



DEPARTMENT OF MATHEMATICS

Inverse Laplace Transform:

If the Laplace transform of $f(t)$ is $F(s)$ i.e., $L[f(t)] = F(s)$. Then $f(t)$ is called an inverse Laplace transform of $F(s)$ and is written as $f(t) = L^{-1}[F(s)]$ where L^{-1} is called the inverse Laplace transform operator.



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Table of Inverse Laplace Transforms:

| $\mathcal{L}[f(t)] = F(s)$ | $\mathcal{L}^{-1}[F(s)] = f(t)$ |
|---|--|
| ① $\mathcal{L}(1) = \frac{1}{s}$ | $\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$ |
| ② $\mathcal{L}(t) = \frac{1}{s^2}$ | $\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t$ |
| ③ $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$ | $\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$ |
| ④ $\mathcal{L}(e^{at}) = \frac{1}{s-a}$ | $\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$ |
| ⑤ $\mathcal{L}(e^{-at}) = \frac{1}{s+a}$ | $\mathcal{L}^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$ |
| ⑥ $\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$ | $\mathcal{L}^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$ |
| ⑦ $\mathcal{L}\left(\frac{\sin at}{a}\right) = \frac{1}{s^2+a^2}$ | $\mathcal{L}^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}$ |
| ⑧ $\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$ | $\mathcal{L}^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$ |
| ⑨ $\mathcal{L}(\sinh at) = \frac{a}{s^2-a^2}$ | $\mathcal{L}^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$ |
| ⑩ $\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}$ | $\mathcal{L}^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$ |
| ⑪ $\mathcal{L}[\delta(t)] = 1$ | $\mathcal{L}^{-1}(1) = \delta(t)$ |



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Important Results in Laplace Transforms:

If a and b are constants while $F(s)$ and $G(s)$ are the Laplace transform of $f(t)$ & $g(t)$ respectively.

1. Linearity property:

$$\mathcal{L}^{-1}[a F(s) + b G(s)] = a \mathcal{L}^{-1}[F(s)] + b \mathcal{L}^{-1}[G(s)]$$

2. First Shifting property:

$$(i) \mathcal{L}^{-1}[F(s+a)] = e^{-at} \mathcal{L}^{-1}[F(s)]$$

$$(ii) \mathcal{L}^{-1}[F(s-a)] = e^{at} \mathcal{L}^{-1}[F(s)]$$

3. Second Shifting property:

If $\mathcal{L}^{-1}[F(s)] = f(t)$ then

$$\mathcal{L}^{-1}[e^{-as} F(s)] = g(t) \text{ where}$$

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t \leq a \end{cases}$$

4. Change of Scale property:

$$\mathcal{L}^{-1}[F(ks)] = \frac{1}{k} f\left(\frac{t}{k}\right)$$

5. Multiplication by s :

If $\mathcal{L}^{-1}[F(s)] = f(t)$ and $f(0) = 0$

$$\text{Then } \mathcal{L}^{-1}[s F(s)] = \frac{d}{dt} \mathcal{L}^{-1}[F(s)]$$

Note:

If $f(0) \neq 0$ then $\mathcal{L}^{-1}[s F(s)] = \frac{d}{dt} \mathcal{L}^{-1}[F(s)] + f(0) \delta(t)$.



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Problem Identification:

If $L^{-1} \left[\frac{s}{\text{quadratic equation}} \right]$ then use result 5.

If $L^{-1} \left[\frac{s}{\text{Linear equation}} \right]$ then we use the above note.

6. Division by s:

$$L^{-1} \left[\frac{F(s)}{s} \right] = \int_0^t L^{-1} [F(s)] dt$$

7. Inverse Laplace transform of derivatives:

$$\text{If } L^{-1} [F(s)] = f(t) \text{ then } L^{-1} [F'(s)] = -t L^{-1} [F(s)]$$

Problem Identification:

If $L^{-1} \left[\frac{s + \text{any term}}{(\text{quadratic eqn})^2} \right]$ then we use the above result.

8. Note:

$$\text{If } L^{-1} [F(s)] = f(t) \text{ then } L^{-1} [F(s)] = -\frac{1}{t} L^{-1} \left[\frac{d}{ds} F(s) \right]$$

Problem Identification:

If $L^{-1} [\log \text{ function or cot function or tan fn}]$ then we use the above result.