



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

(10) To find $\mathcal{L}(\cosh at)$:

$$\begin{aligned}\mathcal{L}(\cosh at) &= \mathcal{L}\left\{\frac{1}{2}\left[e^{at} + e^{-at}\right]\right\} \\ &= \frac{1}{2}\mathcal{L}(e^{at}) + \frac{1}{2}\mathcal{L}(e^{-at}) \\ &= \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\} \\ &= \frac{1}{2} \cdot \frac{2s}{s^2 - a^2} = \frac{s}{s^2 - a^2}\end{aligned}$$

$$\boxed{\mathcal{L}(\cosh at) = \frac{s}{s^2 - a^2} \text{ for } s^2 > a^2}$$

PROBLEMS:

(1) Find $\mathcal{L}(t^8)$

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(t^8) = \frac{8!}{s^{8+1}} = \frac{46320}{s^9}$$

✓ (2) Find $\mathcal{L}(t+1)^2$

$$\mathcal{L}[(t+1)^2] = \mathcal{L}(t^2 + 2t + 1)$$

$$\begin{aligned}&= \mathcal{L}(t^2) + 2\mathcal{L}(t) + \mathcal{L}(1) \\ &= \frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}\end{aligned}$$

(3) Find $\mathcal{L}\left(\frac{1}{\sqrt{t}}\right)$

$$\begin{aligned}\mathcal{L}\left(\frac{1}{\sqrt{t}}\right) &= \mathcal{L}(t^{-1/2}) \\ &= \frac{\Gamma(-1/2 + 1)}{s^{-1/2 + 1}} = \frac{\Gamma(1/2)}{s^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{s}}\end{aligned}$$



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④ $\mathcal{L}(\sqrt{t})$

$$\mathcal{L}(\sqrt{t}) = \mathcal{L}(t^{1/2})$$

$$\Gamma_{n+1} = n \Gamma_n \quad \& \quad \Gamma_{1/2} = \sqrt{\pi}$$

$$= \frac{\Gamma_{1/2+1}}{s^{1/2+1}} = \frac{1/2 \Gamma_{1/2}}{s \sqrt{s}} = \frac{1/2 \cdot \sqrt{\pi}}{s^{3/2}}$$

$$= \frac{\sqrt{\pi}}{2 s^{3/2}}$$

⑤ $\mathcal{L}(t^{5/2})$

$$\mathcal{L}(t^{5/2}) = \frac{\Gamma_{5/2+1}}{s^{5/2+1}} = \frac{5/2 \Gamma_{5/2}}{s^{7/2}}$$

$$= \frac{(5/2 \cdot 3/2 \cdot 1/2 \Gamma_{1/2})}{s^{7/2}} = \frac{15 \sqrt{\pi}}{8 s^{7/2}}$$

⑥ $\mathcal{L}(e^{5t})$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\mathcal{L}(e^{5t}) = \frac{1}{s-5}$$

⑦ $\mathcal{L}(e^t)$

$$\mathcal{L}(e^t) = \frac{1}{s-1}$$

⑧ $\mathcal{L}(e^{-7t})$

$$\mathcal{L}(e^{-7t}) = \frac{1}{s+7}$$

⑨ $\mathcal{L}(e^{-t})$

$$\mathcal{L}(e^{-t}) = \frac{1}{s+1}$$



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(10) Find $L(\sin 5t)$

$$L(\sin at) = \frac{a}{s^2 + a^2}$$

$$L(\sin 5t) = \frac{5}{s^2 + 5^2} = \frac{5}{s^2 + 25}$$

(11) Find $L(\cos bt)$

$$L(\cos at) = \frac{s}{s^2 + a^2}$$

$$L(\cos bt) = \frac{s}{s^2 + b^2} = \frac{s}{s^2 + 36}$$

✓ (12) Find $L(\sin^2 2t)$

$$\sin^2 t = \frac{1 - \cos 2t}{2}$$

$$L(\sin^2 2t) = L\left[\frac{1 - \cos 2(2t)}{2}\right]$$

$$= \frac{1}{2} L(1 - \cos 4t)$$

$$= \frac{1}{2} [L(1) - L(\cos 4t)]$$

$$= \frac{1}{2} \left[\frac{1}{s} - \frac{s}{s^2 + 16} \right]$$

✓ (13) Find $L(\cos^2 3t)$

$$\cos^2 t = \frac{1 + \cos 2t}{2}$$

$$L(\cos^2 3t) = L\left[\frac{1 + \cos 2(3t)}{2}\right]$$

$$= \frac{1}{2} L(1 + \cos 6t)$$

$$= \frac{1}{2} [L(1) + L(\cos 6t)]$$

$$= \frac{1}{2} \left[\frac{1}{s} + \frac{s}{s^2 + 36} \right]$$



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(14) Find $L(\cos^3 2t)$

$$\cos^3 \theta = \frac{1}{4} (\cos 3\theta + 3 \cos \theta)$$

$$L[\cos^3 2t] = L\left[\frac{\cos 3(2t) + 3 \cos(2t)}{4}\right]$$

$$= \frac{1}{4} \{ L(\cos 6t) + 3 L(\cos 2t) \}$$

$$= \frac{1}{4} \left\{ \frac{s}{s^2+36} + 3 \cdot \frac{s}{s^2+4} \right\}$$

$$= \frac{1}{4} \left\{ \frac{s}{s^2+36} + \frac{3s}{s^2+4} \right\}$$

(15) Find $L(\sin^3 3t)$

$$\sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4}$$

$$L(\sin^3 3t) = L\left[\frac{3 \sin 3t - \sin 9t}{4}\right]$$

$$= \frac{1}{4} \{ 3 L(\sin 3t) - L(\sin 9t) \}$$

$$= \frac{1}{4} \left\{ 3 \left(\frac{3}{s^2+9} \right) - \frac{9}{s^2+81} \right\}$$

$$= \frac{9}{4} \left\{ \frac{1}{s^2+9} - \frac{1}{s^2+81} \right\}$$

(16) Find $L(\sin 2t \cos 3t)$

$$\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$L(\sin 2t \cos 3t) = L\left[\frac{\sin(2t+3t) + \sin(2t-3t)}{2}\right]$$



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$$\begin{aligned} &= \frac{1}{2} \{ \mathcal{L}(\sin 5t) + \mathcal{L}(\sin(-t)) \} \\ &= \frac{1}{2} \{ \mathcal{L}(\sin 5t) - \mathcal{L}(\sin t) \} \\ &= \frac{1}{2} \left\{ \frac{5}{s^2 + 25} - \frac{1}{s^2 + 1} \right\} \end{aligned}$$