



Regular Expression

- Set of strings – Algebraic Fashion
- Set of strings – language
 - Example1
 - $L = \{ab, abb, abbb, abbbb, \dots\}$? Regular language
 - (ab^+) ? Regular Expression
 - Example2
 - $R.L = \{0, 1, 00, 11\}$
 - $R.E = (0+1+00+11)$
 - Example 3
 - $R.L = \{0, 1, 00, 11, 000, 111, 0000, 1111, \dots\}$
 - $R.E = (0^+ + 1^+)$



Regular Expression

Regular Expression

- Any terminals (a,b,c,...,^)
- Union of two R.E $\Rightarrow$ R.E (R1,R2 $\Rightarrow$ R1+R2)
- Concatenation of two R.E $\Rightarrow$ R.E (R1,R2 $\Rightarrow$ R1.R2)
- Iteration of R.E is an R.E (R $\Rightarrow$ R*) ($a^* \Rightarrow$ ^,a,aa,aaa,aaaa,...)

- Examples

Sets	Regular Language
{0,1,2}	$R=0+1+2$
{^,ab}	$R=^+ab$
{abb,a,b,ba,.....}	$R=a^*b^*$
{^,0,00,000,0000,..}	$R = 0^*$
{1,11,111,1111,.....}	$R = 1^+$



Regular Expression - Examples



- Set of strings over $\{0,1\}$ that end in 3 consecutives 1's
 - R.L = $\{111,0111,1111,00111,10111,....\}$
 - R.E = $(0+1)^*111$
- Set of strings over $\{a,b\}$ that has atleast 1 a $\square$ ($\geq 1a$)
 - R.L = $\{a,ba,ab,aa,aaa,aba,bbbbba,bba,bbbaa,bbaaa,.....\}$
 - R.E = $(a+b)^* a (a+b)^*$
- Set of strings over $\{a,b\}$ that has atmost 1 a $\square$ ($\leq 1a$)
 - R.L = $\{a,ba,ab,bba,abb,bbba,abbb,.....\}$
 - $b^* a b^*$
- R.L = $\{c,cc,ccc,cccc,.....\}$ $\square$ R.E = (c^+)



Regular Expression - Examples

- Set of string over $\{a,b\}$ which has atleast 1 a $\Rightarrow (a+b)^*a(a+b)^*$
- Set of string over $\{a,b\}$ which has atmost 1 a $\Rightarrow b^*ab^*$
- Set of strings over $\{0,1\}$ which starts with 0 and ends with 1
 - R.L = $\{01,001,011,0111,01101,.....\}$
 - R.E = $0(0+1)^*1$
- Set of strings over $\{0,1\}$ which has consecutives 11 in it
 - R.L = $\{11,011,0110,1110,01101,....\}$
 - R.E = $(0+1)^*11(0+1)^*$
- Set of Strings over $\{0,1\}$ which doesn't contain a substring 110
 - R.L = $\{0,1,001,0111,0011,01010,....\}$
 - R.E = $(0+10)^*1^*$



