SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution
Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 5 – DSP APPLICATIONS

TOPIC – MULTIRATE DSP – DOWNSAMPLING (OR) DECIMATION

INTRODUCTION

- The processing of a discrete time signal at different sampling rates in different parts of a system is called **multirate DSP**
- The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called **multirate DSP Systems**
- The process of converting a signal from one sampling rate to another sampling rate is called **sampling rate conversion**
- There are two general methods for sampling rate conversion. In the first method, the discrete signal is converted to analog signal using a D/A converter and the analog signal is resampled at the desired rate using an A/D converter

INTRODUCTION

- The advantage in this method is that the new sampling rate need not have any relation to the old sampling rate. The disadvantage of this method is distortion during D/A and A/D process
- In the second method, the sampling rate conversion is entirely performed in the digital domain, using Interpolators and Decimators
- The advantage in rate conversion in the digital domain is that the signal distortion in D/A and A/D process are avoided or eliminated
- There are two ways for sampling rate conversion in the digital domain. They are **1. Downsampling or Decimation & 2. Upsampling or Interpolation**

INTRODUCTION

- **Downsampling or decimation** is the process of reducing the sampling rate by an integer factor D
- **Upsampling or interpolation** is the process of increasing the sampling rate by an integer factor I
- **Advantages of Multirate Processing:**

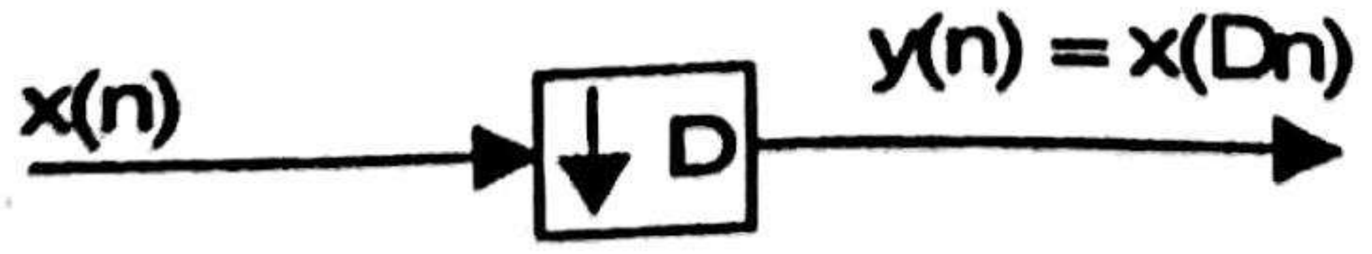
1. The reduction in number of computations
2. The reduction in memory requirement (or storage) for filter coefficients and intermediate results
3. The reduction in the order of the system
4. The finite word length effects are reduced

APPLICATIONS OF MULTIRATE DSP SYSTEMS

- Sub-band coding of speech signals and image compression
- QMF(Quadrature Mirror Filters) for realizing alias–free LTI multirate systems
- Narrowband FIR and IIR filters for various applications
- Digital transmultiplexers for converting TDM signals to FDM signals and vice versa
- Oversampling A/D and D/A converters for high quality digital audio systems and data loggers (or digital storage systems)
- In digital audio systems the sampling rates of broadcasted signal, CD (Compact Disc), MPEG (Motion Picture Expert Group) Standard CD, etc., are different. Hence to access signals from all these devices, sampling rate converters are needed in digital audio systems

DOWNSAMPLING (OR) DECIMATION

- **Downsampling (or decimation)** is the process of reducing the samples of the discrete time signal
- Let, x(n) = Discrete time signal
- D = Sampling rate reduction factor (and D is an integer)
- Now, x(Dn) = Downsampled version of x(n)
- The device which performs the process of downsampling is called a downsampler (or decimator)
- The downsampler can be represented as



EXAMPLE

Consider the discrete time signal,

$$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

Determine the downsampled version of the signals for the sampling rate reduction factors.

a) $D = 2$ b) $D = 3$ c) $D = 4$.

Solution

Given that,

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$\therefore$ When $n = 0$, $x(n) = x(0) = 1$

When $n = 1$, $x(n) = x(1) = 2$

When $n = 2$, $x(n) = x(2) = 3$

When $n = 3$, $x(n) = x(3) = 4$

When $n = 4$, $x(n) = x(4) = 5$

When $n = 5$, $x(n) = x(5) = 6$

When $n = 6$, $x(n) = x(6) = 7$

When $n = 7$, $x(n) = x(7) = 8$

When $n = 8$, $x(n) = x(8) = 9$

When $n = 9$, $x(n) = x(9) = 10$

When $n = 10$, $x(n) = x(10) = 11$

When $n = 11$, $x(n) = x(11) = 12$

EXAMPLE

a) Sampling rate reduction factor, D = 2.

