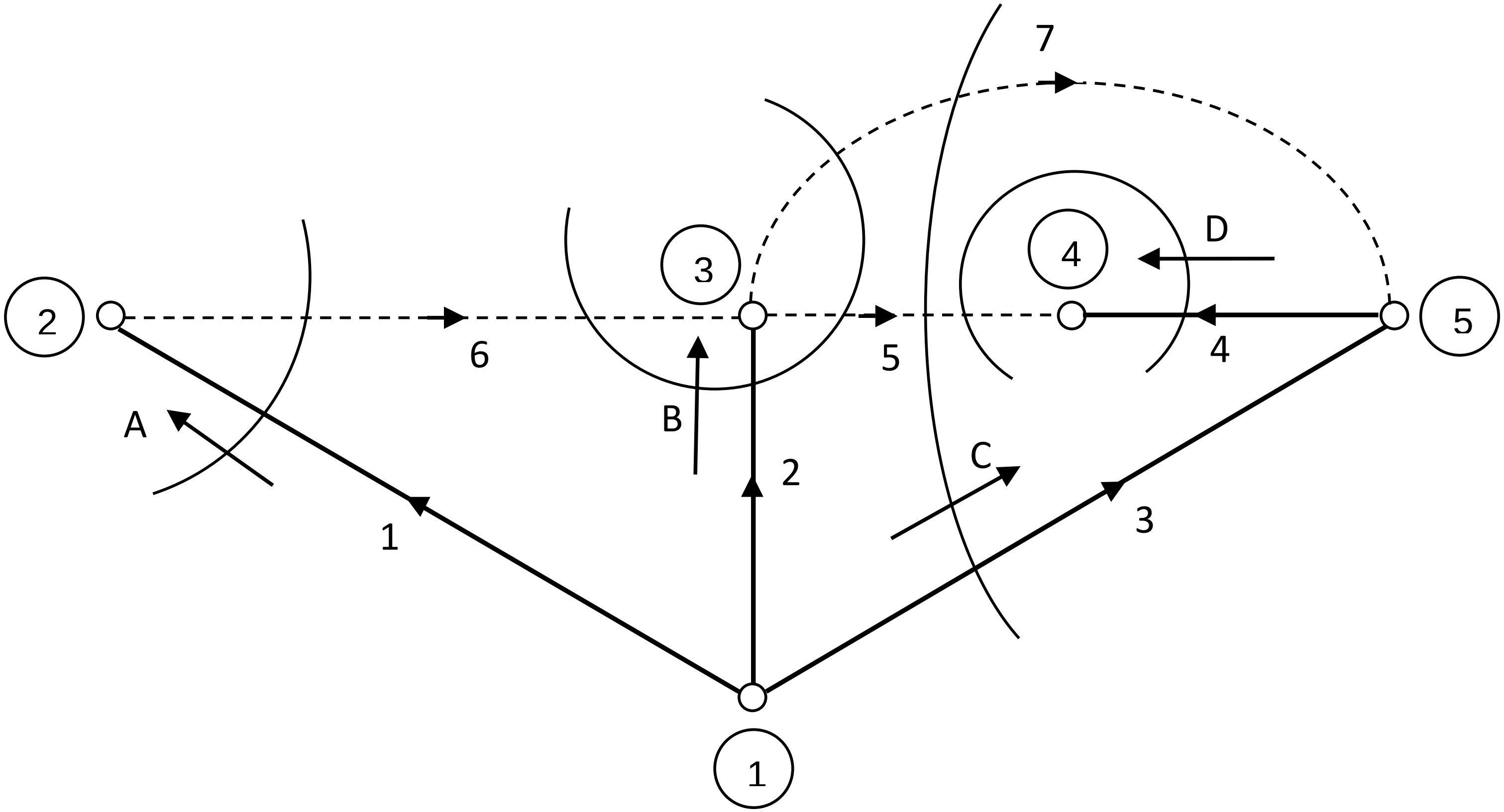


Fig. 9.11 Basic cut-sets.

It is possible to express the element voltages in terms of the tree branch voltages. Let $v_1, v_2, \ldots, v_7$ be the voltages of elements 1, 2, ….., 7. Let e_1, e_2, e_3 and e_4 be voltages of the tree branches 1, 2, 3 and 4.These voltages are marked in Fig. 9.12.

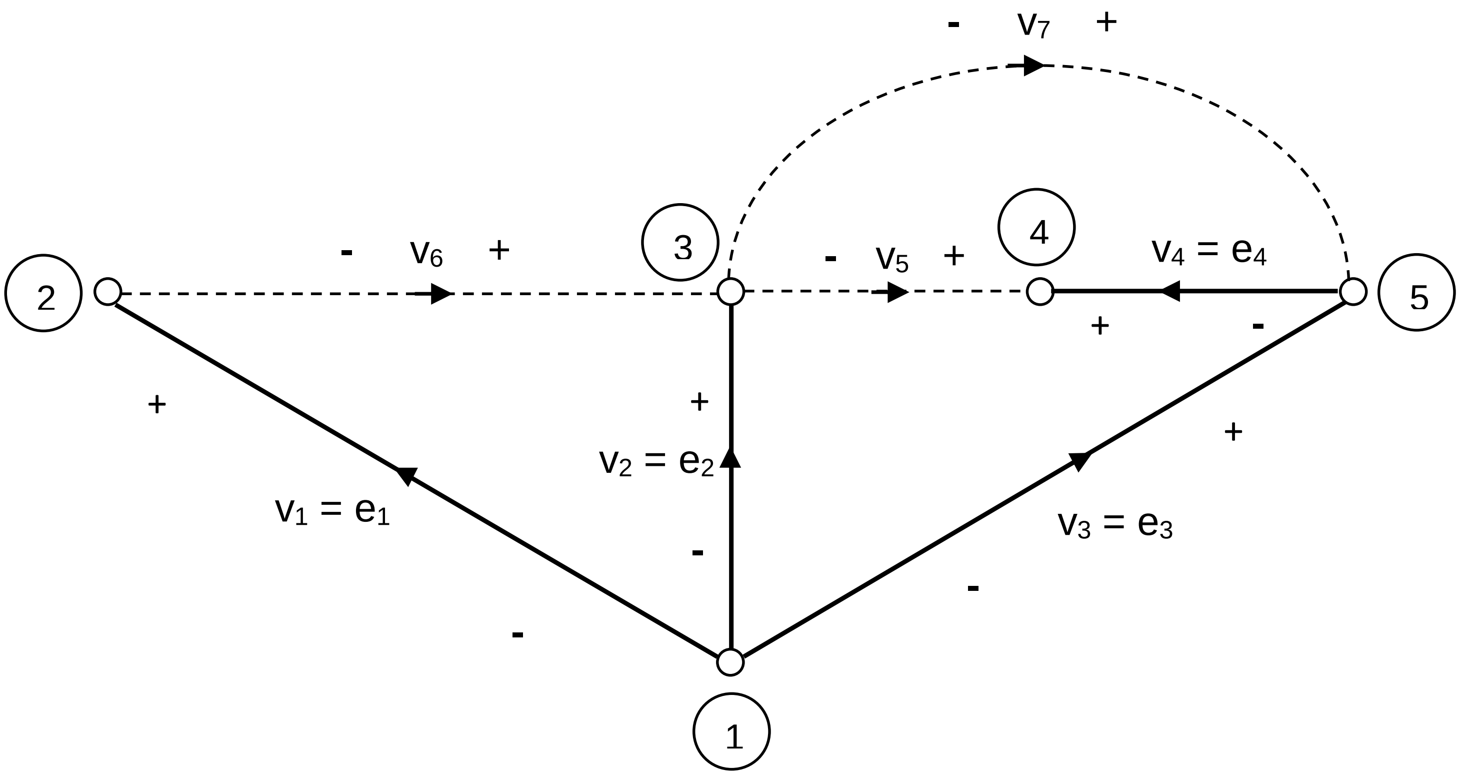
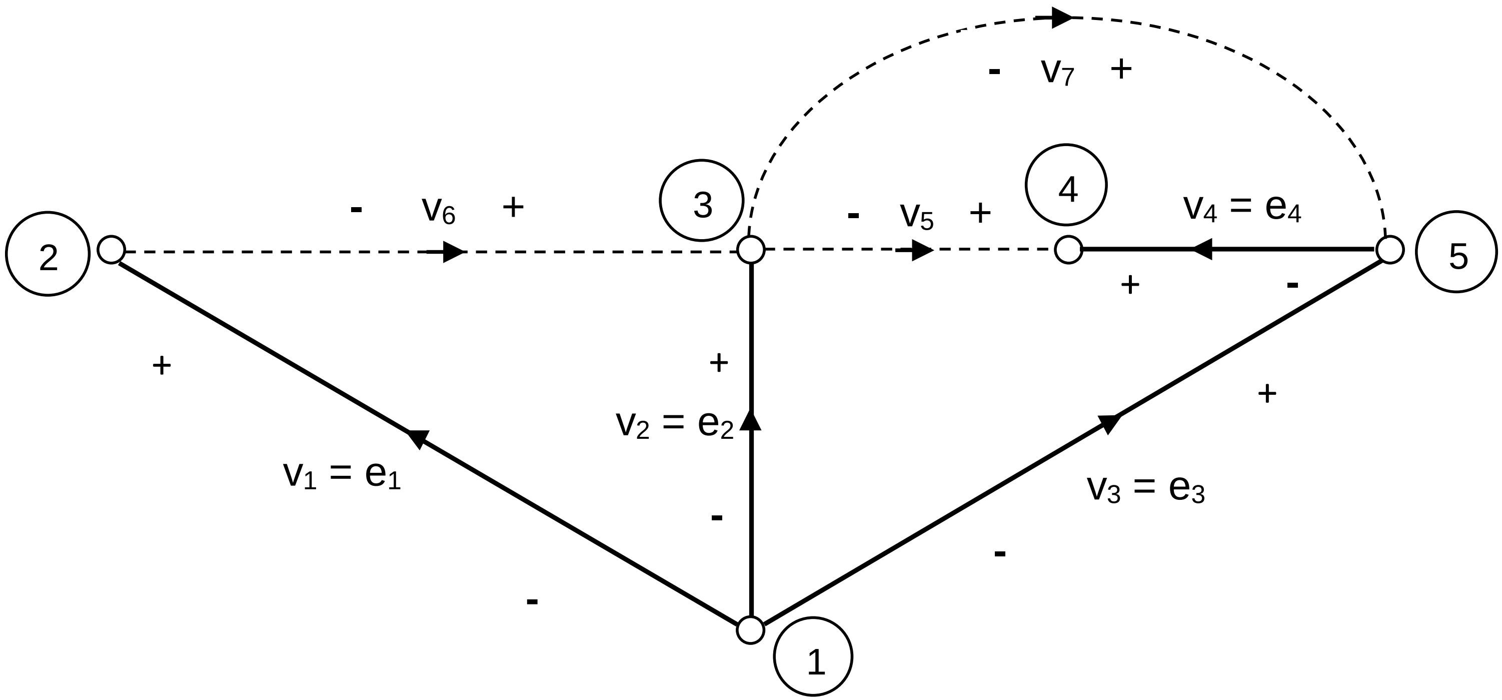


Fig. 9.12 Tree branch and element voltages.