Identities of a Regular Expression

- Identities of Regular Expression:-

$$I_1 \quad \emptyset + R = R$$

$$I_2 \quad \emptyset R = R \emptyset = \emptyset$$

$$I_3 \quad \Lambda R = R \Lambda = R$$

$$I_4 \quad \Lambda^* = \Lambda \text{ and } \emptyset^* = \Lambda$$

$$I_5 \quad R + R = R$$

$$I_6 \quad R^* R^* = R^*$$

$$I_7 \quad R R^* = R^* R$$

$$I_8 \quad (R^*)^* = R^*$$

$$I_9 \quad \Lambda + R R^* = R^* = \Lambda + R^* R$$

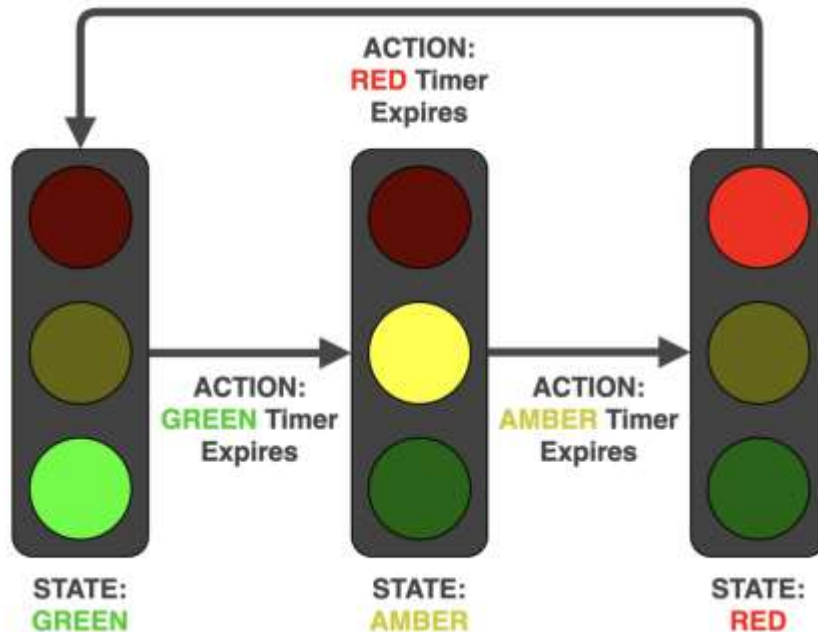
$$I_{10} \quad (PQ)^* P = P(QP)^*$$

$$I_{11} \quad (P + Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$$

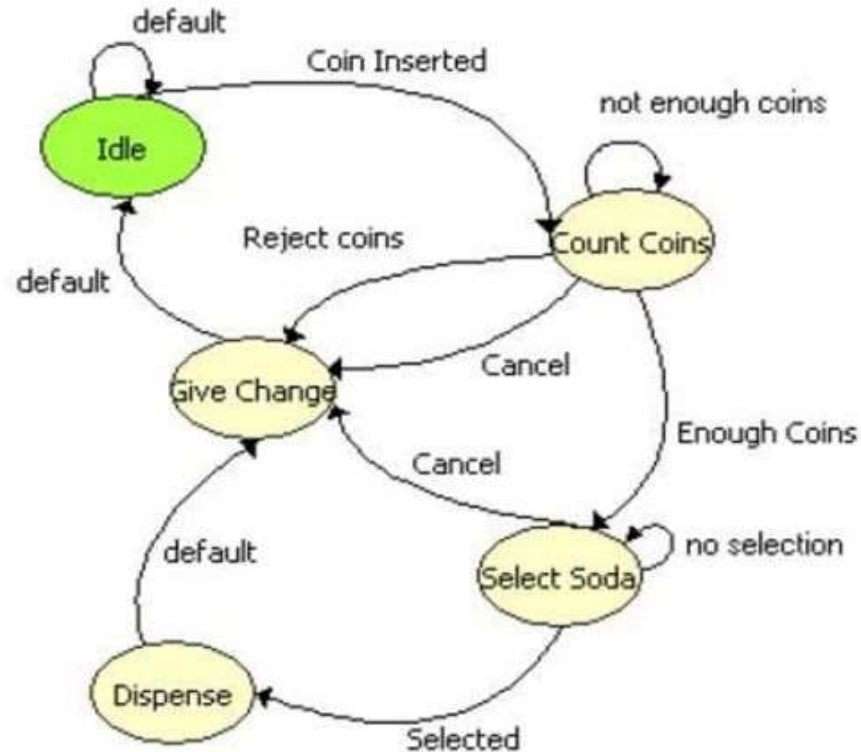
$$I_{12} \quad (P + Q)R = PR + QR \quad \text{and} \quad R(P + Q) = RP + RQ$$

Finite State Automata

- Finite Automata – set of states and rules – transition – input
- 1State $\Rightarrow$ 1 state
- Examples :Vending machine, Turnstile
 - Traffic Light



