SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECE304-SMART SENSORS AND DEVICES

III ECE / V SEMESTER

UNIT 1 – OVERVIEW OF MEASUREMENTS AND SENSORS

TOPIC –ERROR ANALYSIS

ERROR ANALYSIS

Accuracy

- Closeness to the true value
- Measurement Accuracy – determines the closeness of the measured value to the true value
- Instrument Accuracy – related to the worst accuracy obtainable within the dynamic range of the instrument in a specific operating environment

error = (measured value) – (true value)

correction = (true value) – (measured value)

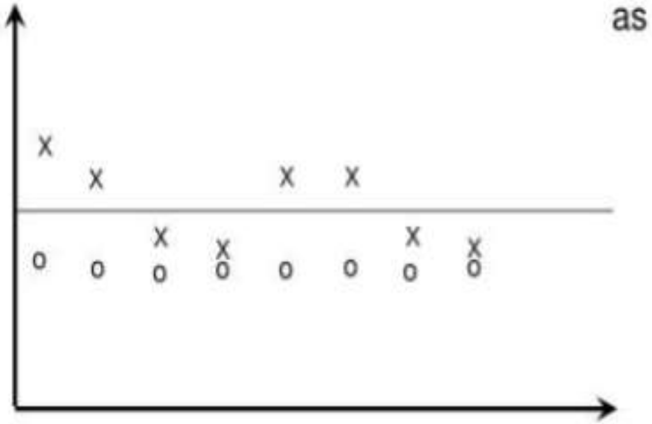
Causes for error: instrument instability, external noise (disturbances), poor calibration, poor analytical models, parameter changes due to environmental changes, and improper use of instrument

Error Analysis

- Errors can be classified as deterministic (systematic) and random (stochastic)
- Deterministic errors are caused by well-defined factors such as
  - Nonlinearities
  - Offsets in readings
- Can be accounted for by proper calibration and analysis practices – calibration charts and error ratings
- Random errors are caused by uncertain factors such as
  - Noise
  - Unknown random variations in the operating environment
- Statistical analysis using a sufficiently large number of data is necessary to estimate random errors

PRECISION

Reproducibility (or repeatability) of an instrument reading determines the precision



Instrument error may be represented as a random variable with

- Mean μ_e
- Standard deviation σ_e

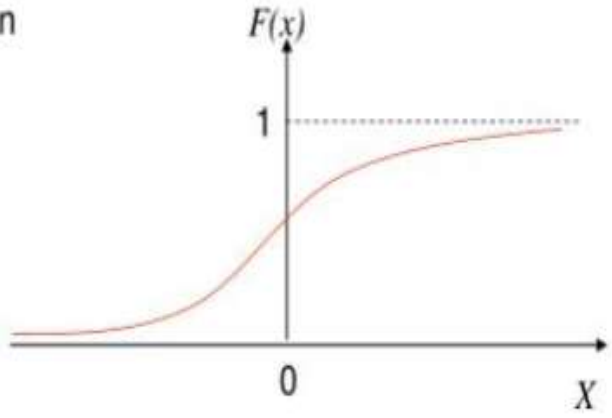
If the standard deviation is zero the error is deterministic or repeatable

Precision = (measurement range)/σ_e

Review of Probability and Statistics

Cumulative Probability Density Function

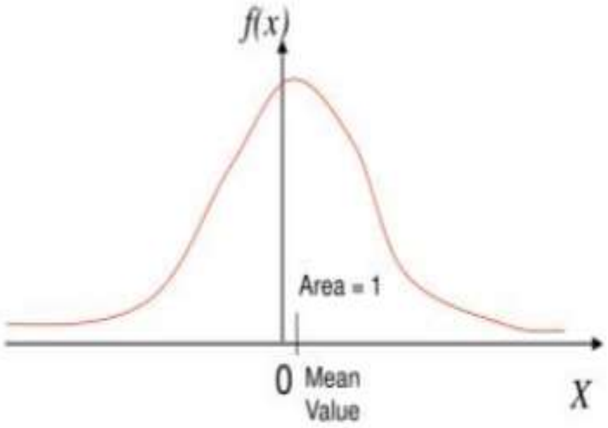
$$F(x) = P(X \le x)$$



Probability Density Function

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = \int_{-\infty}^{x} f(x)dx$$



Probability that a random variable falls within two values

$$P(a < X \le b) = F(b) - F(a)$$

$$= \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$$

$$= \int_{a}^{b} f(x)dx$$

Mean Value (Expected Value)

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

Mean Square Value

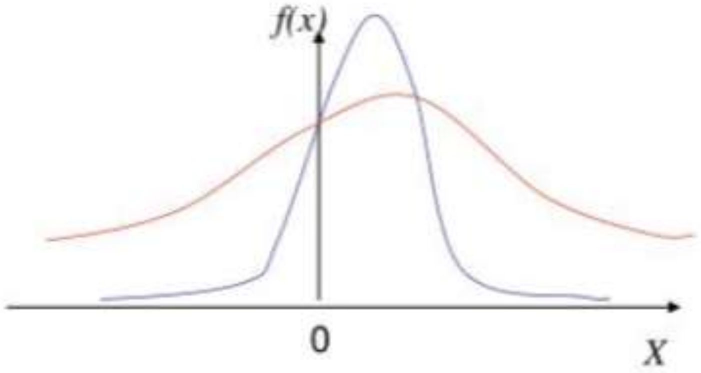
$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

Variance and Standard Deviation

$$Var(X) = \sigma^2 = E\{[X - E(X)]^2\}$$

$$\sigma^2 = E(X^2) - \mu^2$$

$$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$



Some Properties

If $f(x)$ is pdf of X the mean and variance of $aX + b$

$$a\mu + b$$

$$a^2 Var(X)$$

Independent Random Variables

Random variables X_1 and X_2 are said to be independent if the event X_1 assumes a certain value is completely independent from the event X_2 assumes a certain value.

For independent Random variables X_1 and X_2

$$E(X_1 X_2) = E(X_1)E(X_2)$$

$$Var(X_1 + X_2) = Var(X_1) + Var(X_2)$$

Sample Mean

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

Sample Variance

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2$$

Unbiased Estimates

$$E(\overline{X}) = \mu$$

$$E(S^2) = \sigma^2$$

Gaussian (Normal) Distribution

- Most extensively used probability distribution in engineering applications
- Probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Central Limit Theorem

A random variable that is formed by summing a very large number of independent random variables takes Gaussian distribution in the limit.

Standard Normal Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad Z = \frac{X-\mu}{\sigma} \qquad E(Z)=0, \quad Var(Z)=1$$