It is obvious that

$v_1 = e_1$; $v_2 = e_2$; $v_3 = e_3$ and $v_4 = e_4$



Further, using KVL, other element voltages can be obtained in terms of the tree branch voltages as given below.

$$v_5 = e_4 + e_3 - e_2$$

$$v_6 = e_2 - e_1$$

$$v_7 = e_3 - e_2$$

Thus, the element voltages are:

$v_1 = e_1$; $v_2 = e_2$; $v_3 = e_3$ and $v_4 = e_4$

$v_5 = e_4 + e_3 - e_2$

$v_6 = e_2 - e_1$

$v_7 = e_3 - e_2$

The above relations can be shown as in Table 9.2 which is referred as CUT-SET SCHEDULE.

TABLE 9.2 Cut-set schedule.

Tree branch voltages	Element voltages						
	v_1	v_2	v_3	v_4	v_5	v_6	v_7
e_1	1	0	0	0	0	-1	0
e_2	0	1	0	0	- 1	1	-1
e_3	0	0	1	0	1	0	1
e_4	0	0	0	1	1	0	0

Each element Voltage can be read:

For example, $v_5 = e_4 + e_3 - e_2$

A TIE-SET is a set of elements that from a closed loop

Consider the oriented graph shown in Fig. 9.13. Set of elements 1, 6, 2 and 2, 5, 4, 3 are examples two tie-sets as each set of elements form a closed loop. It is possible to identify more number of tie-sets.

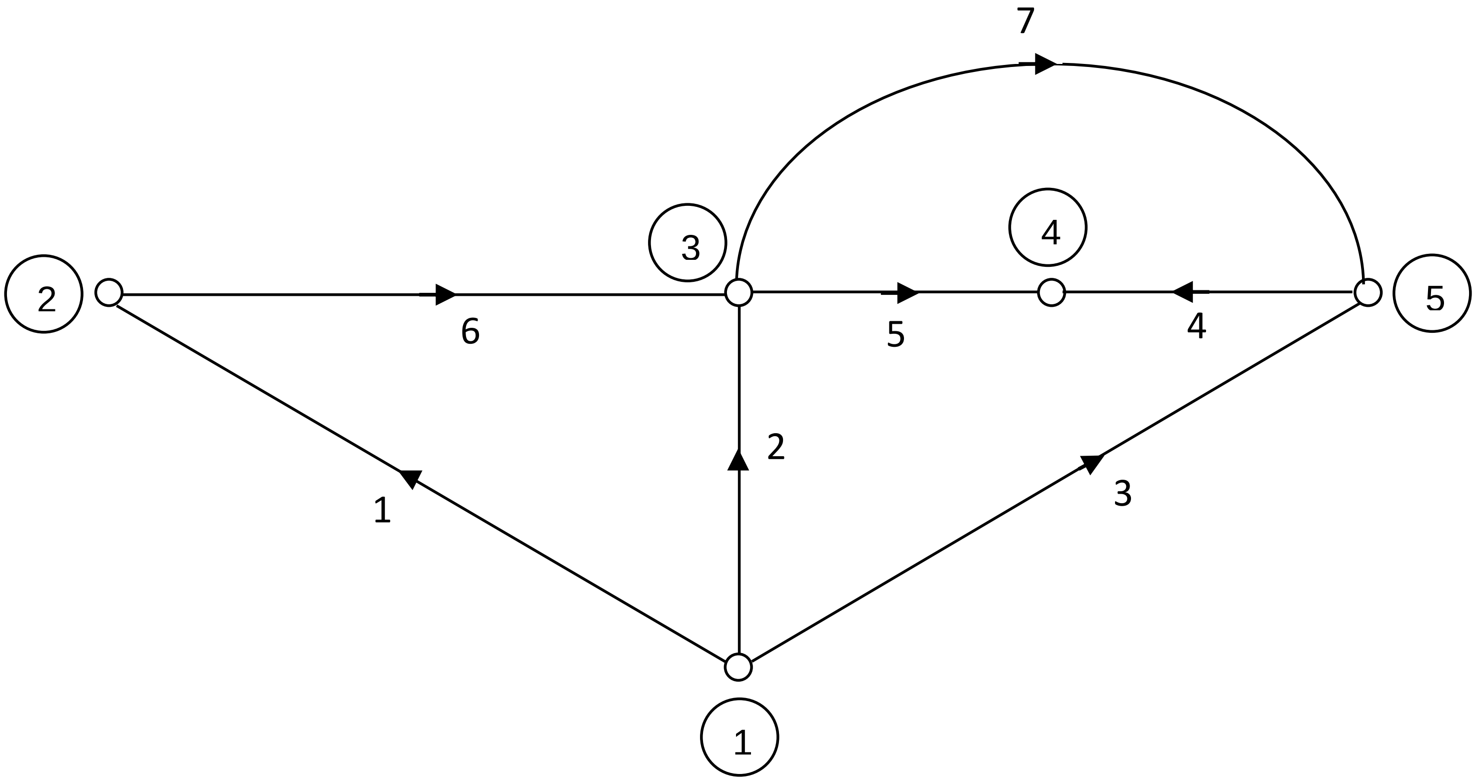
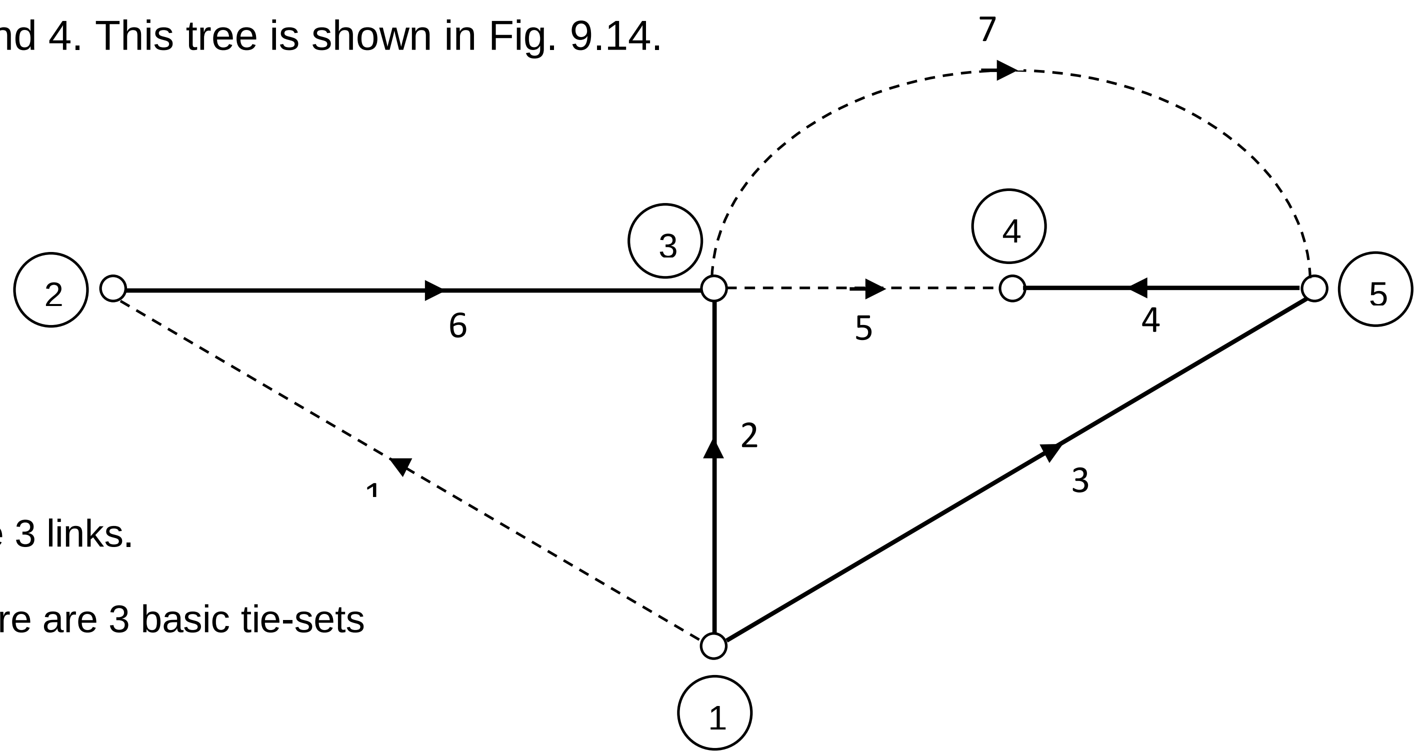


Fig. 9.13 Oriented connected graph.

A unique independent group of tie-sets may be chosen if each tie-set contains only one link. Such independent tie-sets are called BASIC TIE-SETS.

The number of basic tie-set is equal to the number of links. Orientation of a basic tie-set is chosen to be the same as that of its link. Basic tie-sets will be different for different tree selected.

Basic tie-sets depend on the tree selected. Let us consider a tree consisting elements 6, 2, 3 and 4. This tree is shown in Fig. 9.14.



There are 3 links.

Thus, there are 3 basic tie-sets

Fig. 9.14 A tree and co-tree of a connected graph.

Corresponding to one link, there will be one basic tie-set. Orientation of basic tie-set is the same as the orientation of the corresponding link. Fig. 9.15 shows the three basic tie-sets A, B and C corresponding to the tree marked in Fig. 9.14.

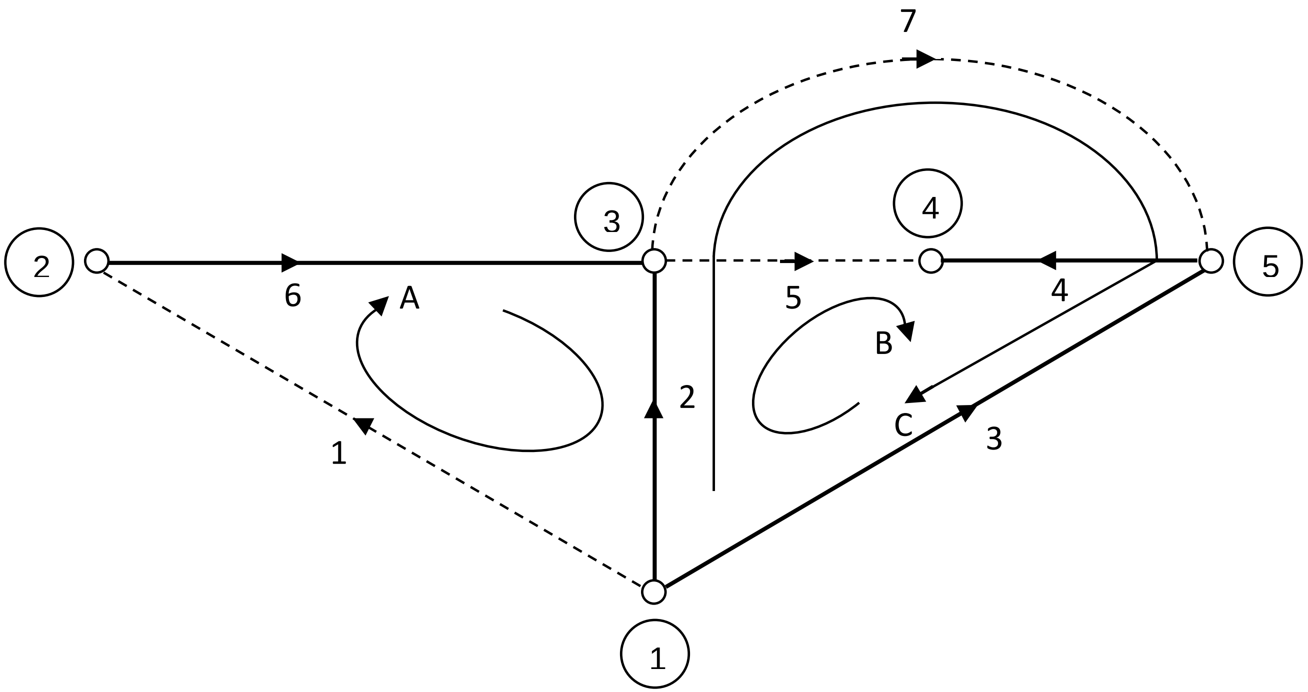


Fig. 9.15 Basic tie-sets.