Now, $x(Dn) = x(2n)$ = Discrete time signal decimated by reduction factor 2.

Let, $x(2n) = x_{D2}(n)$

$\therefore$ When $n = 0$, $x_{D2}(n) = x_{D2}(0) = x(2 \times 0) = x(0) = 1$

When $n = 1$, $x_{D2}(n) = x_{D2}(1) = x(2 \times 1) = x(2) = 3$

When $n = 2$, $x_{D2}(n) = x_{D2}(2) = x(2 \times 2) = x(4) = 5$

When $n = 3$, $x_{D2}(n) = x_{D2}(3) = x(2 \times 3) = x(6) = 7$

When $n = 4$, $x_{D2}(n) = x_{D2}(4) = x(2 \times 4) = x(8) = 9$

When $n = 5$, $x_{D2}(n) = x_{D2}(5) = x(2 \times 5) = x(10) = 11$

$$\therefore\ x(2n) = x_{D2}(n) = \{\underset{\uparrow}{1}, 3, 5, 7, 9, 11\}$$

EXAMPLE

b) Sampling rate reduction factor, D = 3.

Now, $x(Dn) = x(3n) =$ Discrete time signal decimated by reduction factor 3.

Let, $x(3n) = x_{D3}(n)$

$\therefore$ When $n = 0$, $x_{D3}(n) = x_{D3}(0) = x(3 \times 0) = x(0) = 1$

When $n = 1$, $x_{D3}(n) = x_{D3}(1) = x(3 \times 1) = x(3) = 4$

When $n = 2$, $x_{D3}(n) = x_{D3}(2) = x(3 \times 2) = x(6) = 7$

When $n = 3$, $x_{D3}(n) = x_{D3}(3) = x(3 \times 3) = x(9) = 10$

$$\therefore\ x(3n) = x_{D3}(n) = \{\underset{\uparrow}{1}, 4, 7, 10\}$$

EXAMPLE

c) Sampling rate reduction factor, D = 4.

Now, $x(Dn) = x(4n)$ = Discrete time signal decimated by reduction factor 4.

Let, $x(4n) = x_{D4}(n)$

$\therefore$ When $n = 0$, $x_{D4}(n) = x_{D4}(0) = x(4 \times 0) = x(0) = 1$

When $n = 1$, $x_{D4}(n) = x_{D4}(1) = x(4 \times 1) = x(4) = 5$

When $n = 2$, $x_{D4}(n) = x_{D4}(2) = x(4 \times 2) = x(8) = 9$

$$\therefore\ x(4n) = x_{D4}(n) = \{\underset{\uparrow}{1}, 5, 9\}$$

EXAMPLE

Consider the discrete time signal shown in fig 1.

Sketch the downsampled version of the signals for the sampling rate reduction factors, **a)** D = 2 **b)** D = 3.

