SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

$$= \frac{\sin 3t}{3} \left[\frac{e^{-t}}{10} (-\cos 3t + 3\sin 3t) - \frac{1}{10}(-1) \right]$$

$$- \frac{\cos 3t}{3} \left[\frac{e^{-t}}{10} (-\sin 3t - 3\cos 3t) - \frac{1}{10}(-3) \right]$$

$$= \left[\frac{e^{-t}}{30} \cdot 3 (\sin^2 3t + \cos^2 3t) \right] + \left[\frac{1}{30} (\sin 3t + 3\cos 3t) \right]$$

$$= \frac{e^{-t}}{10} + \frac{1}{30} (\sin 3t + 3\cos 3t)$$

Applications of Laplace transforms to Differential Equations:

If $L[f(t)] = F(s)$ then

$$L[y'(t)] = s L(y) - y(0)$$

$$L[y''(t)] = s^2 L(y) - s y(0) - y'(0)$$

① Solve the differential equations using LT $y'' + 4y' + 4y = e^{-t}$ given that $y(0) = 0$ and $y'(0) = 0$.

Soln:

$$y'' + 4y' + 4y = e^{-t}$$

Taking LT on both sides,

$$L(y'' + 4y' + 4y) = L(e^{-t})$$

$$L(y'') + 4L(y') + 4L(y) = \frac{1}{s+1}$$

$$[s^2 L(y) - s y(0) - y'(0)] + 4[s L(y) - y(0)]$$
$$+ 4 L(y) = \frac{1}{s+1}$$

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DEPATMENT OF MATHEMATICS

Given: $y(0) = 0$, $y'(0) = 0$

$\Rightarrow [s^2 L(y) - s \times 0 - 0] + 4[s L(y) - 0] + 4 L(y) = \dfrac{1}{s+1}$

$\Rightarrow s^2 L(y) + 4 s L(y) + 4 L(y) = \dfrac{1}{s+1}$

$\Rightarrow (s^2 + 4s + 4) L(y) = \dfrac{1}{s+1}$

$\Rightarrow L(y)(s+2)^2 = \dfrac{1}{s+1}$

$L(y) = \dfrac{1}{(s+1)(s+2)^2}$

$y = L^{-1}\left[\dfrac{1}{(s+1)(s+2)^2}\right]$

$\dfrac{1}{(s+1)(s+2)^2} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{(s+2)^2}$

$1 = A(s+2)^2 + B(s+2)(s+1) + C(s+1)$

Put $s = -2 \Rightarrow C = -1$

Put $s = -1 \Rightarrow A = 1$

Put $s = 0 \Rightarrow B = -1$

$\dfrac{1}{(s+1)(s+2)^2} = \dfrac{1}{s+1} - \dfrac{1}{s+2} - \dfrac{1}{(s+2)^2}$

$y = L^{-1}\left(\dfrac{1}{(s+1)(s+2)^2}\right)$

$= L^{-1}\left(\dfrac{1}{s+1}\right) - L^{-1}\left(\dfrac{1}{s+2}\right) - L^{-1}\left(\dfrac{1}{(s+2)^2}\right)$

$\left[F_{(s+a)} = e^{-t} - e^{-2t} - t e^{-2t}\right]$

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DEPARTMENT OF MATHEMATICS

② Solve using LT $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 9y = 2e^{-3t}$,

$y(0) = 1$, $y'(0) = -2$

Soln:

$$y'' + 6y' + 9y = 2e^{-3t}$$

$$L(y'' + 6y' + 9y) = 2L(e^{-3t})$$

$$L(y'') + 6L(y') + 9L(y) = 2L(e^{-3t})$$

$$\{s^2 L(y) - s\,y(0) - y'(0)\} + 6\{s L(y) - y(0)\} + 9L(y) = \dfrac{2}{s+3}$$

$$\{s^2 L(y) - s + 2\} + 6\{s L(y) - 1\} + 9L(y) = \dfrac{2}{s+3}$$

$$s^2 L(y) + 6sL(y) + 9L(y) - (s+4) = \dfrac{2}{s+3}$$

$$(s^2 + 6s + 9) L(y) = \dfrac{2}{s+3} + s + 4$$

$$(s+3)^2 L(y) = \dfrac{2}{s+3} + s + 4$$

$$L(y) = \dfrac{2}{(s+3)^3} + \dfrac{s+4}{(s+3)^2}$$

$$y = L^{-1}\left[\dfrac{2}{(s+3)^3} + \dfrac{s+4}{(s+3)^2}\right]$$

$$= 2 L^{-1}\left(\dfrac{1}{(s+3)^3}\right) + L^{-1}\left(\dfrac{s+4}{(s+3)^2}\right)$$

$$= 2 L^{-1}\left(\dfrac{1}{(s+3)^3}\right) + L^{-1}\left[\dfrac{1}{s+3} + \dfrac{1}{(s+3)^2}\right]$$

$$= 2 e^{-3t} L^{-1}\left(\dfrac{1}{s^3}\right) + L^{-1}\left(\dfrac{1}{s+3}\right) + L^{-1}\left(\dfrac{1}{(s+3)^2}\right)$$

$$= 2e^{-3t}\, t^2 + e^{-3t} + t e^{-3t}$$