It is possible to express the element currents in terms of the link currents. Let i_1, i_2, …., i_7 be the currents in elements 1, 2, ….., 7. . Let i_A, i_B, and i_C be the link currents in links A, B and C respectively. It is obvious that

$i_1 = i_A$ $\qquad$ $i_5 = i_B$ $\qquad$ $i_7 = i_C$

$i_1 = i_A$ **$i_5 = i_B$** **$i_7 = i_C$**

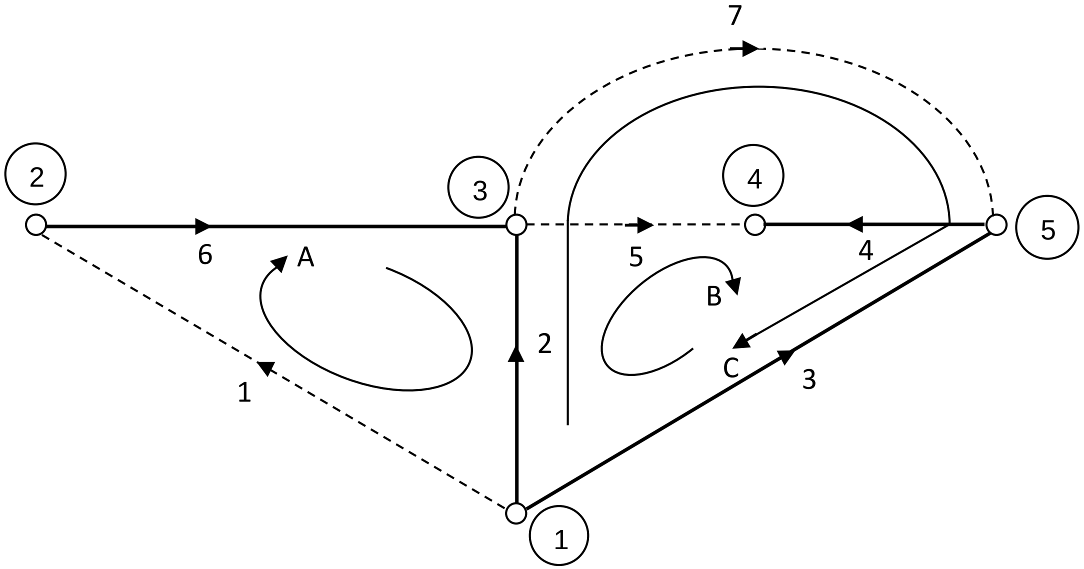
Using KCL, other element currents can be expressed in terms of the link currents as shown below.

$i_2 = - i_A + i_B + i_C$

$i_3 = - i_B - i_C$

$i_4 = - i_B$

$i_6 = i_A$



The above relations can be shown as in Table 9.3 which is referred as TIE-SET SCHEDULE. Here elements are arranged in the order 1, 5, 7, 2, 3, 4 and 6.

TABLE 9.3 Tie-set schedule.

Link currents	Element currents						
	i_1	i_5	i_7	i_2	i_3	i_4	i_6
i_A	1	0	0	-1	0	0	1
i_B	0	1	0	1	- 1	-1	0
i_C	0	0	1	1	-1	0	0

$i_2 = - i_A + i_B + i_C$

Example 9.1

Consider the oriented graph shown in Fig. 9.16.

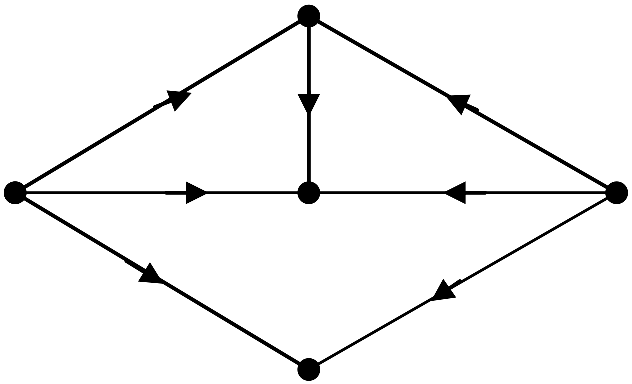


Fig. 9.16 Oriented graph for Example 9.1.

(a) How many elements and nodes are there?

(b) Number the elements and nodes. Draw two different trees and give the element numbers of tree branches and links.

Solution

(a) Given graph has 7 elements and 5 nodes.

(b) One way of numbering the elements and nodes is shown in Fig. 9.17.

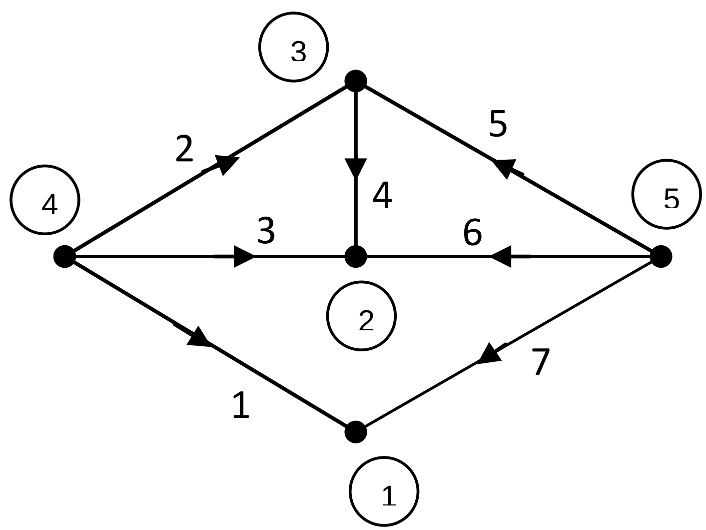


Fig. 9.17 Oriented graph with element and node numbers.

Two different trees are shown in Fig. 9.18.

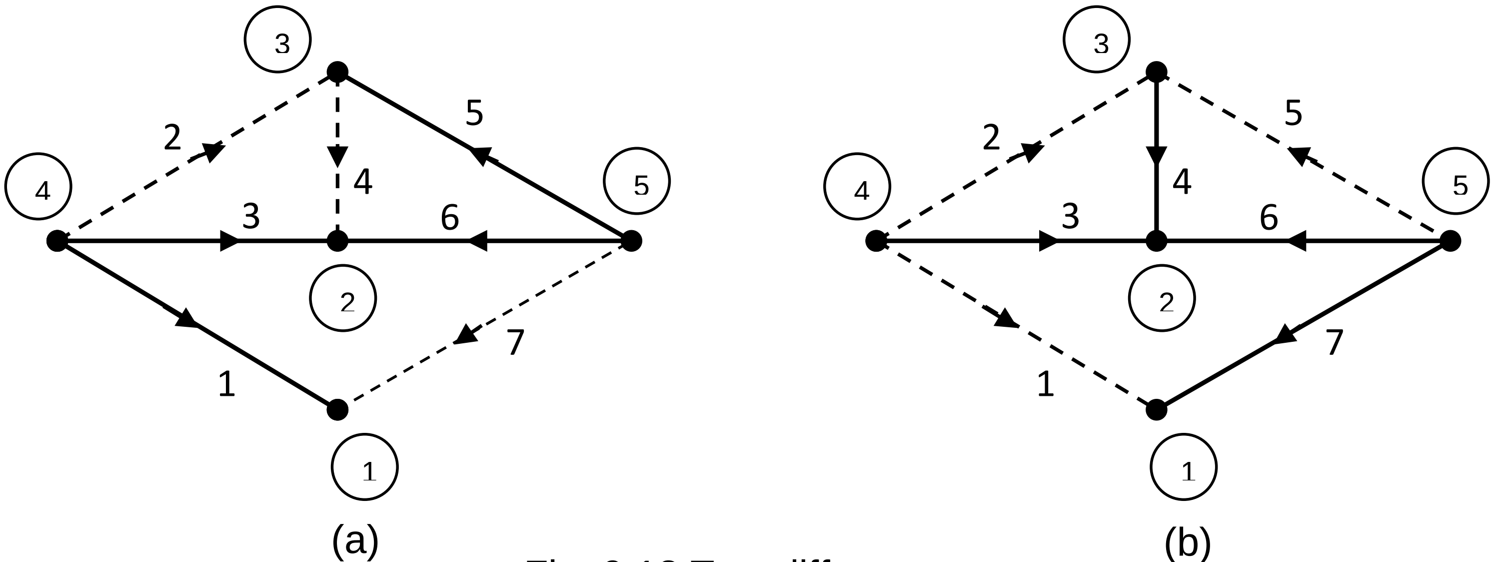


Fig. 9.18 Two different trees

For the tree shown in Fig. 9.18 (a):

Tree branches are 1, 3, 5 and 6

Links are 2, 4 and 7.

For the tree shown in Fig. 9.18 (b):

Tree branches are 3, 4, 6 and 7

Links are 1, 2 and 5.

Example 9.2

For the network graph shown in Fig. 9.19, taking elements 5, 6, 7 and 8 as tree branches obtain the cut-set and tie-set schedules.

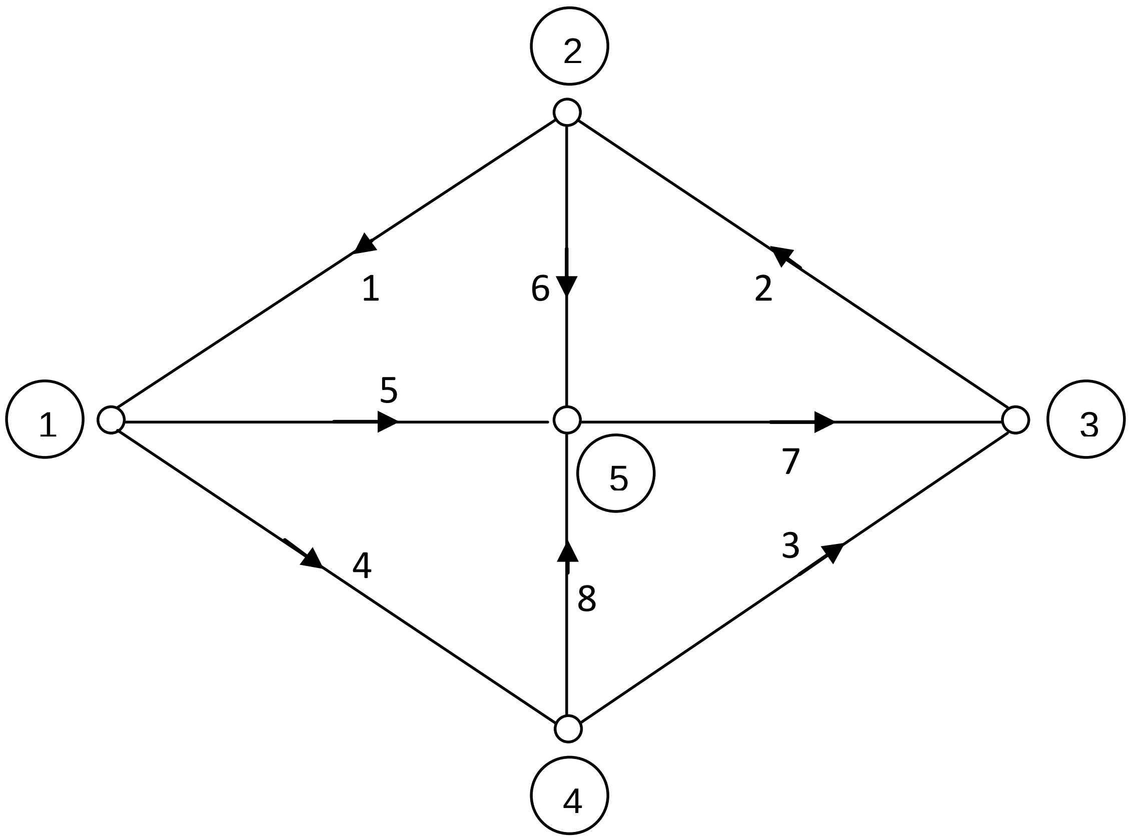


Fig. 9.19 Oriented graph for Example 9.2.