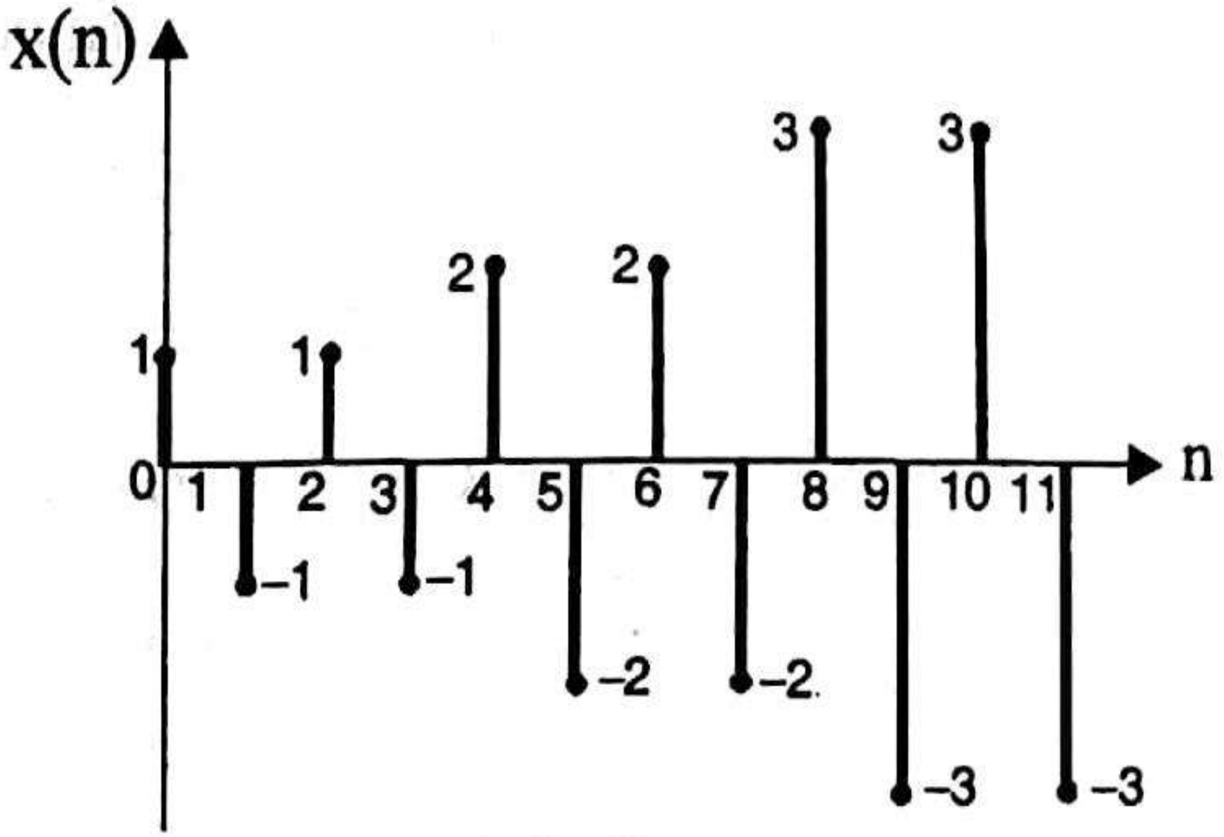


Fig 1.

Solution

From fig 1, we can write the samples of the given sequence as shown below.

$$x(n) = \{\underset{\uparrow}{1}, -1, 1, -1, 2, -2, 2, -2, 3, -3, 3, -3\}$$

∴ When n = 0, x(n) = x(0) = 1	When n = 4, x(n) = x(4) = 2	When n = 8, x(n) = x(8) = 3
When n = 1, x(n) = x(1) = −1	When n = 5, x(n) = x(5) = −2	When n = 9, x(n) = x(9) = −3
When n = 2, x(n) = x(2) = 1	When n = 6, x(n) = x(6) = 2	When n = 10, x(n) = x(10) = 3
When n = 3, x(n) = x(3) = −1	When n = 7, x(n) = x(7) = −2	When n = 11, x(n) = x(11) = −3

EXAMPLE

a) Sampling rate reduction factor, D = 2.

Let, $x_{D2}(n)$ = Discrete time signal decimated by reduction factor 2.

Now, $x_{D2}(n) = x(Dn) = x(2n)$

$\therefore$ When $n = 0$, $x_{D2}(n) = x_{D2}(0) = x(2 \times 0) = x(0) = 1$

When $n = 1$, $x_{D2}(n) = x_{D2}(1) = x(2 \times 1) = x(2) = 1$

When $n = 2$, $x_{D2}(n) = x_{D2}(2) = x(2 \times 2) = x(4) = 2$

When $n = 3$, $x_{D2}(n) = x_{D2}(3) = x(2 \times 3) = x(6) = 2$

When $n = 4$, $x_{D2}(n) = x_{D2}(4) = x(2 \times 4) = x(8) = 3$

When $n = 5$, $x_{D2}(n) = x_{D2}(5) = x(2 \times 5) = x(10) = 3$

$$\therefore\ x(2n) = x_{D2}(n) = \{\underset{\uparrow}{1}, 1, 2, 2, 3, 3\} \qquad .....(1)$$

EXAMPLE

b) Sampling rate reduction factor, D = 3.

- Let, $x_{D3}(n)$ = Discrete time signal decimated by reduction factor 3.