State Diagram of a Simple Soda Vending Machine

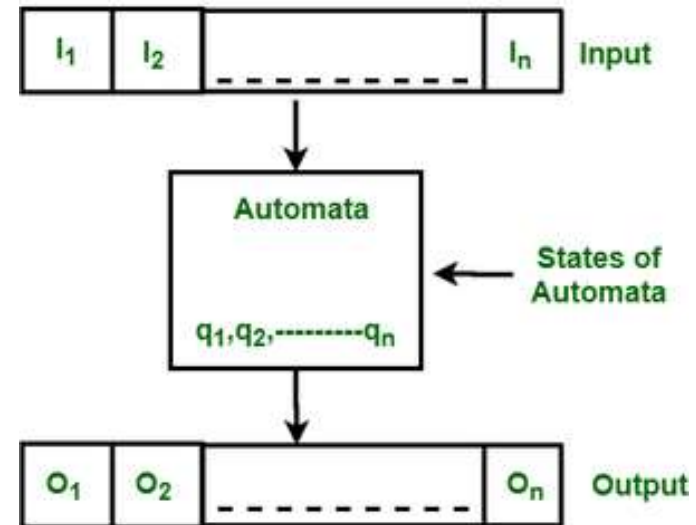




Finite State Automata

- ***FSA – Lexical Analysis of Compiler***

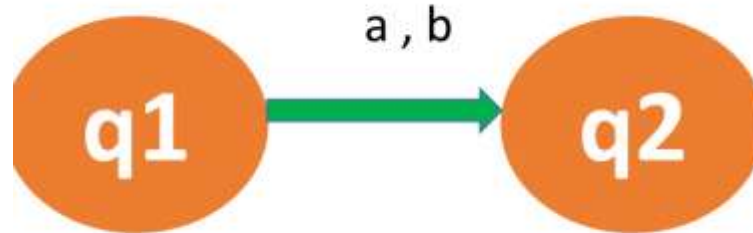
- FA – Tuple – $\{Q, \Sigma, q, F, \delta\}$
- Q – set of states
- Σ – set of input symbols
- q – initial state
- F – set of final states
- δ – Transitions





