SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 641 035

DEPARTMENT OF MATHEMATICS

23MAT203-PROBABILITY AND RANDOM PROCESSES

Unit V

1. Prove that, if the input to a time – invariant, stable linear system is a WSS process, then the output will also be a WSS process.

2. Prove that (i) $R_{XY}((\tau) = R_{XX}(\tau) * h(\tau)$ (ii) $R_{YY}((\tau) = R_{XY}(\tau) * h(-\tau)$ (iii) $S_{XY}(\omega) = S_{XX}(\omega)H(\omega)$ (iv) $S_{YY}(\omega) = S_{XY}(\omega)H^{*}(\omega)$ (v) $S_{YY}(\omega) = S_{XX}(\omega)|H(\omega)|^{2}$

3. X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary Random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$.

4. An LTI system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{YY}(\tau)$ corresponding to an input X(t).

5. Assume a random process X(t) is given as input to a system with transfer function $H(\omega)$ =1 for $-\omega_0 < \omega < \omega_0$.If the autocorrelation function of the input process $\frac{N_0}{2}\ \delta(r)$. Point out the autocorrelation function of the output process.

6. Let X(t) be a stationary process with mean 0 and autocorrelation function $e^{-2|c|}$. If X(t) is the input to a linear system and Y(t) is the output process ,Calculate (i) E[Y(t)] (ii) S_{YY}(□) and (iii) $R_{YY}(|r|)$, if the system function $H(\omega) = \frac{1}{\omega+2i}$.

7. Let X(t) be the input voltage to a circuit system and Y(t) be the output voltage.If X(t)is a stationary random process with mean 0 and autocorrelationfunction $R_{XX}(\tau) = e^{-\alpha|\tau|}$. .
   i) E[Y(t)]
   ii) S_{XX}(□) and
   The spectral density of Y(t) if the power transfer function $H(\omega) =$

$\frac{R}{R+iL\omega}$

8. A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t}, t \geq 0$. The auto correlation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process Y(t).

9. Find the power spectral density of a random telegraph signal.

10. If X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_y and power spectrum $S_{yy}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega+2i}$.
11. Two random processes X(t) and Y(t) are jointly wide-sense stationary. Prove that the cross-spectral density $S_{XY}(f)$ is the Fourier transform of the cross-correlation function $R_{XY}(\tau)$. (Gate ECE 2024)

12. Compute the auto-correlation function of a random process for an LTI system with $H(\omega) = \frac{1}{1+j\omega}$, find output PSD if input X(t) has $R_{XX}(\tau) = e^{-|\tau|}$.(Gate ECE 2025)

13. Design a problem where you determine the output auto correlation of an LTI system given its impulse response $h(t) = e^{-t}u(t)$ and input auto correlation $R_{XX}(\tau) = \delta(\tau)$. (Leetcode Hard)

14. Show that if the input {X(t)} is a WSS process then the output {Y(t)} is also a WSS process.

15. Two jointly WSS processes X(t) and Y(t) have cross-correlation $R_{XY}(\tau) = 5e^{-2|\tau|}\sin(3\tau)$. Compute the cross spectral density. (GATE ECE 2025)

16. Sensor network captures $X(t)$ with $R_{XX}(\tau) = 2e^{-3|\tau|}$. Find $S_{XX}(\omega)$. (Leetcode /GATE ECE 2024)