Now, $x_{D3}(n) = x(Dn) = x(3n)$

$\therefore$ When $n = 0$, $x_{D3}(n) = x_{D3}(0) = x(3 \times 0) = x(0) = 1$

When $n = 1$, $x_{D3}(n) = x_{D3}(1) = x(3 \times 1) = x(3) = -1$

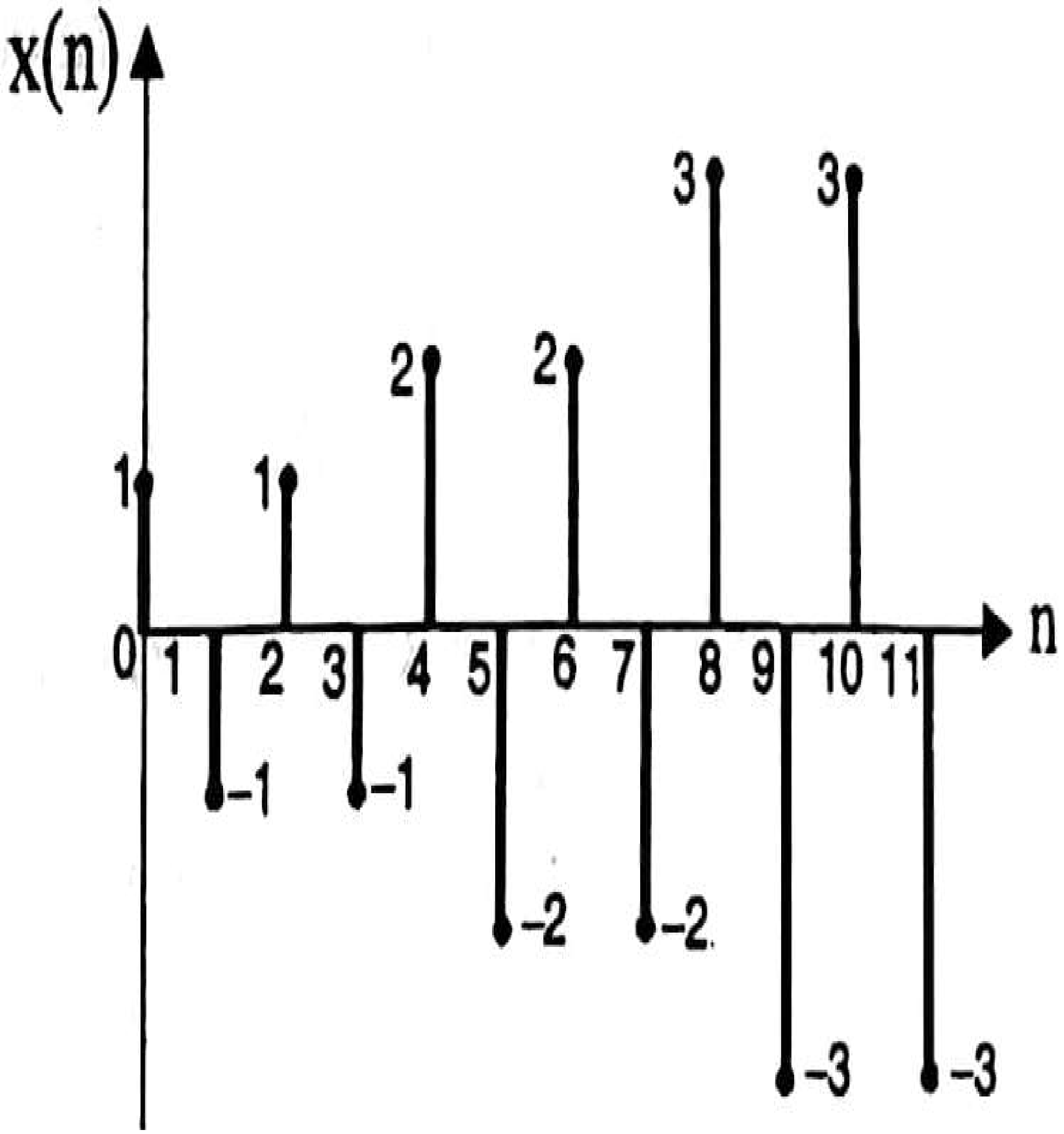
When $n = 2$, $x_{D3}(n) = x_{D3}(2) = x(3 \times 2) = x(6) = 2$