Solution

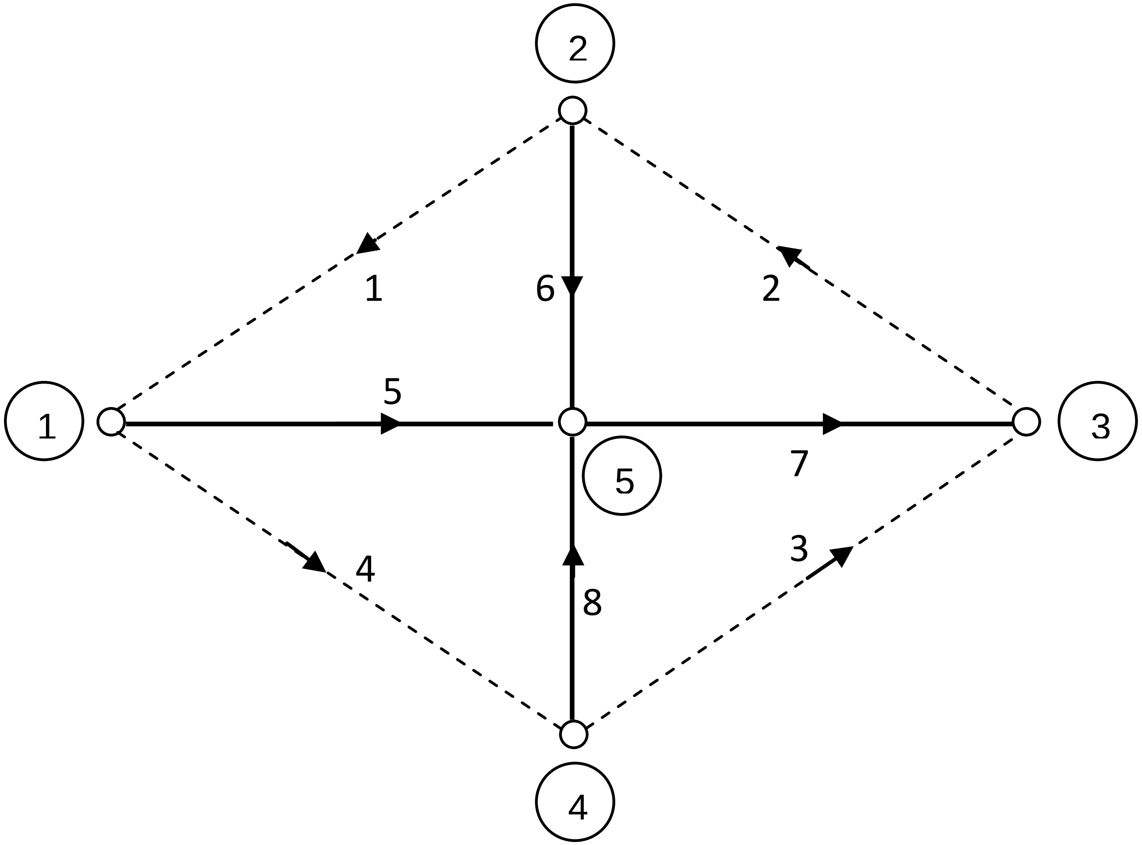


Fig. 9.20 Tree branches and links - Example 9.2.

Tree branches and the links are shown in Fig. 9.20

Let v_1, v_2, v_3, v_4, v_5, v_6, v_7 and v_8 be the element voltages and e_5, e_6, e_7 and e_8 be the tree branch voltages.

Element and tree branch voltages are shown in Fig. 10.21.

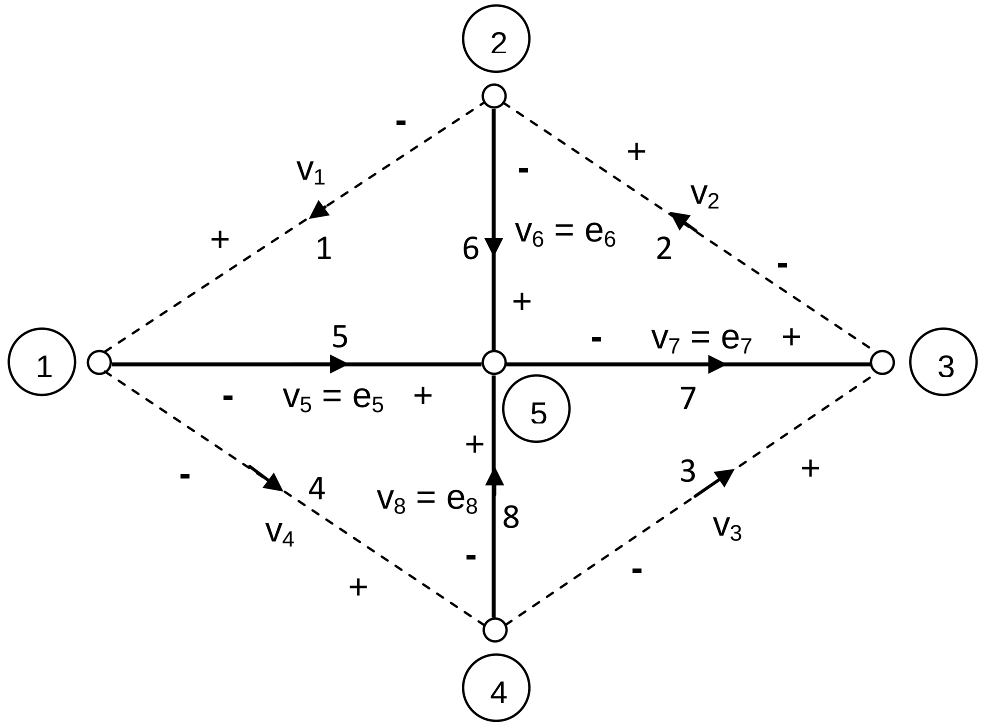


Fig. 10.21 Element and tree branch voltages - Example 10.2.

Referring to Fig. 10.21

$v_5 = e_5$ $\quad$ $v_1 = - e_5 + e_6$

$v_6 = e_6$ $\quad$ $v_2 = - e_6 - e_7$

$v_7 = e_7$ $\quad$ $v_3 = e_7 + e_8$

$v_8 = e_8$ $\quad$ $v_4 = - e_8 + e_5$

Referring to Fig. 9.21

$v_5 = e_5$ $\quad$ $v_1 = - e_5 + e_6$

$v_6 = e_6$ $\quad$ $v_2 = - e_6 - e_7$

$v_7 = e_7$ $\quad$ $v_3 = e_7 + e_8$

$v_8 = e_8$ $\quad$ $v_4 = - e_8 + e_5$

Cut-set schedule is

Tree branch voltages	Element voltages							
	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
e_5	-1	0	0	1	1	0	0	0
e_6	1	-1	0	0	0	1	0	0
e_7	0	-1	1	0	0	0	1	0
e_8	0	0	1	-1	0	0	0	1

Basic tie-sets are shown in Fig. 9.22.

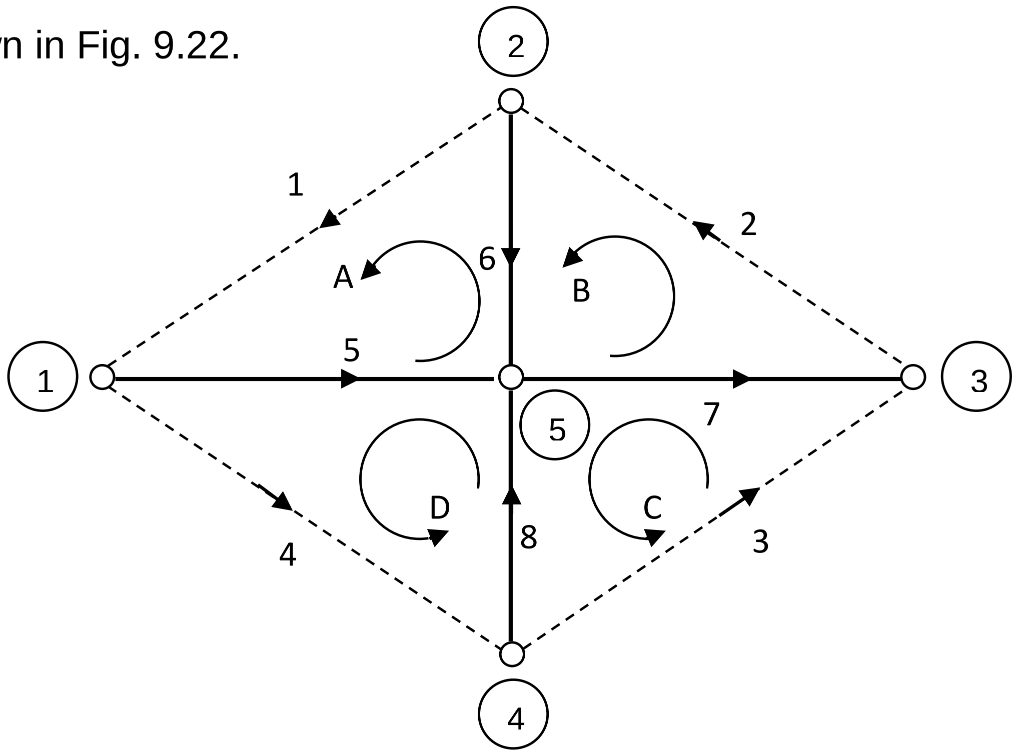


Fig. 9.22 Basic tie-sets - Example 9.2.

Let i_1, i_2, i_3, i_4, i_5, i_6, i_7 and i_8 be the element currents and i_A, i_B, i_C and i_D be the link currents.

