SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 641 035

DEPARTMENT OF MATHEMATICS

23MAT203-PROBABILITY AND RANDOM PROCESSES

Unit IV

1. State and prove Wiener -Khinchine theorem.

2. Define spectral density of a stationary random process $X(t)$. Prove that for a real random process X(t), the power spectral density is an even function.

3. Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1 - |\tau| & for\ |\tau| \leq 1 \\ 0, & elsewhere \end{cases}$.

4. Find the power spectral density function whose auto correlation is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0 \tau)$.
5. A random process {X(t)} is given by $X(t) = Acospt + Bsinpt$, where A and B are independent random variables such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$. Find the power spectral density of the process.

6. The auto correlation function of a random process is given by $R(\tau) = \begin{cases} \lambda^2 & |\tau| > \varepsilon \\ \lambda^2 + \frac{\lambda}{\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right) & |\tau| \leq \varepsilon \end{cases}$. Find the power spectral density of the process.

7. The auto correlation function of a WSS process with autocorrelation function $R(\tau) = \alpha^2 e^{-2\lambda\sqrt{|\tau|}}$, determine the power spectral density of the process.

8. Determine the power spectral density of a WSS process X(t) which has an auto correlation $R_{XX}(\tau) = \begin{cases} \left[1 - \frac{|\tau|}{T}\right], -T \leq t \leq T \\ 0 \quad otherwise \end{cases}$. Show that the process is mean ergodic.

9. The power spectral density function of a zero mean WSS process {X(t)} is given by $S(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & otherwise \end{cases}$. Find $R(\tau)$. Show that X(t) and $X(t + \frac{\pi}{\omega_0})$ are uncorrelated.

10. Consider two random processes $X(t) = 3\ \cos(\omega t + \theta)$ and $Y(t) = 2\cos(\omega t + \varphi)$ where $\varphi = \theta - \frac{\pi}{2}$ & θ is uniformly distributed random variable over $(0,2\ \pi)$ and also Verify that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$

11. Two jointly WSS processes X(t) and Y(t) have Cross-correlation$R_{XY}(\tau) = 5e^{-2|\tau|}\sin(\ 3\tau)$.(GATE ECE 2025)

12. Two jointly WSS processes X(t) and Y(t) have cross-correlation $R_{XY}(\tau) = 5e^{-3|\tau|}$. Verify if $R_{XY}(\tau)$ satisfies any two properties of cross-correlation.

(Gate ECE 2025)

13. The power spectral density of a WSS process is given by

$$S_{XX}(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), & |\omega| \leq a \\ 0, & \text{otherwise} \end{cases}$$

. Find the auto correlation function.

14. Compute the auto-correlation function of a random process $X(t) = A\cos(\omega t + \theta)$, where A and ω are constants and θ is uniformly distributed in $[0,2\pi]$. (Gate ECE 2023)

15. Two sensors measure signals $X(t)$ and $Y(t)$ with cross-correlation $R_{XX}(\tau) = e^{-|\tau|}$. Determine if the signals are jointly wide-sense stationary (WSS). (Infosys)

Unit V

1. Prove that, if the input to a time – invariant, stable linear system is a WSS process, then the output will also be a WSS process.

2. Prove that (i) $R_{XY}((\tau) = R_{XX}(\tau) * h(\tau)$ (ii) $R_{YY}((\tau) = R_{XY}(\tau) * h(-\tau)$ (iii) $S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$ (iv) $S_{YY}(\omega) = S_{XY}(\omega)H^{*}(\omega)$ (v) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^2$

3. X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary Random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$.

4. An LTI system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{YY}(\tau)$ corresponding to an input X(t).

5. Assume a random process X(t) is given as input to a system with transfer function $H(\omega)$ =1 for $-\omega_0 < \omega < \omega_0$.If the autocorrelation function of the input process $\frac{N_0}{2}\ \delta(r)$. Point out the autocorrelation function of the output process.

6. Let X(t) be a stationary process with mean 0 and autocorrelation function $e^{-2|c|}$. If X(t) is the input to a linear system and Y(t) is the output process ,Calculate (i) E[Y(t)] (ii) S_{YY}(□) and (iii) $R_{YY}(|r|)$, if the system function $H(\omega) = \frac{1}{\omega+2i}$.

7. A wide sense stationary random process {X(t)} with autocorrelation $R_{XX}(\tau) = Ae^{-a|\tau|}$, where A and a are real positive constants, is applied to the input of a linear transmission input system with impulse response $h(t) = e^{-bt}\ u(t)$ Where b is a real positive constant. Give the power spectral density of the output Y(t) of the system.

8. A linear system is described by the impulse response $h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}u(t)$. Assume an input process whose autocorrelation function is $B\delta(\tau)$.Point out the mean and
autocorrelation function of the output function.

9. If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & |\omega - \omega_0| < \omega_B \\ 0, & elsewhere \end{cases}$$

Identify the auto correlation function of N(t).

10. Let X(t) be the input voltage to a circuit system and Y(t) be the output voltage.If X(t)is a stationary random process with mean 0 and autocorrelationfunction $R_{XX}(\tau) = Ae^{-a|\tau|}$.

Identify
(i) E[Y(t)]
(ii) S_{XX}(□) and
The spectral density of Y(t) if the power transfer function

$$H(\omega) = \frac{R}{R + iL\omega}$$

11. A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t}, t \geq 0$. The auto correlation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process Y(t).

12. Find the power spectral density of a random telegraph signal.

13. If X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_y and power spectrum $S_{yy}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega + 2i}$.

14. If X(t) is the input and Y(t) is the output of the system. The autocorrelation of X(t) is $R_{XX}(\tau) = 3.\delta(\tau)$. Find the power spectral

density, autocorrelation function and mean square value of the output Y(t) with $H(\omega) = \frac{1}{6+j\omega}$

15. Analyse the mean of the output of a linear system is given by $\mu_Y = H(0)\mu_X$, where X(t) is a WSS.