When $n = 3$, $x_{D3}(n) = x_{D3}(3) = x(3 \times 3) = x(9) = -3$

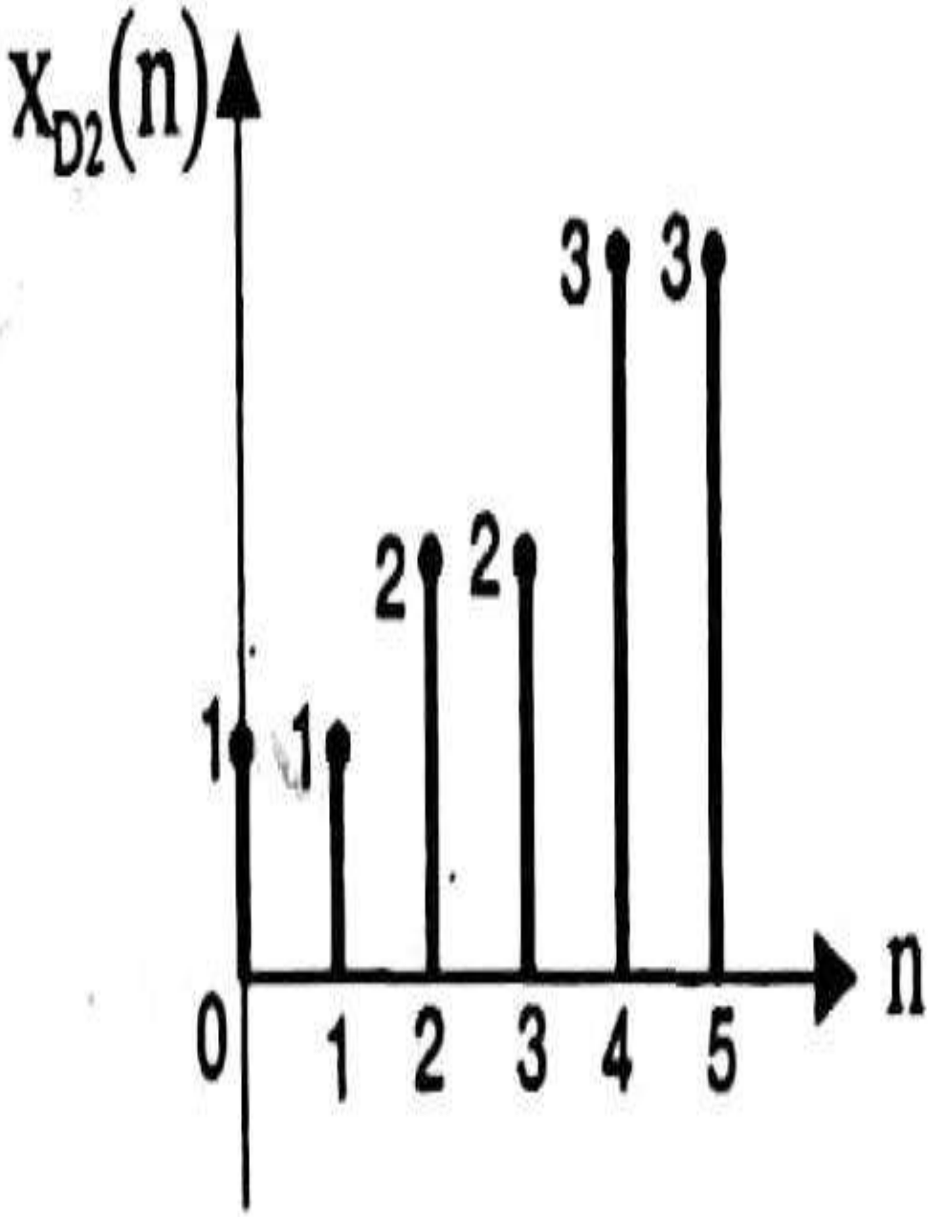
$$\therefore \; x(3n) = x_{D3}(n) = \{\underset{\uparrow}{1}, -1, 2, -3\} \qquad .....(2)$$