Referring to Fig.10.22

$i_1 = i_A$

$i_2 = i_B$

$i_3 = i_C$

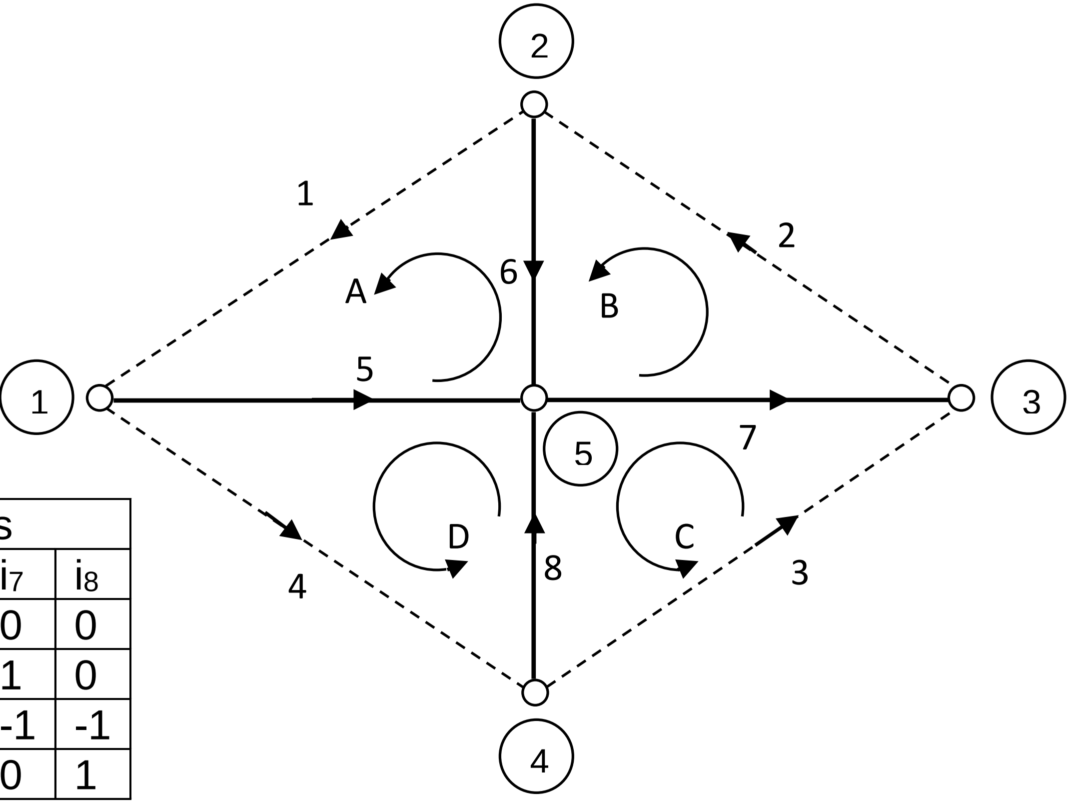
$i_4 = i_D$

$i_5 = i_A - i_D$

$i_6 = - i_A + i_B$

$i_7 = i_B - i_C$

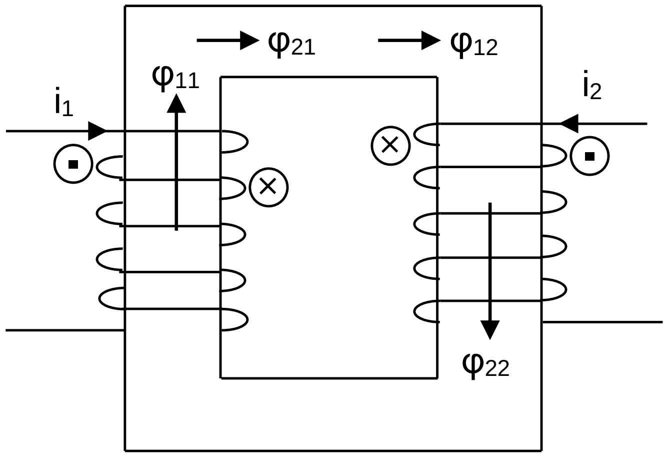
$i_8 = - i_C + i_D$

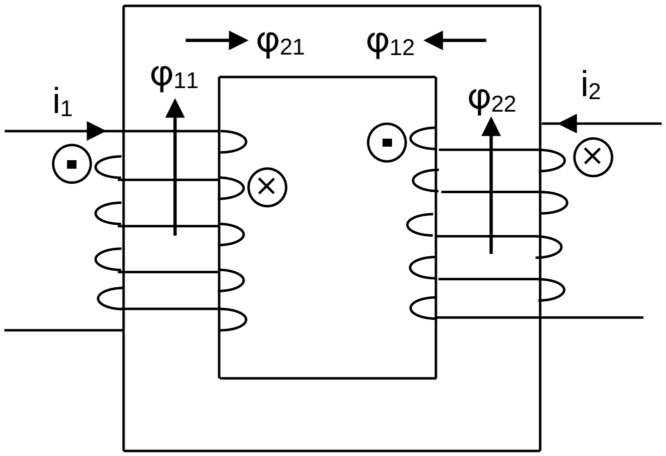


Tie-set schedule is

Link currents	Element currents							
	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
i_A	1	0	0	0	1	-1	0	0
i_B	0	1	0	0	0	1	1	0
i_C	0	0	1	0	0	0	-1	-1
i_D	0	0	0	1	-1	0	0	1

4.13.4 DOT CONVENTION - MAGNETICALLY COUPLED CIRCUIT





φ_{21} = Part of φ_{11} that links both the coils 1 and 2

φ_{12} = Part of φ_{22} that links both the coils 1 and 2

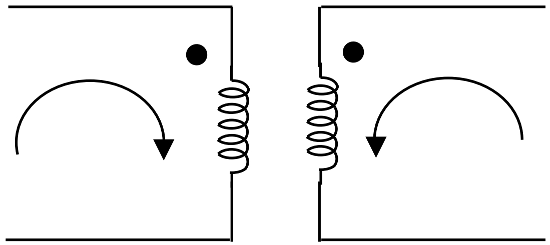
In the first case mutual fluxes φ_{21} and φ_{12} are aiding each other.

In the second case mutual fluxes φ_{21} and φ_{12} are opposing each other.

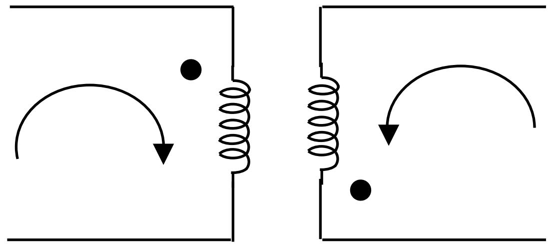
The sign of **mutual inductance** depends on the winding direction and the current direction. Dot convention is used to provide the necessary information.

In the dot convention, depending on the physical orientation of the coils, in each coil, a dot is placed at one end of the terminal, such that positive currents flowing into both the dots or positive currents flowing out of both the dots will result in mutual fluxes aiding each other. *This implies that the voltage due to mutual inductance, will have the same sign as the voltage due to self inductance*. Such a case is shown in Fig. 4.79 (a) along with the corresponding representation of coils in the circuit.

On the other hand, if *a* positive current in one coil flows into the dot and the positive current in the other coil flows out of the dot, then the mutual fluxes are opposing each other. *This implies that the voltage due to mutual inductance, have sign opposite of the voltage due to self inductance*. This case is shown in Fig. 4.79 (b) along with the corresponding representation of coils in the circuit.



(a)



(b)

Fig. 4.79 Sign for Mutual inductance.

While writing the voltage equations of each mesh, corresponding to each inductance, we need to consider two terms, one due to self inductance and the other due to mutual inductance.

Example 4.39

Write the mesh current equations for the circuit shown in Fig. 4.80. Take the initial voltage across the capacitor as zero.

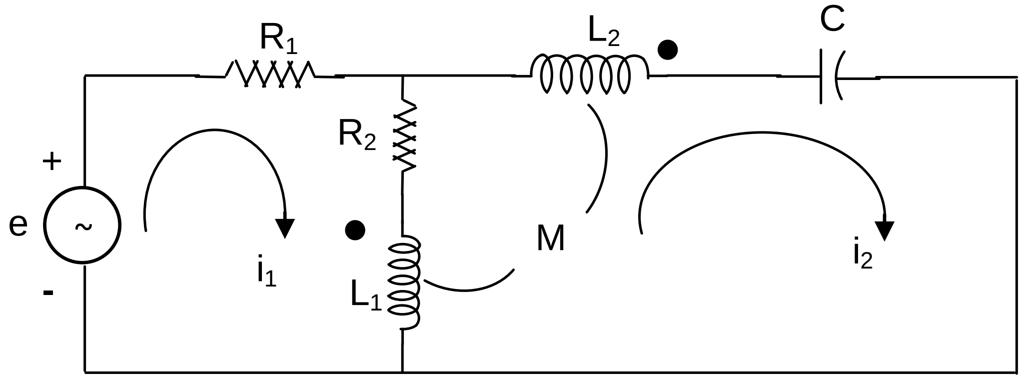


Fig. 4.80 Circuit for Example 4.39.

Solution:

The mesh current equations are:

$$R_1\, i_1 + R_2\,(i_1 - i_2) + L_1 \frac{d}{dt}(i_1 - i_2) - M\frac{di_2}{dt} = e$$

$$L_2 \frac{di_2}{dt} + M\frac{d}{dt}(i_2 - i_1) + \frac{1}{C}\int_0^t i_2\, dt' + L_1 \frac{d}{dt}(i_2 - i_1) + M\frac{di_2}{dt} + R_2\,(i_2 - i_1) = 0$$

Example 4.40 Two coils of self inductances L_1 and L_2 which are mutually coupled with mutual inductance M are connected in series. Show that, depending on the type of series connection, the equivalent inductance is either $L_1 + L_2 + 2M$ or $L_1 + L_2 - 2M$.

Solution: Two types of series connection of the coils are shown in Fig. 4.81 (a) and (b).

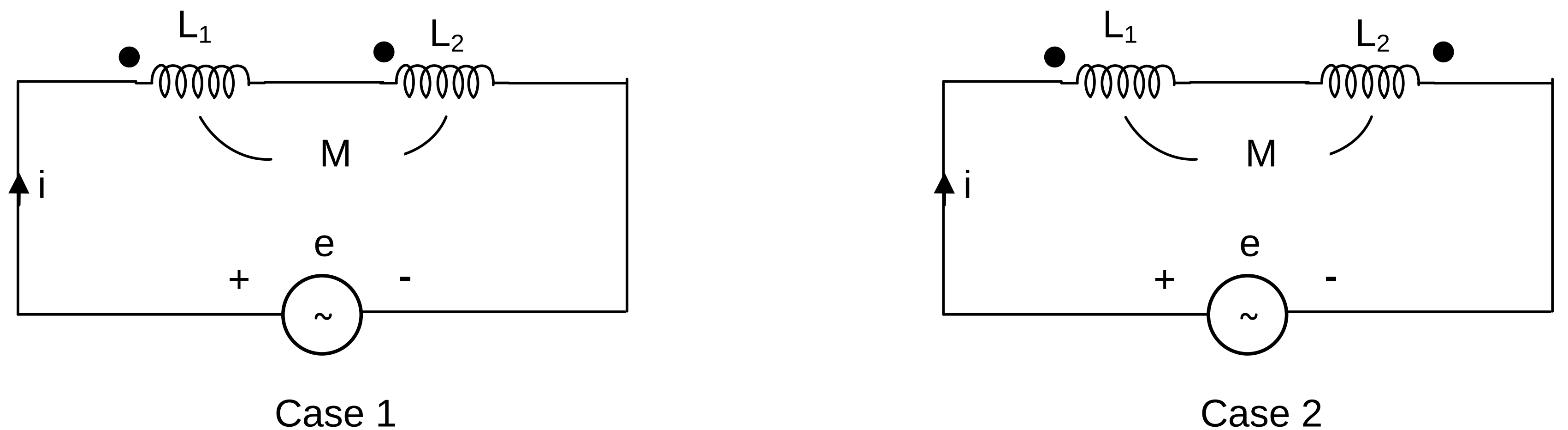
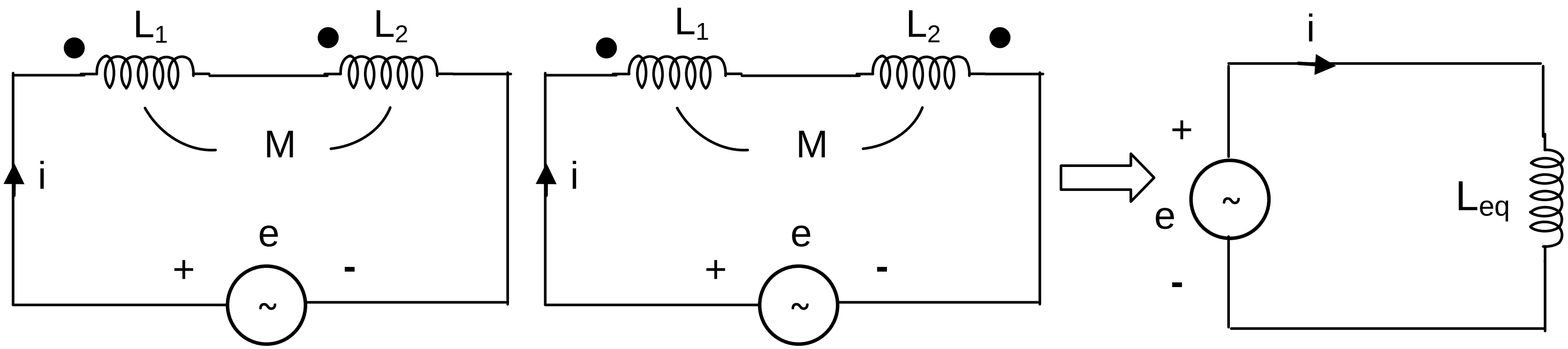


Fig. 4.81 Mutually coupled coils - Example 4.40.