Regular Expression to Finite Automata

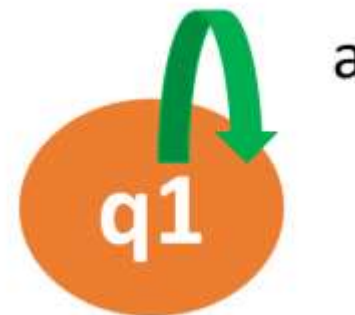
- $a+b$



- ab

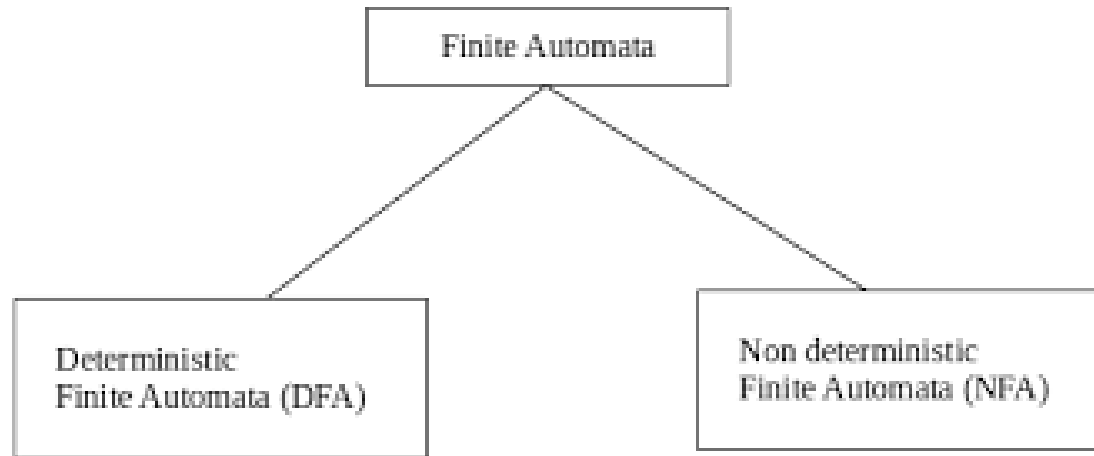


- a^*

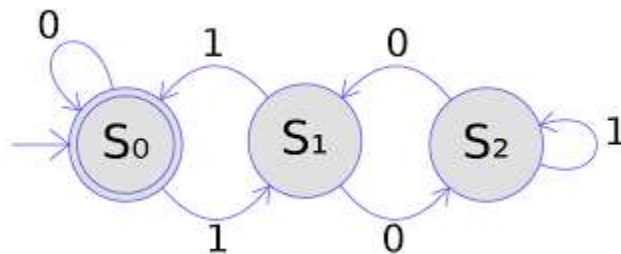




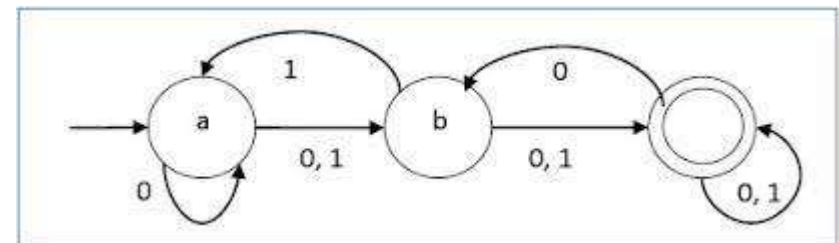
Types of Finite State Automata



Transition $\Rightarrow$ 1 state to single next state for each input symbol

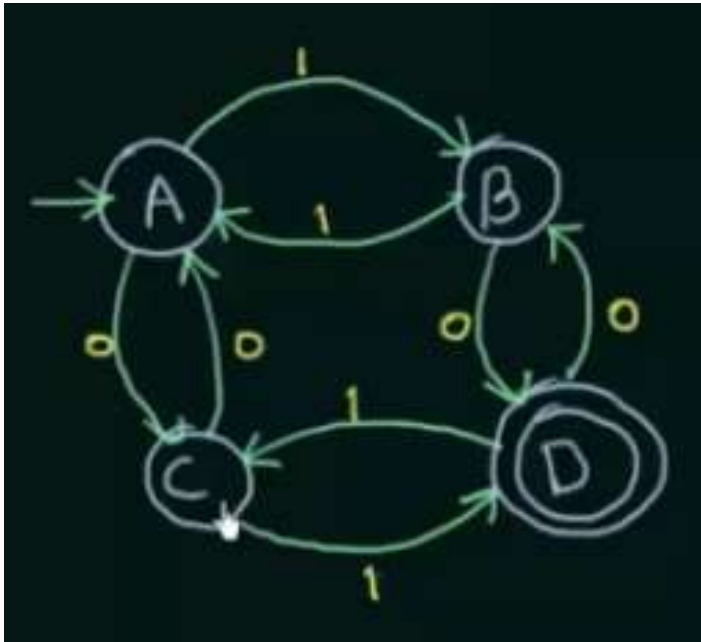


Transition $\Rightarrow$ 1 state to multiple next state for each input symbol





Deterministic Finite Automata (DFA)



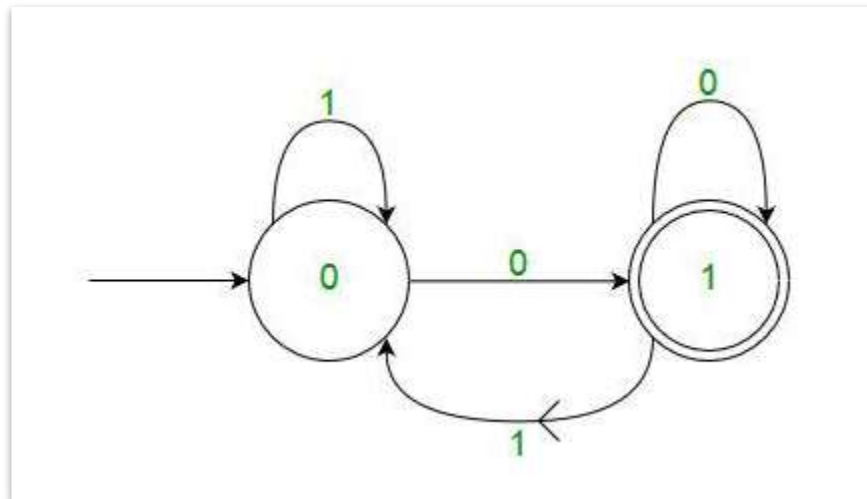
- $\{Q, \Sigma, q, F, \delta\}$
- $Q = \{A, B, C, D\}$
- $\Sigma = \{0, 1\}$
- $Q_0 = A$
- $F = D$
- δ Transition function

	0	1
A	C	B
B	D	A
C	A	D
D	B	C



Deterministic Finite Automata (DFA)