EXAMPLE

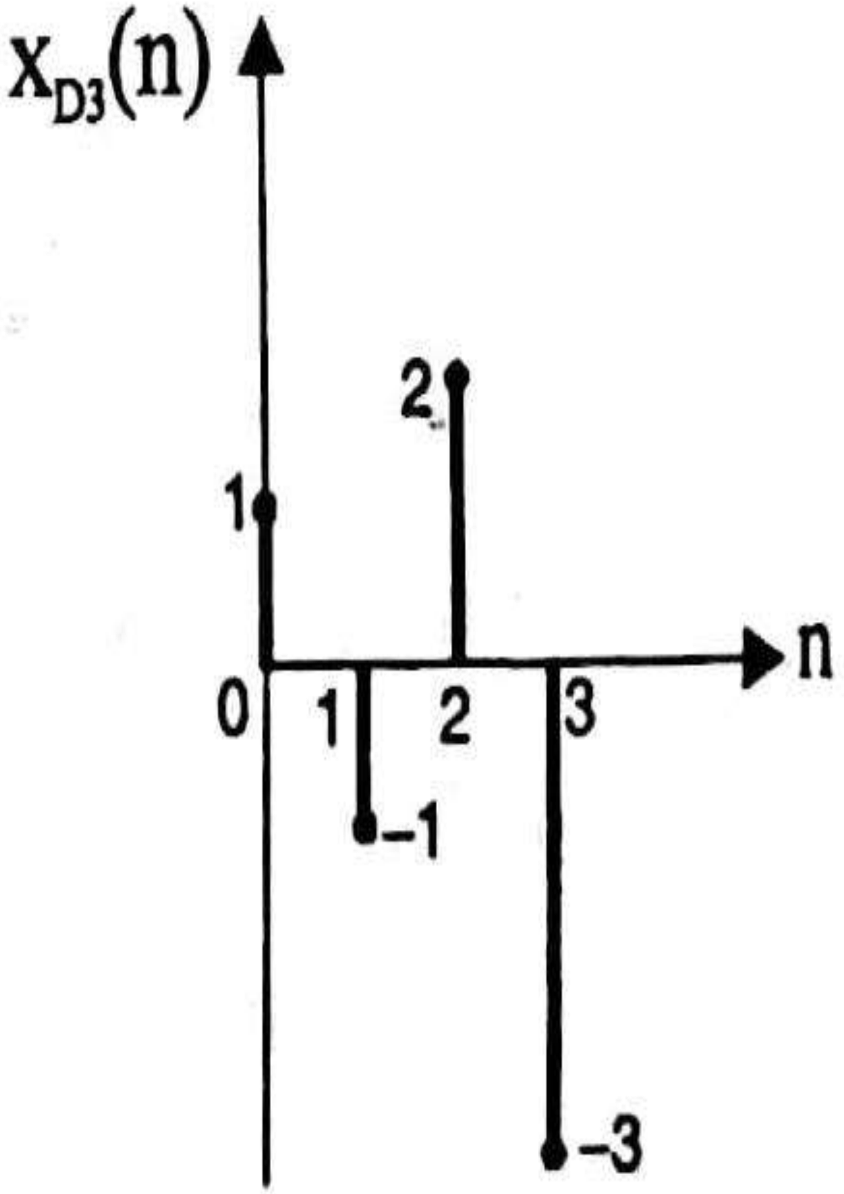
Samples of sequence



x(n) decimated by 2



x(n) decimated by 3



SPECTRUM OF DOWNSAMPLER

- Let x(n) be an input signal to the downsampler and y(n) be the output signal
- Let x'(nD) be the downsampled version of x(n) by an integer factor D

$$y(n) = x'(nD)$$

- Consider a unit pulse train defined as

$$p(n) = 1 \text{ ; for } n = 0, \pm D, \pm 2D, \pm 3D, \ldots\ldots$$
$$= 0 \text{ ; otherwise}$$

- Consider the product of x(n) and p(n)

$$x(n)\,p(n) = x(n) \text{ ; for } n = 0, \pm D, \pm 2D, \ldots\ldots$$
$$= 0 \text{ ; otherwise}$$

- Now x'(n) is the signal obtained after removing all zeros from x(n) p(n)

$$x'(n) = x(n)\,p(n) \text{ ; for } n = 0, \pm D, \pm 2D, \ldots\ldots$$

Z TRANSFORM

$$Y(z) = \sum_{n=-\infty}^{+\infty} y(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} x'(nD)\, z^{-n}$$

On substituting, y(n) = x′(nD), from equation

Let, m = nD ⇒ n = $\frac{m}{D}$

When n = −∞, m = −∞

When n = +∞, m = +∞

$$= \sum_{m=-\infty}^{+\infty} x'(m)\, z^{-\frac{m}{D}}$$

$$= \sum_{n=-\infty}^{+\infty} x'(n)\, z^{-\frac{n}{D}}$$

Let, m = n

$$= \sum_{n=-\infty}^{+\infty} x(n)\, p(n)\, z^{-\frac{n}{D}}$$

On substituting, x′(n) = x(n) p(n), from equation

$$= \sum_{n=-\infty}^{+\infty} x(n) \left[\frac{1}{D} \sum_{k=0}^{D-1} e^{\frac{j2\pi kn}{D}} \right] z^{-\frac{n}{D}}$$

Replacing p(n) by its Fourier series representation.

$$= \frac{1}{D} \sum_{k=0}^{D-1} \left[\sum_{n=-\infty}^{+\infty} x(n)\, e^{\frac{j2\pi kn}{D}}\, z^{-\frac{n}{D}} \right]$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} \left[\sum_{n=-\infty}^{+\infty} x(n) \left[e^{\frac{-j2\pi k}{D}}\, z^{\frac{1}{D}} \right]^{-n} \right]$$

SPECTRUM OF DOWNSAMPLER

Fourier series representation of p(n)

One period of p(n) is,

$$p(n) = \{1, 0, 0, \ldots\ldots 0\}$$

$\uparrow n=0$ $\qquad \uparrow n=D-1$

The Fourier coefficients c_k are given by,

$$c_k = \frac{1}{D}\sum_{n=0}^{D-1} p(n)\, e^{\frac{-j2\pi nk}{D}} = \frac{1}{D}$$

The Fourier series representation of p(n) is

$$p(n) = \sum_{k=0}^{D-1} c_k\, e^{\frac{j2\pi nk}{D}}$$

$$= \sum_{k=0}^{D-1} \frac{1}{D}\, e^{\frac{j2\pi nk}{D}} = \frac{1}{D}\sum_{k=0}^{D-1} e^{\frac{j2\pi nk}{D}}$$