Case 1 Case 2

Consider circuit shown in Case 1. Mesh current equation is

$$L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} = e \quad \text{i.e.} \quad (L_1 + L_2 + 2\,M) \frac{di}{dt} = e$$

Thus the equivalent inductance is obtained as $L_{eq} = L_1 + L_2 + 2\,M$

Consider circuit shown in Case 2. Its mesh current equation is

$$L_1 \frac{di}{dt} - M \frac{di}{dt} + L_2 \frac{di}{dt} - M \frac{di}{dt} = e \quad \text{i.e.} \quad (L_1 + L_2 - 2\,M) \frac{di}{dt} = e$$

Thus the equivalent inductance is obtained as $L_{eq} = L_1 + L_2 - 2\,M$

Example 4.42 Obtain the equivalent inductance of the two inductors connected as shown in Fig. 4.82. Take the coefficient of coupling k as 0.7.

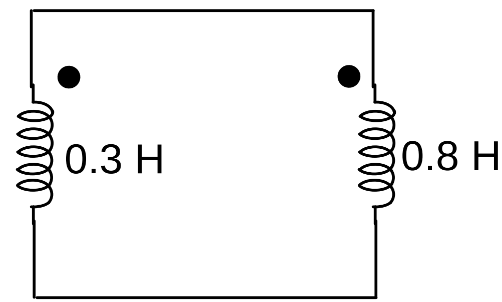


Fig. 4.82 Mutually coupled coils for Example 4.42.

Solution:

Let us apply a voltage e_1 across the parallel combination as shown in Fig. 4.83 (a) whose equivalent circuit is shown in Fig. 4.83 (b).

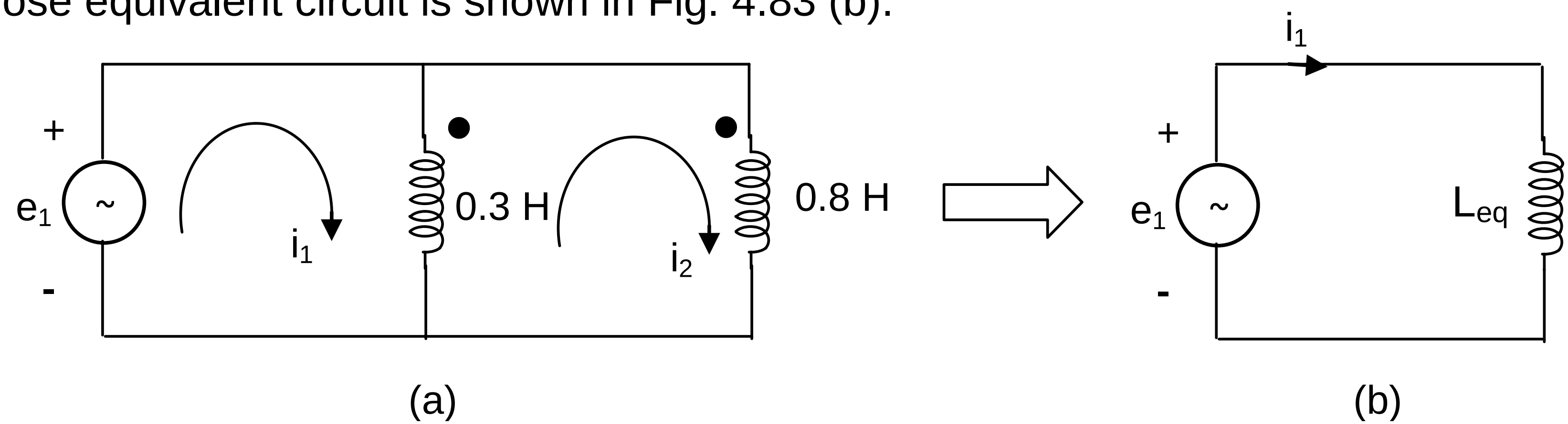
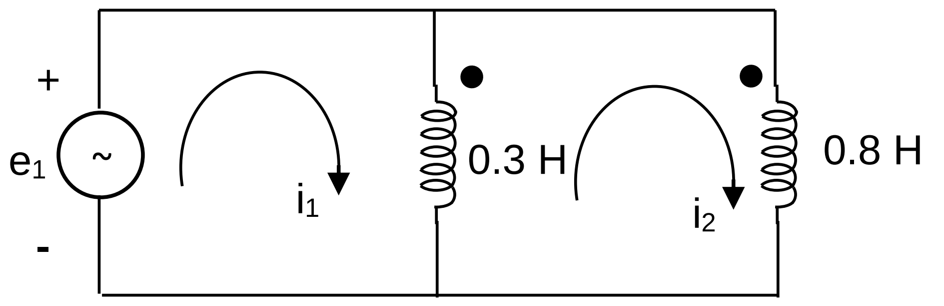


Fig. 4.83 Mutually coupled circuit - Example 4.42.

Mutual inductance $M = k\sqrt{L_1 L_2} = 0.7 \times \sqrt{0.3 \times 0.8} = 0.3429$ H



Mesh current equations are:

$$0.3\frac{d}{dt}(i_1 - i_2) + 0.3429\frac{di_2}{dt} = e_1$$

$$0.8\frac{di_2}{dt} - 0.3429\frac{d}{dt}(i_2 - i_1) + 0.3\frac{d}{dt}(i_2 - i_1) - 0.3429\frac{di_2}{dt} = 0$$

Let $\frac{di_1}{dt} = a$ and $\frac{di_2}{dt} = b$. Then

$$0.3\,(a - b) + 0.3429\,b = e_1$$

$$0.8\,b - 0.3429\,(b - a) + 0.3\,(b - a) - 0.3429\,b = 0$$

$$0.3\,(a - b) + 0.3429\,b = e_1$$

$$0.8\,b - 0.3429\,(b - a) + 0.3\,(b - a) - 0.3429\,b = 0$$

Arranging in matrix form, we get $\begin{bmatrix} 0.3 & 0.0429 \\ 0.0429 & 0.4142 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \end{bmatrix}$

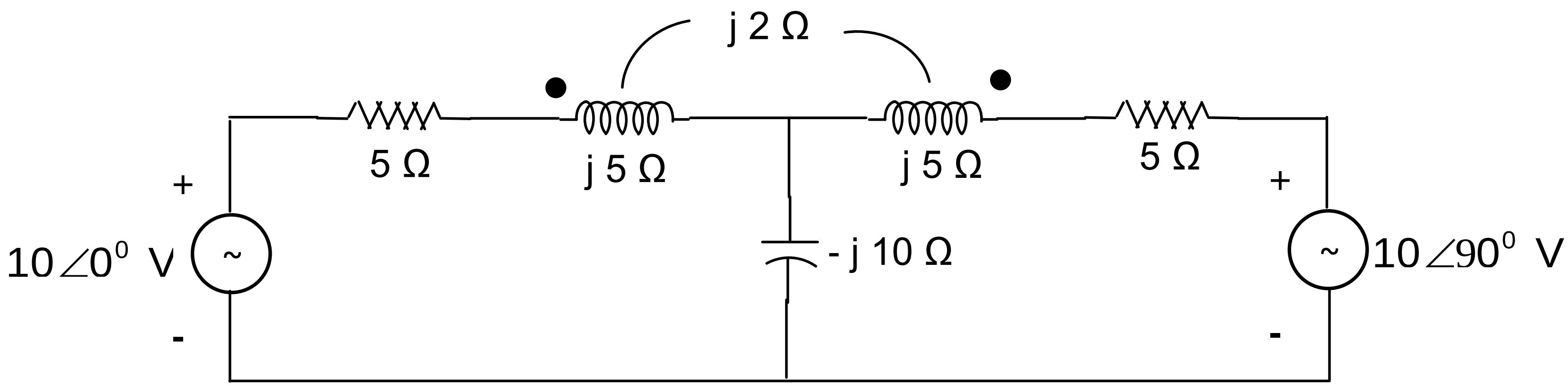
On solving for 'a' $\quad a = \dfrac{\begin{vmatrix} e_1 & 0.0429 \\ 0 & 0.4142 \end{vmatrix}}{0.1224} = \dfrac{0.4142}{0.1224}\, e_1 = 3.384\, e_1$

Thus $e_1 = \dfrac{a}{3.384} = 0.2955\, \dfrac{di_1}{dt}$

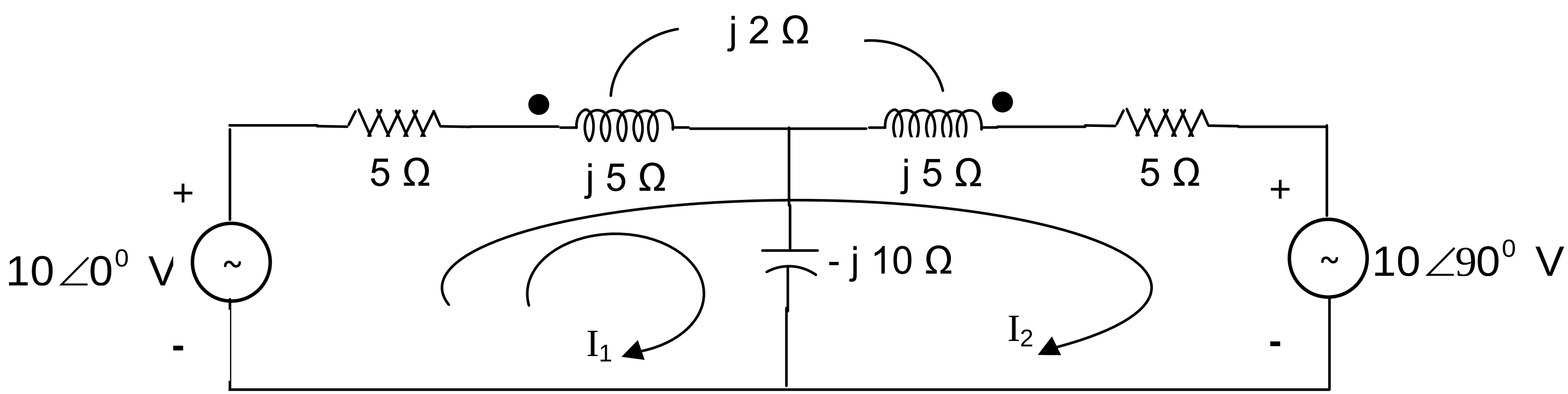
For the equivalent circuit $\quad e_1 = L_{eq}\, \dfrac{di_1}{dt} \qquad$ Therefore $L_{eq} = 0.2955$ H