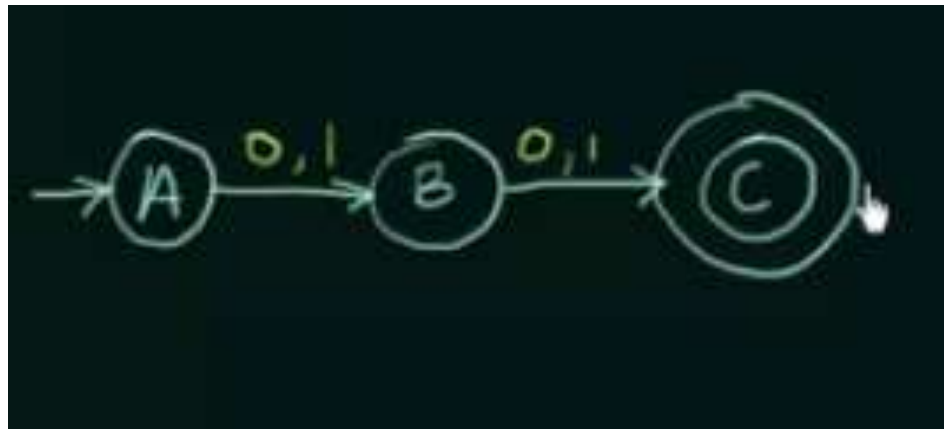
- Example 1
- L1 = Set of all strings that end with '0'
- L1 = {000,000,010,0110,0100,01110,....}





Deterministic Finite Automata (DFA)

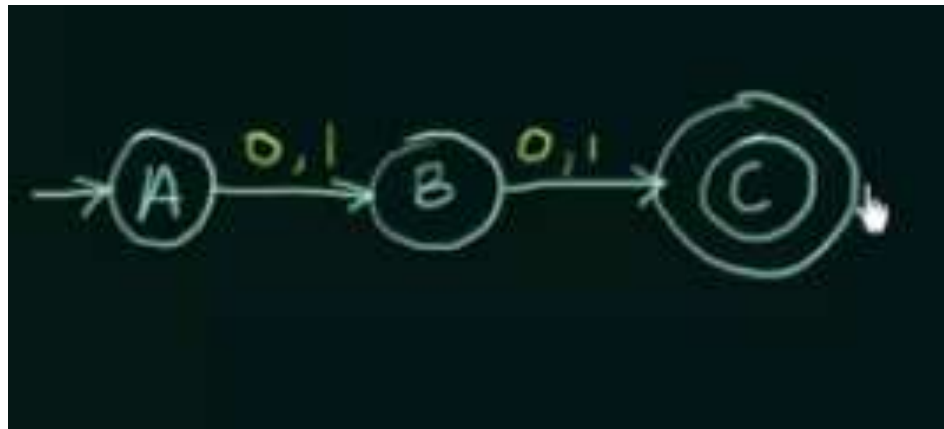
- Example 2
- $L1$ = Set of strings over $\{0,1\}$ of length 2
- $L1 = \{00,11,01,10\}$





Deterministic Finite Automata (DFA)

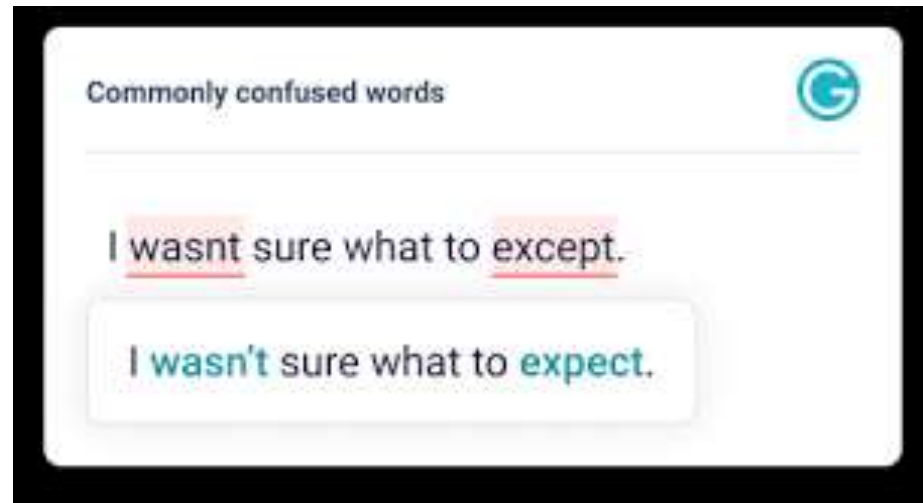
- Example 3
- $L1$ = Set of strings over $\{0,1\}$ of length 2
- $L1 = \{00,11,01,10\}$





DFA - Applications

- Lexical Analysis – compiler
- Spelling Checker
- Search Command





DFA -Examples

- Set of strings over $\{0,1\}$ that start with 0 and end with 1
- Set of strings over $\{a,b\}$ that ends with bb