In equation the terms inside the bracket is similar to Z-transform of y(n) except that, $z \rightarrow e^{\frac{-j2\pi k}{D}} z^{\frac{1}{D}}$, hence Y(z) can be written as shown in equation

$$\therefore Y(z) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{\frac{-j2\pi k}{D}} z^{\frac{1}{D}}\right)$$

where, $X\left(e^{\frac{-j2\pi k}{D}} z^{\frac{1}{D}}\right) = \sum_{n=0}^{+\infty} x(n)\left[e^{\frac{-j2\pi k}{D}} z^{\frac{1}{D}}\right]^{-n}$

SPECTRUM OF DOWNSAMPLER

On substituting, $z = e^{j\omega}$ in equation we get,

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{-j2\pi k/D}\, e^{j\omega/D}\right)$$

$$\therefore\ Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{j(\omega-2\pi k)/D}\right)$$

On expanding above equation

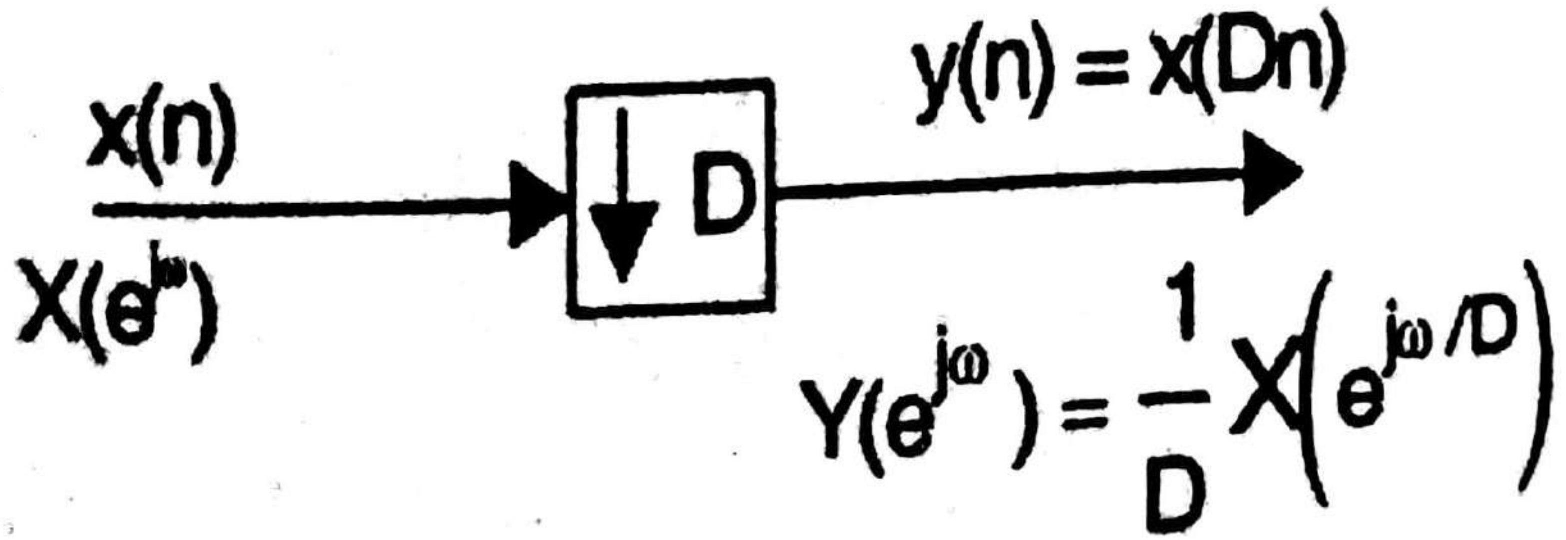
$$Y(e^{j\omega}) = \frac{1}{D}X\left(e^{j\omega/D}\right) + \frac{1}{D}X\left(e^{j(\omega-2\pi)/D}\right) + \frac{1}{D}X\left(e^{j(\omega-4\pi)/D}\right) + \ldots\ldots$$