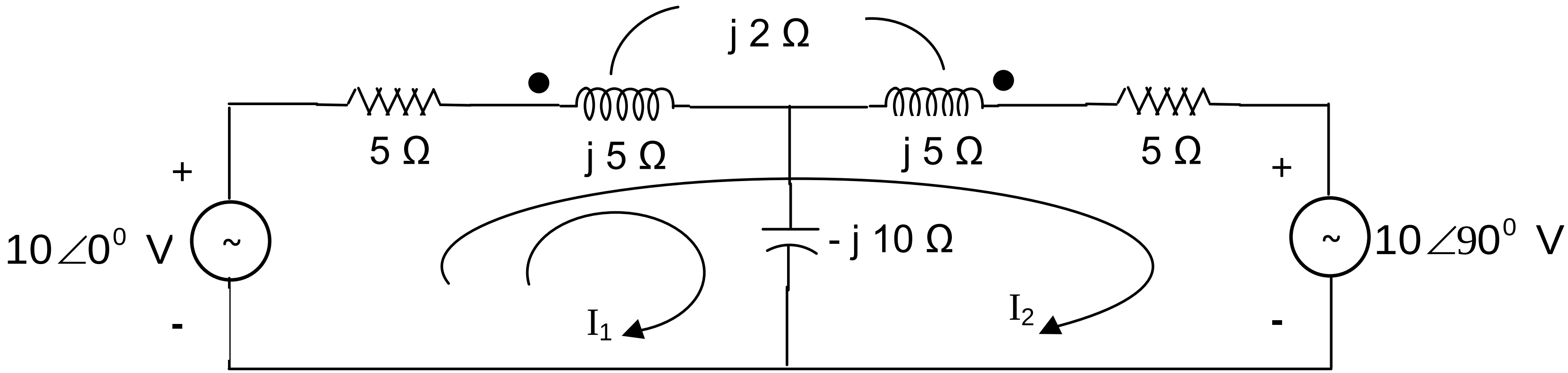
When two inductances L_1 and L_2 having mutual inductance of M are connected in parallel, it can be shown that the equivalent inductance is $\dfrac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$. depending on the sign for mutual inductance.

Example 4.43 Determine the voltage across the capacitor in the circuit shown.



Solution: Mesh currents are assumed as shown in Fig. below.





Mesh current equations are:

$$5\,(I_1 + I_2) + j\,5\,(I_1 + I_2) - j\,2\,I_2 - j\,10 I_1 = 10$$

$$5\,(I_1 + I_2) + j\,5\,(I_1 + I_2) - j\,2\,I_2 + j\,5\,I_2 - j\,2\,(I_1 + I_2) + 5\,I_2 = 10 - j\,10$$

Writing the above in matrix form, we get $\begin{bmatrix} 5 - j5 & 5 + j3 \\ 5 + j3 & 10 + j6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 - j10 \end{bmatrix}$

On solving $I_1 = \dfrac{20 + j\,80}{64 - j50} = 1.0153\ \angle 113.96^0\ \text{A}$

Voltage across the capacitor $= -\,j\,10\,I_1 = 10.153\ \angle 23.96^0\ \text{V}$

Example 4.72

Two coils of self inductances L_1 = 8 H and L_2 = 2 H have a coefficient of coupling 0.5.Find all possible values of equivalent inductances that may be obtained by connecting the coils in series or in parallel.

Solution:

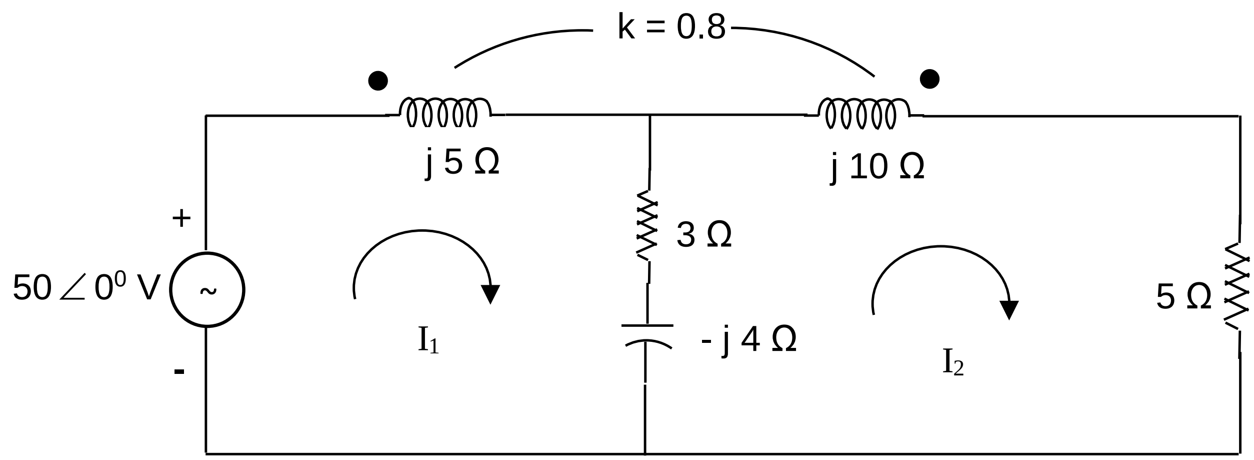
$M = 0.5\sqrt{8 \times 2} = 2\ H$

L_{eq} (series) = $L_1 + L_2 \pm 2M$ = 14 H or 6 H

$$L_{eq}\ (\text{parallel}) = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M} = \frac{12}{14} \text{ or } \frac{12}{6} = 0.8571\ H \text{ or } 2\ H$$

Example 4.73

In the coupled circuit shown, find the voltage across the 5 Ω resistor.



$M = k\sqrt{L_1 L_2}$; Multiplying by ω, $X_m = k\sqrt{X_{L1} X_{L2}} = j\,0.8\sqrt{5 \times 10} = j\,5.65685\ \Omega$

Taking clockwise mesh currents

$j\,5\,I_1 - j\,5.65685\,I_2 + (3 - j\,4)(I_1 - I_2) = 50$

$j\,10\,I_2 - j\,5.65685\,I_1 + 5\,I_2 + (3 - j\,4)(I_2 - I_1) = 0$

$$\begin{bmatrix} 3+j1 & -3-j1.65685 \\ -3-j1.65685 & 8+j6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$\Delta = 11.7479 + j\,16.054$; $\Delta_2 = 150 + j\,82.925$

Current $I_2 = \Delta_2 / \Delta = 8.6157 \angle -24.87^0$ A Voltage $V_{5\Omega} = 5\,I_2 = 43.0785 \angle -24.87^0$ V

TUNED CIRCUITS

There are many practical applications of coupled circuits, especially in the communication system. Capacitors are associated with the coupling inductors at the input terminals and / or output terminals. Such circuits are called tuned circuits. Depending on the number of resonance circuits present we have single tuned and double tuned circuits.

4.15.1 SINGLE TUNED CIRCUIT

Consider the single tuned circuit shown in Fig. 4.91.

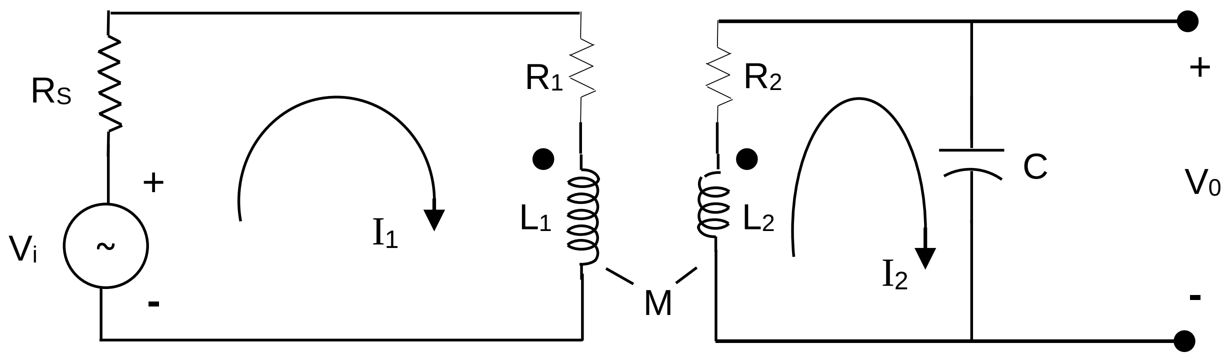
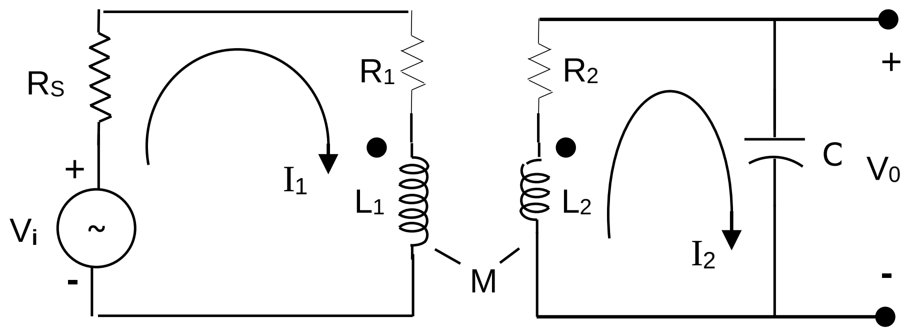


Fig. 4.91 Single tuned circuit.

A parallel resonance circuit is also called as tank circuit. The tank circuit in the secondary side is inductively coupled to a coil in the primary side. The primary is excited by a variable frequency voltage source V_i. Let R_S be the source resistance and R_1 and R_2 be the resistances of the coils 1 and 2 respectively. Also let L_1 and L_2 be the self-inductances of the coils 1 and 2 respectively.

Let $R_S >> R_1$ and $R_S >> \omega L_1$. Then, $R_S + R_1 + j\omega L_1 \approx R_S$.

