# COMPLEX INTEGRATION

## Cauchy's integration theorem:

If a function $f(z)$ is analytic and its derivative $f'(z)$ is continuous with all points inside and on a simple closed curve $C$, then

$$\int_C f(z)\,dz = 0.$$

## Cauchy's integral formula:

If $f(z)$ is analytic inside and on a simple closed curve $C$ and 'a' be any point inside $C$, then

$$\int_C \frac{f(z)}{z-a}\,dz = 2\pi i\, f(a).$$

where integration being taken in the anti-clockwise direction around $C$.

1. Evaluate $\int_C \frac{\cos \pi z}{z-1}\, dz$ where $C$ is $|z| = 2$

Soln:

$f(z) = \cos \pi z$

$a = 1$

$C : |z| = 2.$

$a = 1$ lies inside $C$, $|1| < 2$

By Cauchy's integral formula,

$$\int_C \frac{f(z)}{z-a}\, dz = 2\pi i f(a)$$

$$\int_C \frac{\cos \pi z}{z-1}\, dz = 2\pi i f(1)$$

$$= -2\pi i$$

$[\because f(z) = \cos \pi z$, $f(1) = \cos \pi = -1]$

2. Evaluate $\int_C \frac{dz}{(z-3)^2}$ where $C$ is the circle $|z| = 1$

Soln: $a = 3$ lies outside the circle $|z| = 1$.