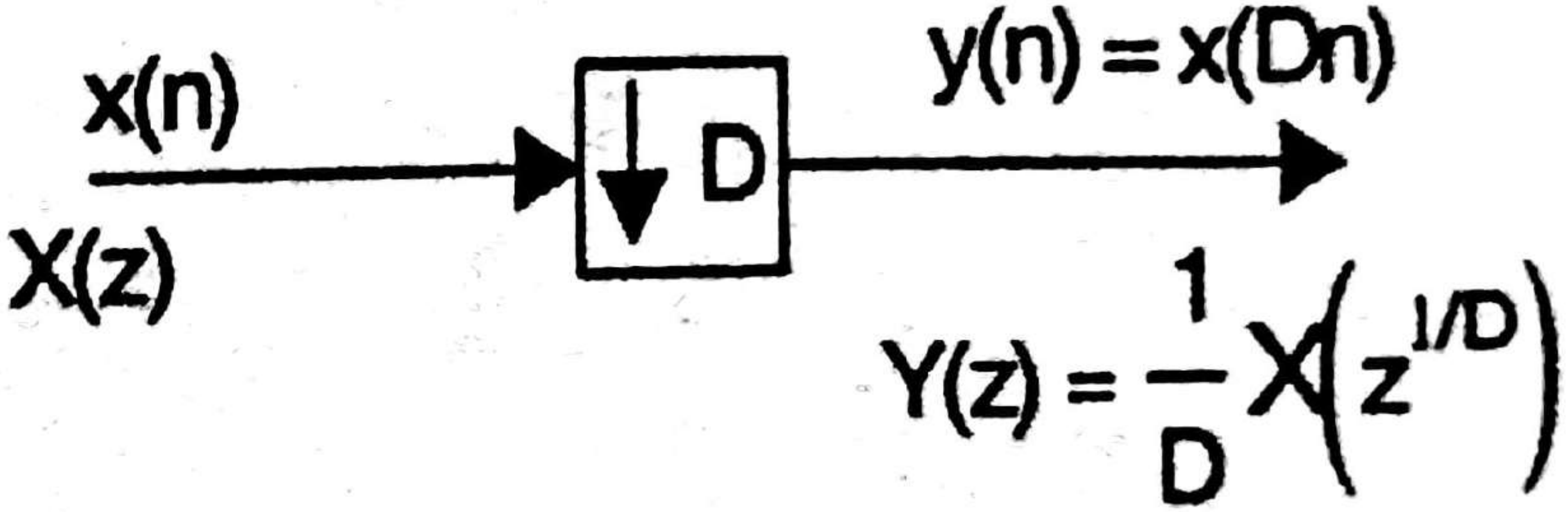
$$\ldots\ldots + \frac{1}{D}X\left(e^{j(\omega-2\pi(D-1))/D}\right)$$

SPECTRUM OF DOWNSAMPLER

$$\therefore\ Y(e^{j\omega}) = \frac{1}{D} X\left(e^{j\omega/D}\right)$$

$$Y(z) = \frac{1}{D} X\left(z^{1/D}\right)$$



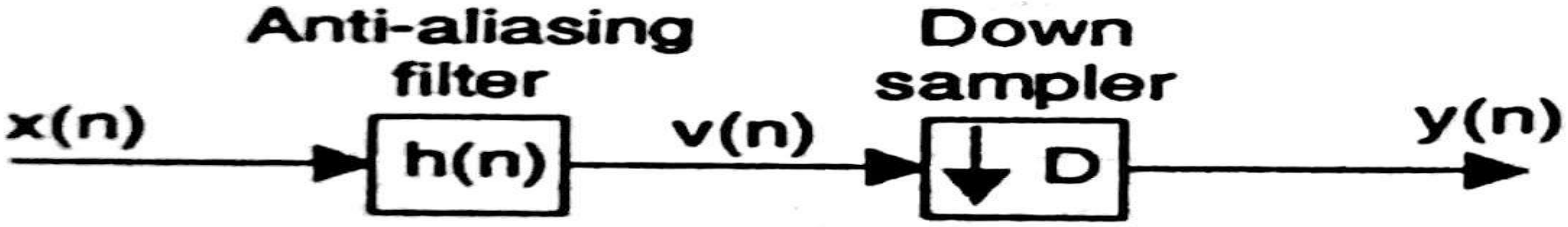


Frequency Domain Representation of downsampler

Z-Domain Representation of downsampler

ANTI-ALIASING FILTER

- When the input signal to the decimator is not bandlimited then the spectrum of decimated signal has aliasing. Inorder to avoid aliasing the input signal should be bandlimited to **π/D** for decimation by a factor D
- Hence the input signal is passed through a lowpass filter with a bandwidth of **π/D** before decimation. Since this lowpass filter is designed to avoid aliasing in the output spectrum of decimator, it is called anti-aliasing filter



ASSESSMENT

1. Define multirate DSP.
2. The discrete time systems that employ sampling rate conversion while processing the discrete time signals are called --------------------
3. What is meant by sampling rate conversion.
4. List the two ways for sampling rate conversion in the digital domain
5. What is meant by downsampling and upsampling?
6. What are the advantages of multirate Processing?
7. Summarize the applications of multirate DSP Systems.
8. Define anti-aliasing filter.

THANK YOU