Two mesh current equations are



$$R_S I_1 - j\omega M I_2 = V_i \tag{4.188}$$

$$\left(R_2 + j\omega L_2 - j\frac{1}{\omega C}\right) I_2 - j\omega M I_1 = 0 \tag{4.189}$$

Matrix form of above two equations is

$$\begin{bmatrix} R_S & -j\omega M \\ -j\omega M & (R_2 + j\omega L_2 - \frac{j}{\omega C}) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_i \\ 0 \end{bmatrix} \tag{4.190}$$

We like to obtain an expression for the output voltage V_0. This voltage depends on the current I_2. Solving for the current I_2 we get

$$I_2 = \frac{\begin{vmatrix} R_2 & V_i \\ -j\omega M & 0 \end{vmatrix}}{\begin{vmatrix} R_S & -j\omega M \\ -j\omega M & (R_2 + j\omega L_2 - \frac{j}{\omega C}) \end{vmatrix}} = \frac{j\omega M V_i}{R_S (R_2 + j\omega L_2 - \frac{j}{\omega C}) + \omega^2 M^2} \tag{4.191}$$

Output voltage $V_0 = (-\frac{j}{\omega C})\, I_2$ (4.192)

Thus output voltage $V_0 = \dfrac{\frac{M}{C} V_i}{R_S(R_2 + j\omega L_2 - \frac{j}{\omega C}) + \omega^2 M^2}$ (4.193)

Voltage amplification factor, also called voltage transfer function, A is defined as

$$A = \frac{V_0}{V_i} \tag{4.194}$$

Thus output voltage $V_0 = \dfrac{\frac{M}{C} V_i}{R_S(R_2 + j\omega L_2 - \frac{j}{\omega C}) + \omega^2 M^2}$ (4.193)

Thus $A = \dfrac{\frac{M}{C}}{R_S(R_2 + j\omega L_2 - \frac{j}{\omega C}) + \omega^2 M^2}$ (4.195)

When the secondary side is tuned, the value of the angular frequency ω is set to ω_r such that

$$\omega_r L_2 = \frac{1}{\omega_r C} \quad (4.196)$$

Then, the output voltage V_0 and the amplification factor A are given by

$$V_0 = \frac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^2 M^2} \quad (4.197)$$

$$A = \frac{\frac{M}{C}}{R_S R_2 + \omega_r^2 M^2} \quad (4.198)$$

$$V_0 = \frac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^2 M^2} \qquad (4.197)$$

It is to be noted from eq. (4.197), that the output voltage depends on the mutual inductance M. The value of M can be adjusted to get maximum output voltage. For this purpose, dividing the numerator and denominator by M, the eq. (4.197) is written as

$$V_0 = \frac{\frac{V_i}{C}}{\frac{R_S R_2}{M} + \omega_r^2 M} = \frac{P}{Q} \qquad (4.199)$$

For V_0 to be maximum, Q must be minimum i.e. $\frac{dQ}{dM} = 0$

i.e. $-\frac{R_S R_2}{M^2} + \omega_r^2 = 0$ i.e. $M^2 = \frac{R_S R_2}{\omega_r^2}$

Thus, the optimal value of the mutual inductance M_{opt} is obtained as

$$M_{opt} = \frac{\sqrt{R_S R_2}}{\omega_r} \qquad (4.200)$$

$$V_0 = \frac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^2 M^2} \qquad (4.197) \qquad\qquad M_{opt} = \frac{\sqrt{R_S R_2}}{\omega_r} \qquad (4.200)$$

When the above optimal value is substituted in eq. (4.197), the maximum output voltage $V_{0\,max}$ is obtained as

$$V_{0\,max} = \frac{\frac{\sqrt{R_S R_2}}{\omega_r C} V_i}{R_S R_2 + \omega_r^2 \frac{R_S R_2}{\omega_r^2}} = \frac{\frac{\sqrt{R_S R_2}}{\omega_r C} V_i}{2 R_S R_2} = \frac{V_i}{2 \omega_r C \sqrt{R_S R_2}} \qquad (4.201)$$

Corresponding voltage amplification factor, A_{max} is given by

$$A_{max} = \frac{1}{2 \omega_r C \sqrt{R_S R_2}} \qquad (4.202)$$

We know that mutual inductance $M = k \sqrt{L_1 L_2}$. Once the value of M_{opt} is known, this can be achieved by adjusting the value of the coefficient of coupling k.

$$M_{opt} = \frac{\sqrt{R_S\ R_2}}{\omega_r} \qquad (4.200)$$

The value of the coefficient of coupling of the tuned circuit for getting maximum output voltage at the resonance frequency is called as CRITICAL COUPLING. Thus critical coupling k_{cr} is given by

$$k_{cr} = \frac{M_{opt}}{\sqrt{L_1\ L_2}} = \frac{\sqrt{R_S\ R_2}}{\omega_r \sqrt{L_1\ L_2}} \qquad (4.203)$$

It is to be noted that sometime the value of M cannot be brought to $\frac{\sqrt{R_S\ R_2}}{\omega_r}$, as its value cannot exceed $\sqrt{L_1\ L_2}$ taking k = 1. In such case, the value of M shall be restricted to $\sqrt{L_1\ L_2}$.

Whenever $M > M_{opt}$ or $M < M_{opt}$, the output voltage V_0 will be less than the maximum output voltage $V_{0\ max}$. Similarly, whenever $k > k_{cr}$ or $k < k_{cr}$ the output voltage V_0 will be less than $V_{0\ max}$.

Example 4.47

Consider the single tuned circuit shown in Fig. 4.92. It has L_1 = 25 µH, L_2 = 100 µH and coefficient of coupling as 0.4. Assume $R_S >> \omega_r L_1$. Determine (i) angular resonance frequency (ii) output voltage at resonance (iii) maximum output voltage and the corresponding value of M (iv) critical coupling and (v) output voltage when k = 0.8.

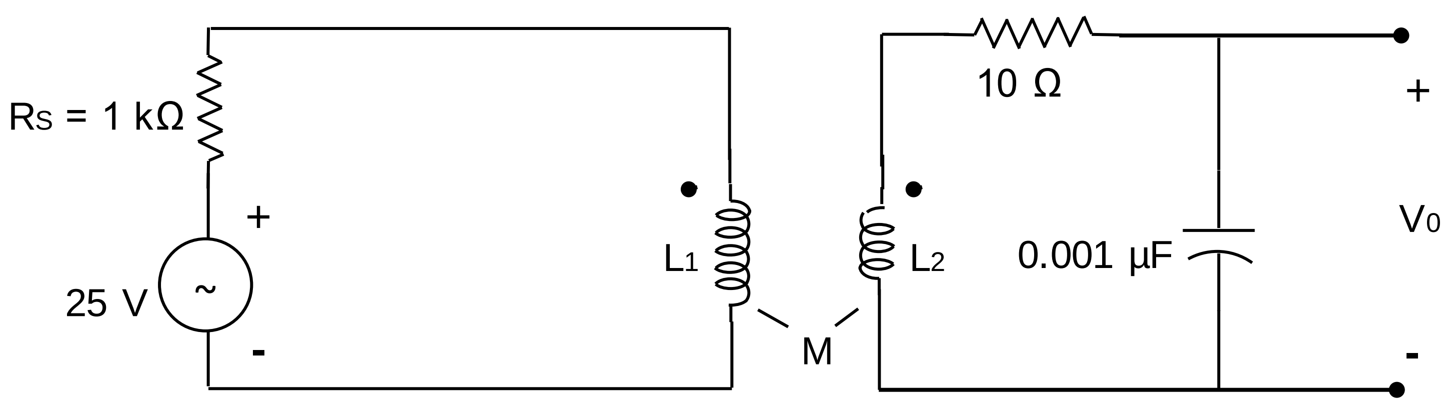


Fig. 4.92 Circuit for Example 4.47

Solution:

i) At resonance $\omega_r L_2 = \dfrac{1}{\omega_r C}$

Angular resonance frequency $\omega_r = \dfrac{1}{\sqrt{L_2 C}} = \dfrac{10^6}{\sqrt{100 \times 0.001}} = 3.1623 \times 10^6$ rad. / sec.

ii) Mutual inductance $M = k\sqrt{L_1 L_2} = 0.4\sqrt{25 \times 100}$ μH = 20 μH

Output voltage $V_0 = \dfrac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^{\,2} M^2} = \dfrac{20000 \times 25}{10 \times 10^3 + 4000} = 35.7143$ V

iii) $M_{opt} = \dfrac{\sqrt{R_S R_2}}{\omega_r} = \dfrac{\sqrt{10^4}}{3.1623 \times 10^6}$ H = 31.6226 μH

Value of M with k = 1, is $\sqrt{25 \times 100}$ μH = 50 μH

Therefore, M_{opt} = 31.6226 μH is feasible.

Maximum output voltage $V_{0\,max} = \dfrac{V_i}{2\,\omega_r\,C\sqrt{R_S R_2}}$

$$= \frac{25}{2 \times 3.1623 \times 10^{6} \times 0.001 \times 10^{-6} \times 100} = 39.5282\ V$$

iv) Critical coupling $k_{cr} = \dfrac{M_{opt}}{\sqrt{L_1 L_2}} = \dfrac{31.6226 \times 10^{-6}}{50 \times 10^{-6}} = 0.63245$

v) When the coefficient of coupling is 0.8

Mutual coupling $M = 0.8\sqrt{25 \times 100}\ \mu H = 40\ \mu H$

Output voltage $V_0 = \dfrac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^{\,2} M^2} = \dfrac{40000 \times 25}{10 \times 10^{3} + 16000} = 23.0769\ V$

Note: In this case R_S = 1000 Ω and $\omega_r L_1 = 3.1623 \times 10^{6} \times 25 \times 10^{-6} = 79.0575\ \Omega$ and hence the assumption of $R_S >> \omega L_1$ is justified.

Example 4.48

Consider the single tuned circuit shown in Fig. 4.93. It has L_1 = 12.5 µH, L_2 = 50 µH and coefficient of coupling as 0.6. Assume $R_S >> \omega_r L_1$. Determine (i) angular resonance frequency (ii) output voltage at resonance (iii) maximum output voltage and the corresponding value of M.

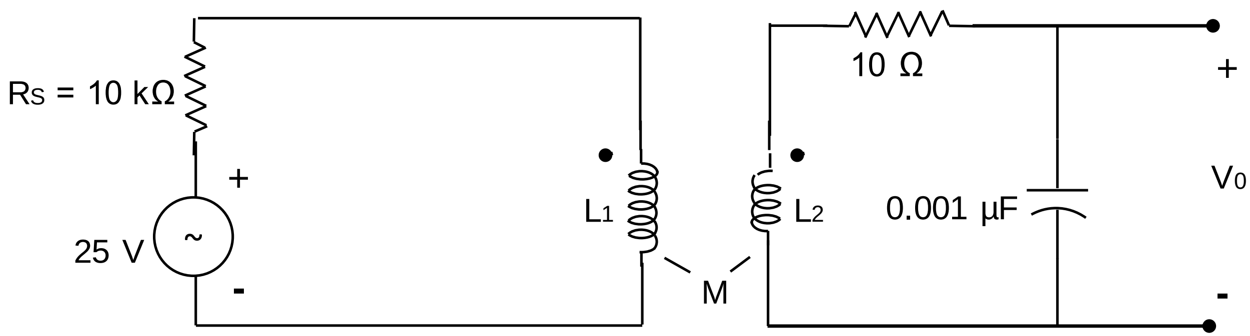


Fig. 4.93 Circuit for Example 4.48.

Solution:

i) Angular resonance frequency $\omega_r = \dfrac{1}{\sqrt{L_2\ C}} = \dfrac{10^6}{\sqrt{50 \text{ x } 0.001}}$ = 4.4721 x 10^6 rad. / sec.

ii) Mutual inductance $M = k\ \sqrt{L_1\ L_2} = 0.6\ \sqrt{12.5 \text{ x } 50}$ µH = 15 µH

Output voltage $V_0 = \dfrac{\dfrac{M}{C} V_i}{R_S R_2 + \omega_r^{\ 2}\ M^2} = \dfrac{15000 \text{ x } 25}{10 \text{ x } 10^4 + 4500}$ =3.5885 V

iii) $M_{opt} = \dfrac{\sqrt{R_S\ R_2}}{\omega_r} = \dfrac{\sqrt{10^5}}{4.4721 x 10^6} H = 70.7112\ \mu H$

Value of M with k = 1, is $\sqrt{12.5 x 50}\ \mu H = 25\ \mu H$

Value of M is limited to 25 μH

Output voltage $V_0 = \dfrac{\frac{M}{C} V_i}{R_S R_2 + \omega_r^{\ 2} M^2} = \dfrac{25000 x 25}{10 x 10^4 + 12500} = 5.5555\ V$

Corresponding value of M = 25 μH